Generative Adversarial Networks

11785 Deep Learning

Fall 2022

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Topics for the week

- Transformers
- GNNs
- VAEs
- GANs
- Connecting the dots

The problem

- From a large collection of images of faces, can a network learn to *generate* new portrait
  - Generate samples from the distribution of “face” images
    - How do we even characterize this distribution?

The problem

- https://thispersondoesnotexist.com

StyleGAN2

Discriminative vs Generative Models

Given a distribution of inputs X and labels Y.

Discriminative models

- Discriminative models learn conditional distribution P(Y | X)

Generative models

- Generative models learn the joint distribution P(Y, X)

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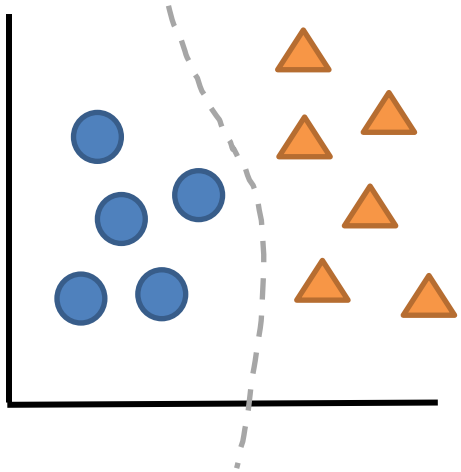

Decision Boundary

Generative models

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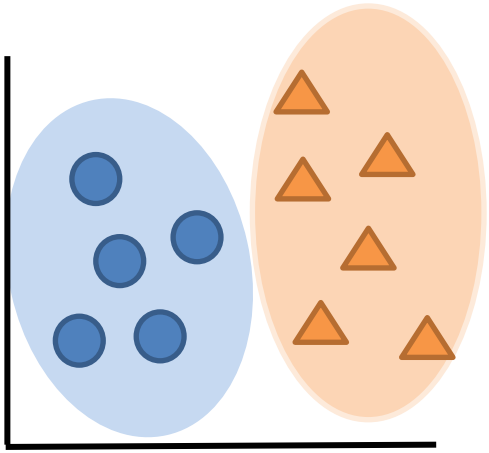
- Learns actual probability distribution of data.



Distributions

Discriminative vs Generative Models

Given a distribution of inputs X and labels Y.

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- Limited scope. Can only be used for classification tasks.
- E.g. Logistic regression, SVM etc.

Generative models

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- Can do both generative and discriminative tasks.
- E.g. Naïve Bayes, Gaussian Mixture Model etc.

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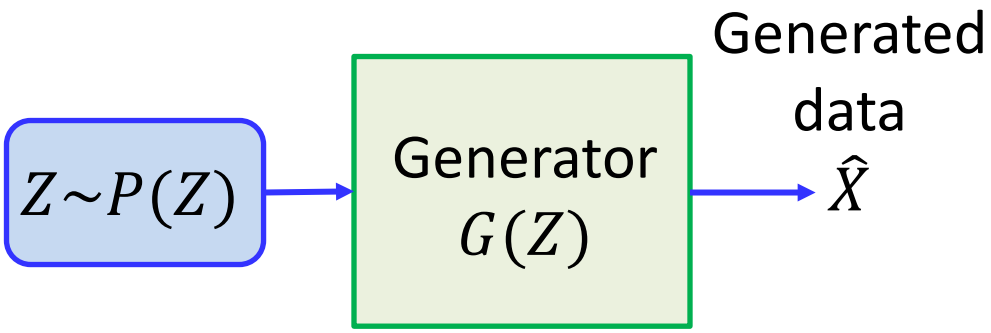
What we have seen: VAE



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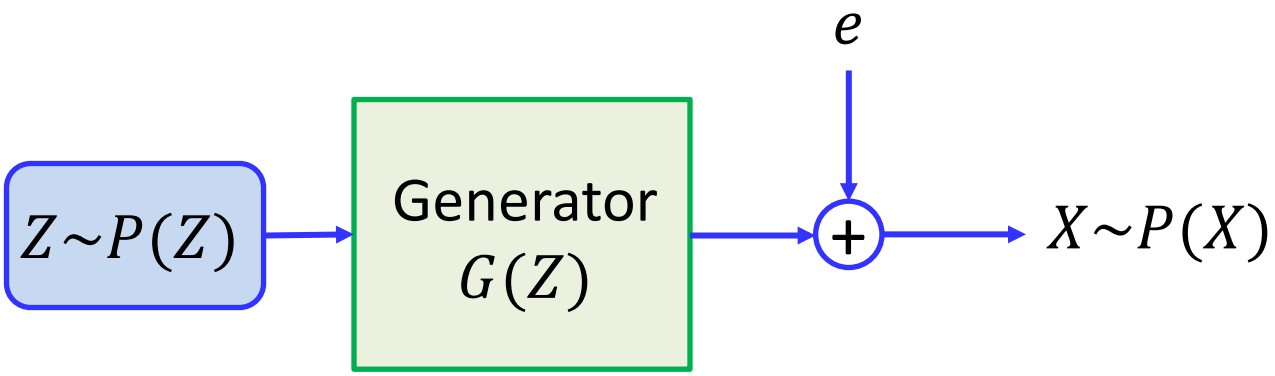

- The decoder of a VAE is a generator
- Trained by maximizing the *likelihood* of the data
  - Likelihood maximization does not actually relate to whether the output *actually* looks like a face

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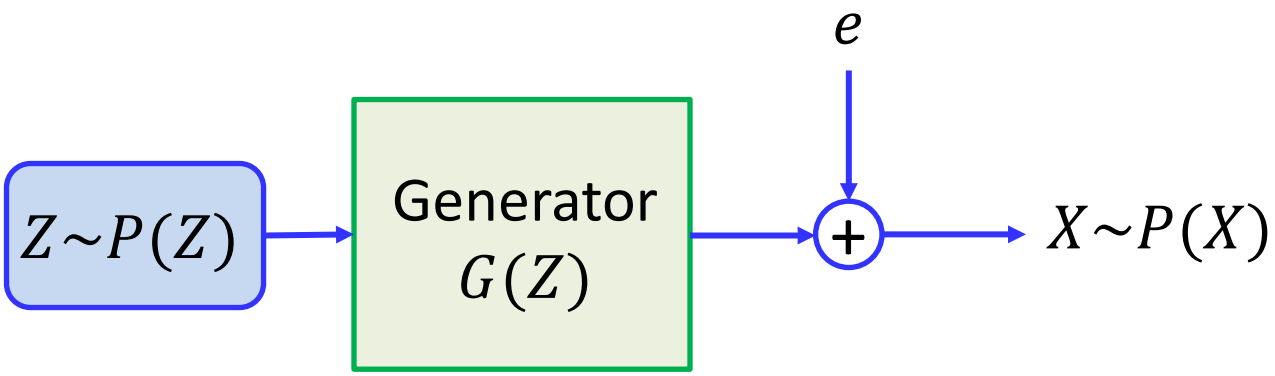
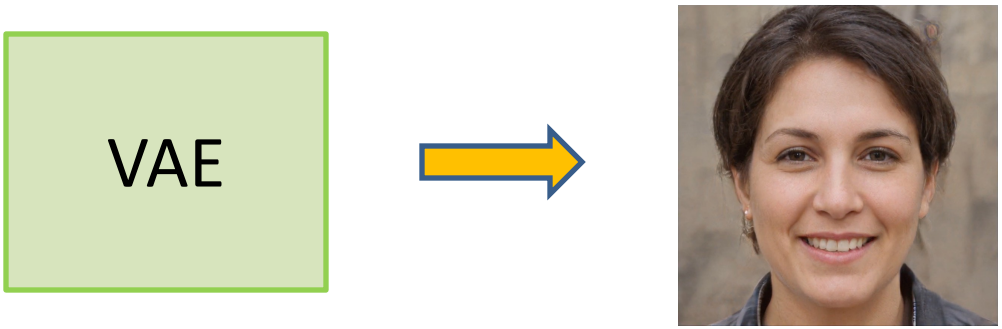


- The decoder of a VAE is a generator
- Trained by maximizing the *likelihood* of the data
  - Likelihood maximization does not actually relate to whether the output *actually* looks like a face
- Can we make the training criterion more direct?

What we have seen: VAE

- What is the simplest way to evaluate a VAE?



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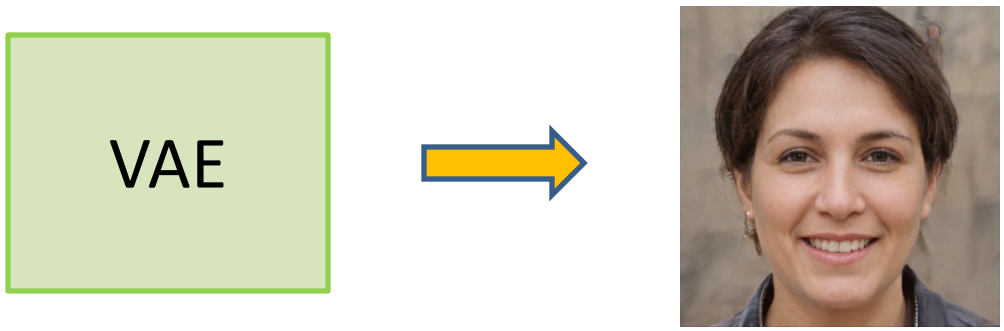

- You eyeball the result 👀 and predict if its Fake/Real

What we have seen: VAE

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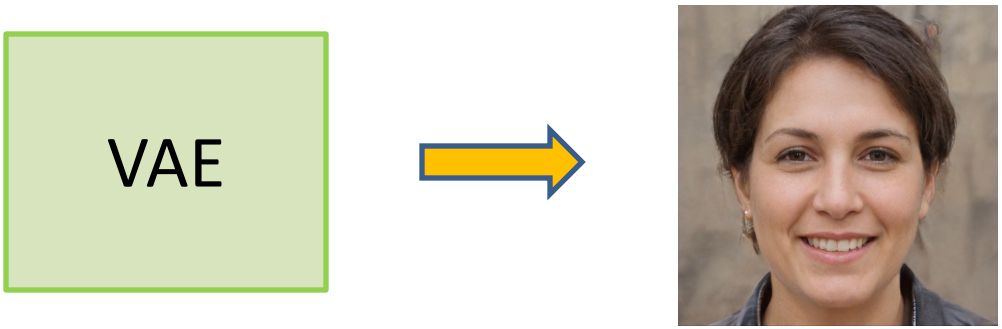
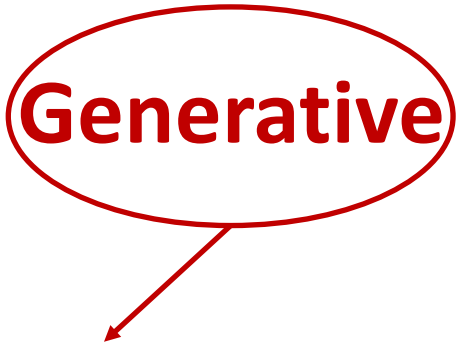


- You eyeball the result 👀 and predict if its Fake/Real
- But YOU are not differentiable 🫠

What are GANs

Generative Adversarial Networks

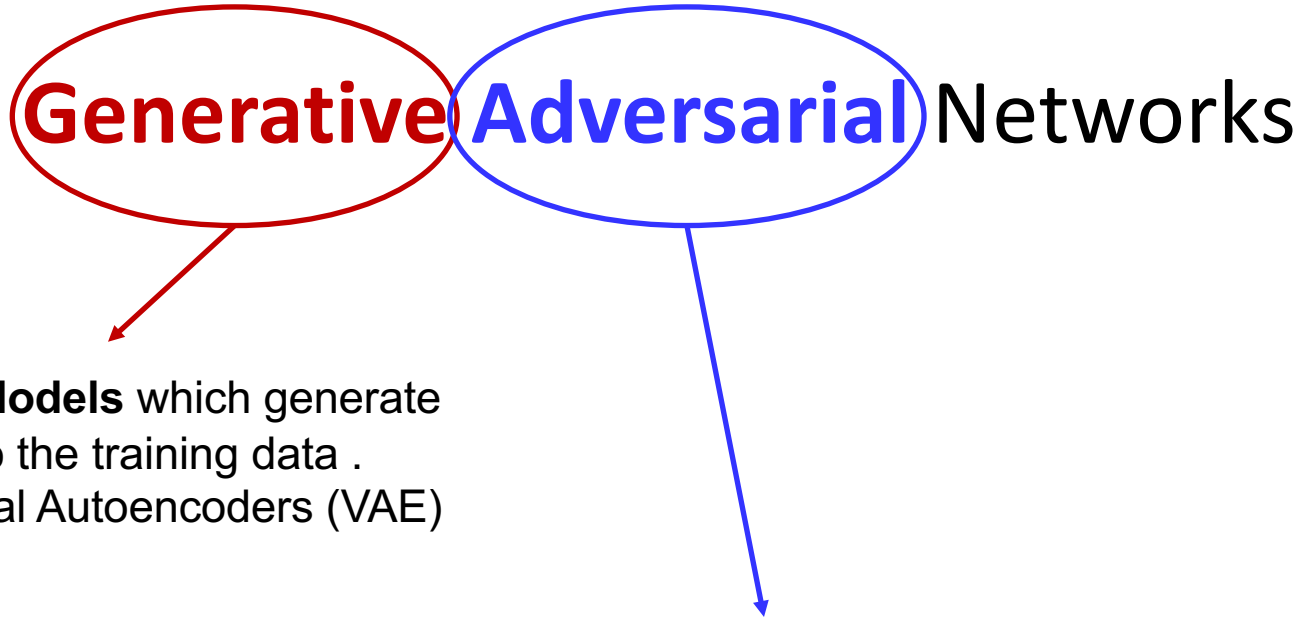
What are GANs



Adversarial Networks

Generative Models which generate data similar to the training data .
E.g. Variational Autoencoders (VAE)

What are GANs



Generative Models which generate data similar to the training data .
E.g. Variational Autoencoders (VAE)

Adversarial Training
GANS are made up of two competing networks (adversaries) that are trying beat each other.

What are GANs

Generative Models which generate data similar to the training data .
E.g. Variational Autoencoders (VAE)

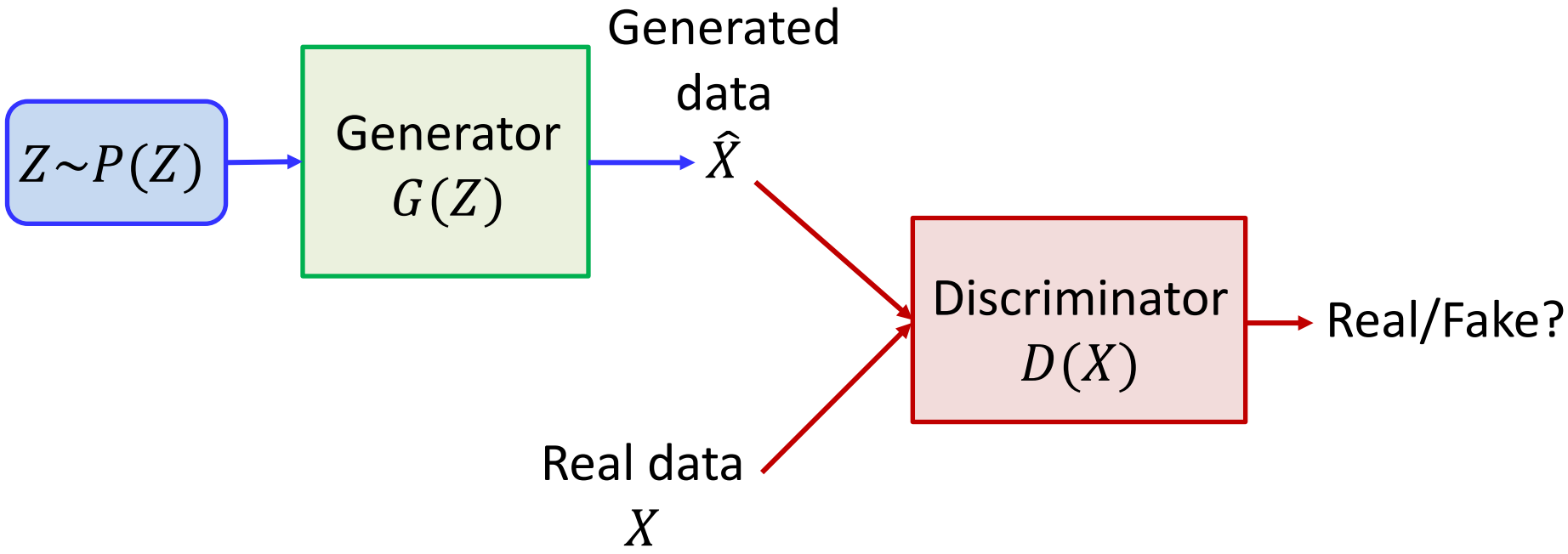
Neural Networks

Adversarial Training
GANS are made up of two competing networks (adversaries) that are trying beat each other.

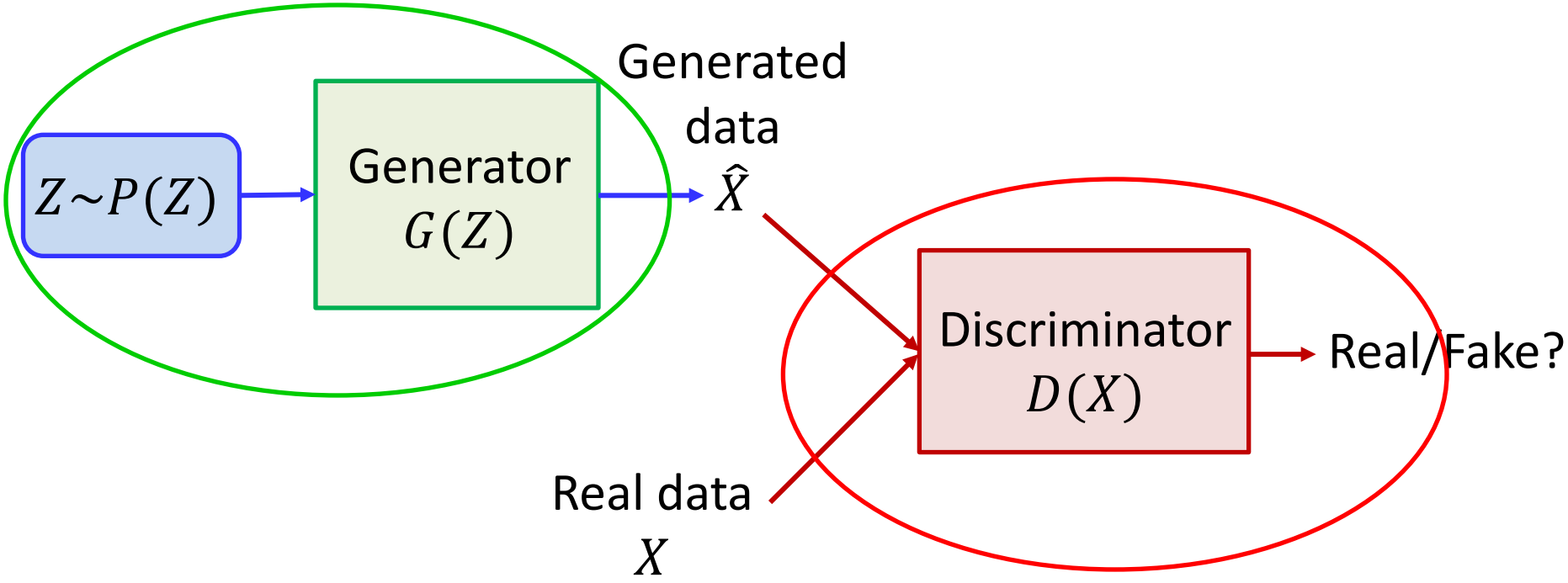
Generative Adversarial Networks

- Introduced in 2014
- Goal is to model $P(X)$, the distribution of training data
  - Model can generate samples from $P(X)$
- Trained using a pair of models acting as “adversaries”
  - A “Generator” that generates data
  - A “Discriminator” that evaluates it

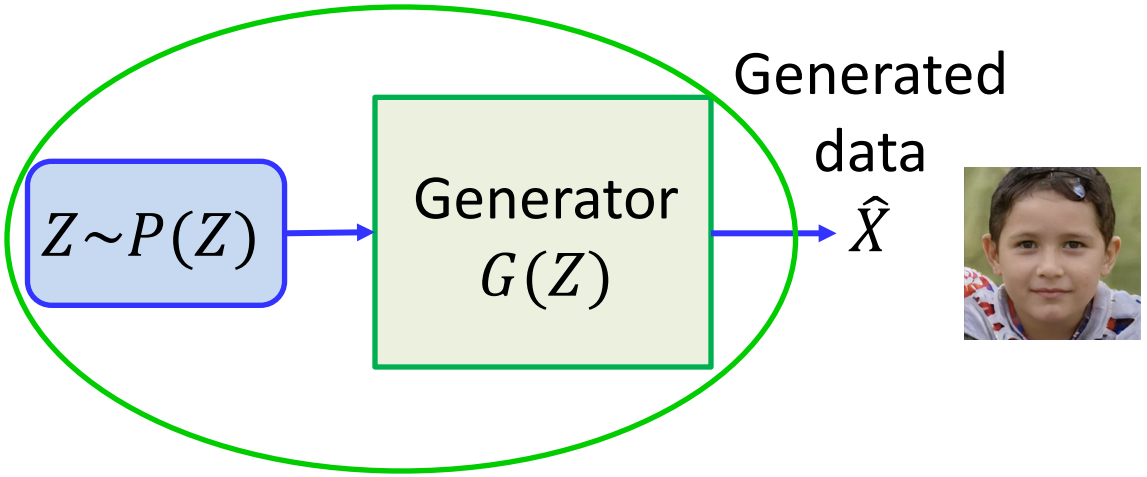
What are GANs?



What are GANs?



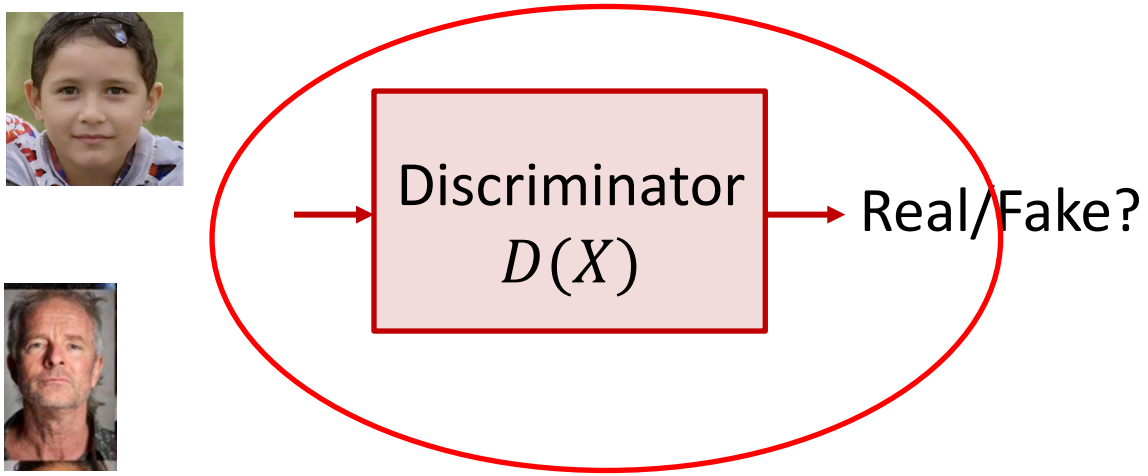
The Generator



- **The generator** produces realistic looking $X = G(z)$ from a latent vector Z
- Generator input Z can be sampled from a known prior, e.g. standard Gaussian

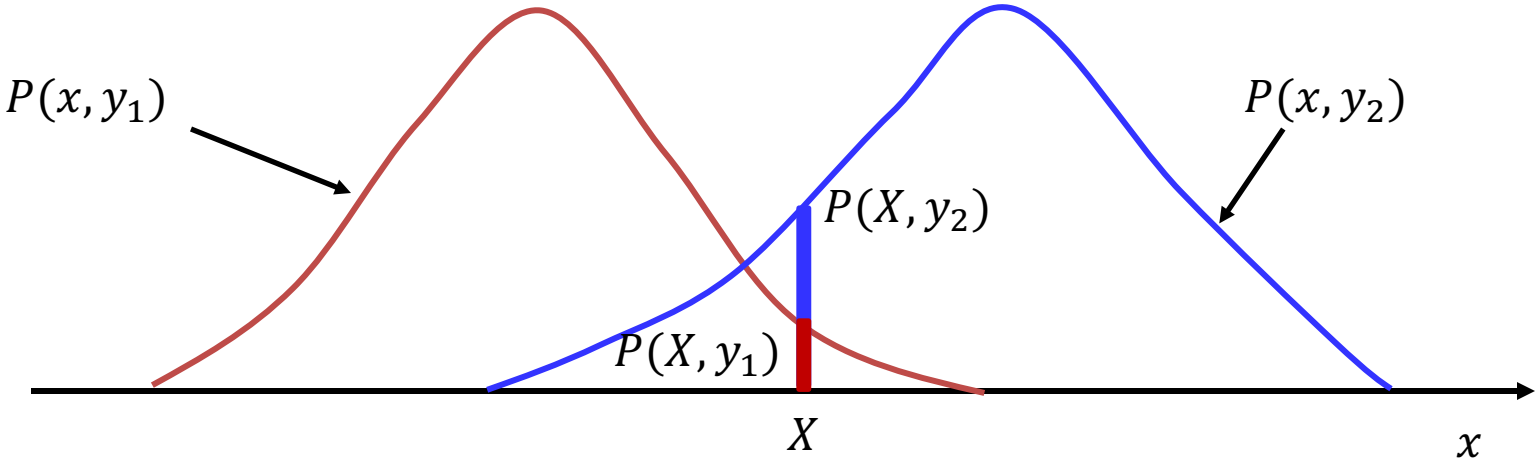
- **Goal**: generated distribution, $P_G(X)$ matches the true data distribution $P_X(X)$
  - $P_G(X)$ is the more "memorable" notation for $P_{\hat{X}}(X)$, the probability that a generated sample $\hat{X}$ takes the value X

The Discriminator



- Discriminator $D(X)$ is trained to tell the difference between real and generated (fake) data
  - Specifically, data produced by the generator
  - If a perfect discriminator is fooled, the generated data cannot be distinguished from real data

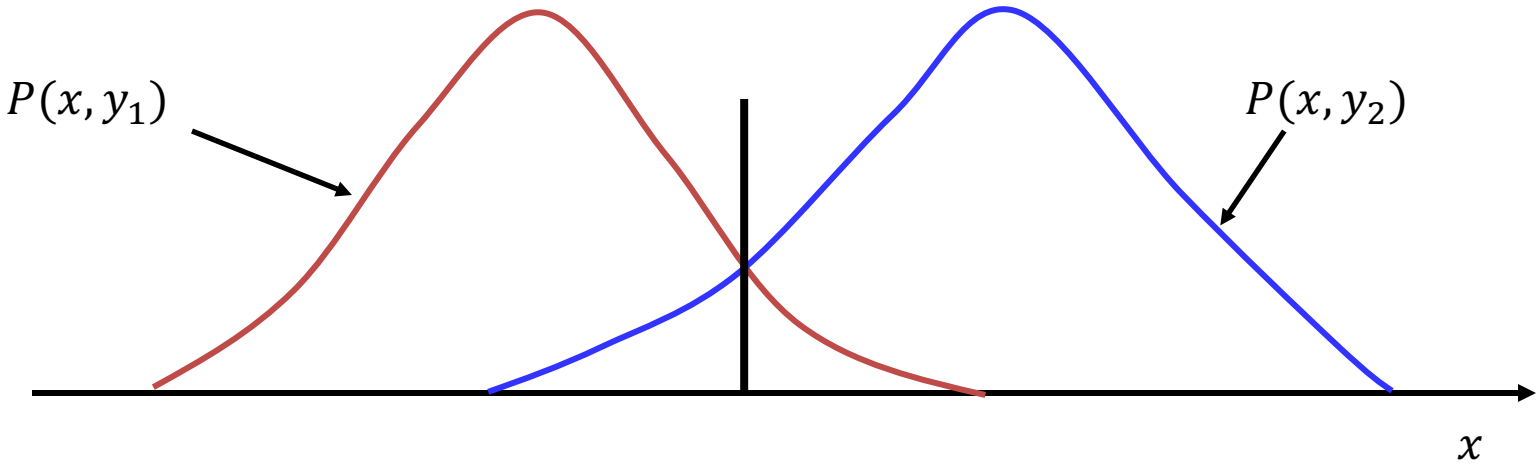
The perfect discriminator



- The a posteriori probability of the classes for any instance $x = X$ is

$$P(y_i|X) = \frac{P(X, y_i)}{P(X, y_1) + P(X, y_2)}$$

The perfect discriminator



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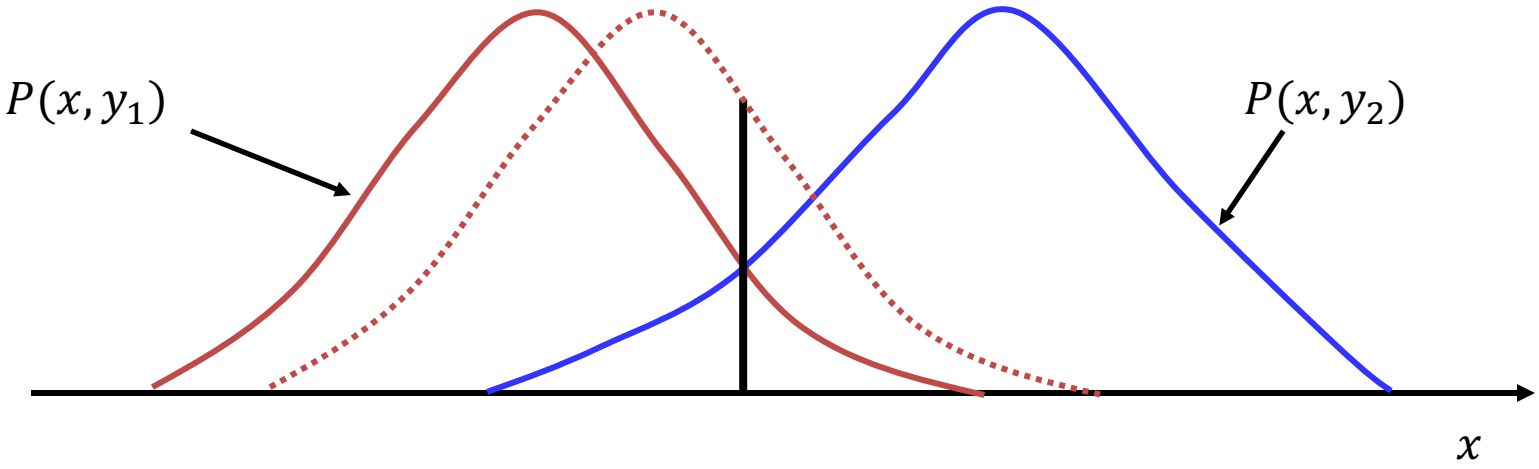
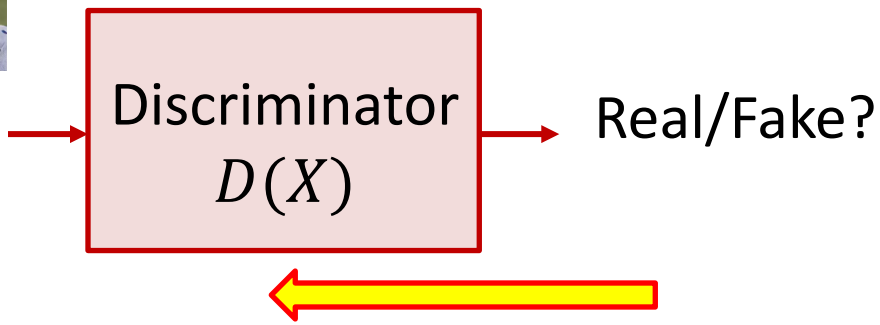
- The perfect decision boundary is where $P(y_1|X) = P(y_2|X)$
  - The perfect discriminator will compute $P(y_i|X)$ for each class
  - It will assign any X to the class with the higher $P(y_i|X)$

Fooling the perfect discriminator



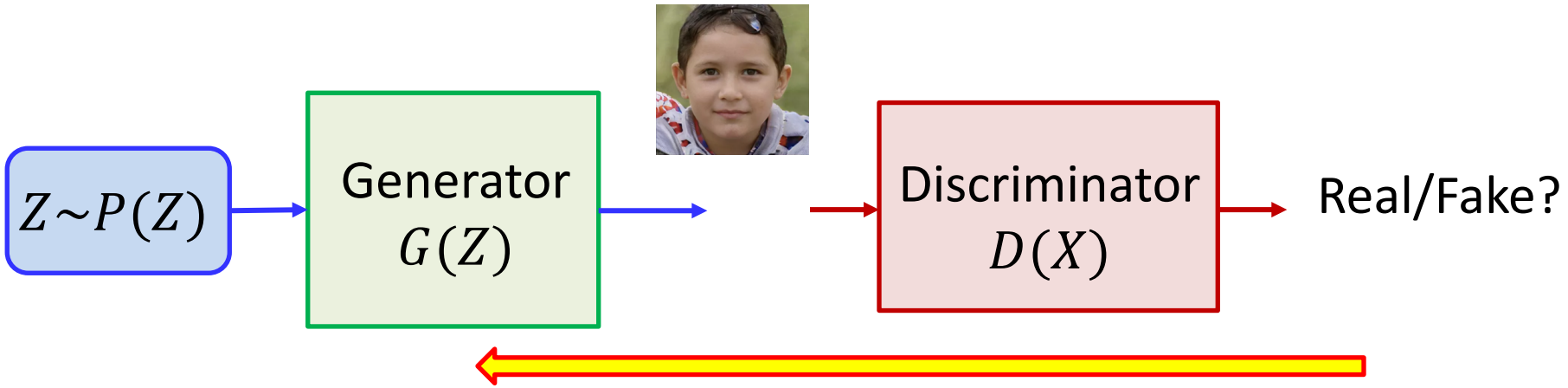
- Relearn generator parameters so that the new distribution of generated data "fools" the discriminator
  - By moving it into the region assigned to the other class by the (perfect) discriminator

Training the discriminator



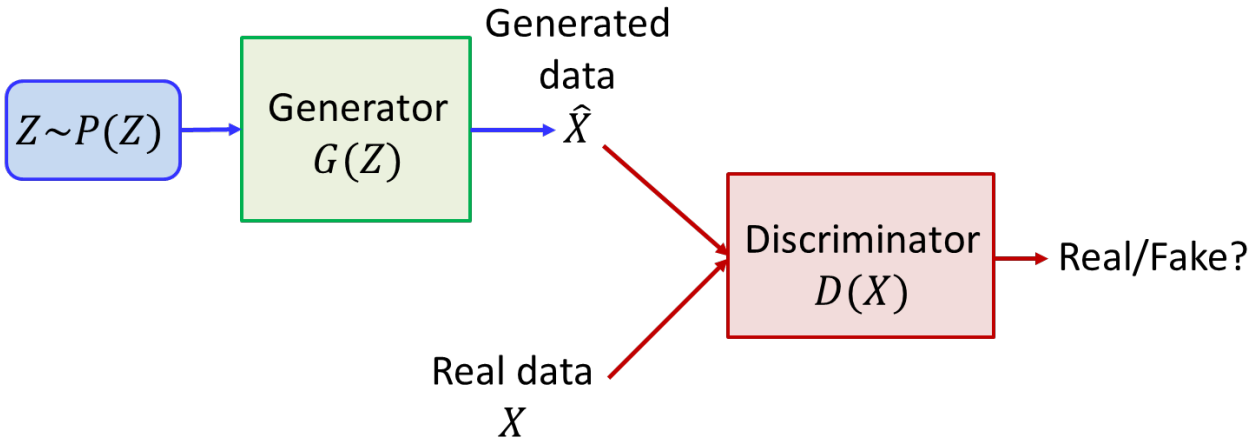
- **Training the discriminator:**
  - The discriminator is provided training examples of real and synthetic faces
  - The discriminator is trained to minimize its classification loss
    - Minimize error between actual and predicted labels

Training the generator



- **Training the generator:**
  - The discriminator's loss is backpropagated to the generator
  - The generator is trained to *maximize* the discriminator loss
    - It is trained to "fool" the discriminator

The GAN formulation



- For real data X, the desired output of the discriminator is $D(X) = 1$
  - The log probability that the instance is real, as computed by the discriminator is log $D(X)$

- For synthetic data $\hat{X}$, the desired output of the discriminator is $D(\hat{X}) = 0$
  - The log probability that the instance is synthetic, as computed by the discriminator, is $\log(1 - D(\hat{X}))$
    - $= \log(1 - G(Z))$

Poll

@1681

Detour: Information

$$\boldsymbol{I}(\boldsymbol{x}) \;=\; -\log(\boldsymbol{P}(\boldsymbol{x}))$$

Information is more where probability is low

“Today I didn’t see a tornado” – low information

“Today I saw a tornado” – high information

Detour: Entropy

$$\boldsymbol{H(x)} = -\sum \boldsymbol{P(x)} \log\big(\boldsymbol{P(x)}\big)$$
$$\boldsymbol{H(x)} = \mathbb{E}_{\boldsymbol{x} \sim \boldsymbol{X}}[-\log(\boldsymbol{P(x)})]$$

Measure of uncertainty – weighted average of information

Detour on a Detour: Softmax

$$\boldsymbol{softmax}(\boldsymbol{x}) = \frac{\boldsymbol{exp}(\boldsymbol{x})}{\sum \boldsymbol{exp}(\boldsymbol{x})}$$

Gives a probability distribution.

Detour on a Detour: Softmax

$$softmax(x) = \frac{exp(x)}{\sum exp(x)}$$

$$Boltzmann(x, \tau) = \frac{exp\left(\frac{x}{\tau}\right)}{\sum exp\left(\frac{x}{\tau}\right)}$$

τ is called a ‘temperature’ parameter.

Detour on a Detour: Softmax

$$\boldsymbol{Boltzmann(x, \tau) = \frac{exp\left(\frac{x}{\tau}\right)}{\sum exp\left(\frac{x}{\tau}\right)}}$$

High value of $\boldsymbol{\tau}$ causes distribution to be more uniform – high entropy – "hot"

Low value "cools" the distribution – low entropy

Detour on a Detour: Softmax

$$\boldsymbol{Boltzmann}(\boldsymbol{x}, \boldsymbol{\tau}) = \frac{\boldsymbol{exp}\left(\frac{\boldsymbol{x}}{\boldsymbol{\tau}}\right)}{\sum \boldsymbol{exp}\left(\frac{\boldsymbol{x}}{\boldsymbol{\tau}}\right)}$$

Try plotting using numpy and matplotlib for various values of $\boldsymbol{\tau}$ `[1e-3, 1e-2,1e-1,1,10,100]`!

You might want to use stable-softmax.

(look it up!)

Detour: Entropy

$$\boldsymbol{H}(\boldsymbol{x}) = -\sum \boldsymbol{P}(\boldsymbol{x}) \log\big(\boldsymbol{P}(\boldsymbol{x})\big)$$
$$\boldsymbol{H}(\boldsymbol{x}) = \mathbb{E}_{\boldsymbol{x} \sim \boldsymbol{X}}[-\log(\boldsymbol{P}(\boldsymbol{x}))]$$

Measure of uncertainty – weighted average of information

Detour: Cross Entropy

Information of event under some distribution Q measured over another distribution P

$$\boldsymbol{XEnt}(\boldsymbol{x};\boldsymbol{P},\boldsymbol{Q}) = -\sum \boldsymbol{P}(\boldsymbol{x})\log\big(\boldsymbol{Q}(\boldsymbol{x})\big)$$

$$\boldsymbol{XEnt}(\boldsymbol{x};\boldsymbol{P},\boldsymbol{Q}) = \mathbb{E}_{\boldsymbol{x}\sim\boldsymbol{P}(\boldsymbol{X})}[-\log(\boldsymbol{Q}(\boldsymbol{x}))]$$

Detour: Binary Cross Entropy

Information of event under some distribution Q measured over another distribution P

When $P(x)$ can be **0** or **1**

$$\boldsymbol{BCE}(\boldsymbol{x};\boldsymbol{P},\boldsymbol{Q})$$
$$= -\sum \boldsymbol{P}(\boldsymbol{x})\log\big(\boldsymbol{Q}(\boldsymbol{x})\big) + \big(\mathbf{1} - \boldsymbol{P}(\boldsymbol{x})\big)\log(\mathbf{1} - \boldsymbol{Q}(\boldsymbol{x}))$$

$$\boldsymbol{BCE}(\boldsymbol{x};\boldsymbol{P},\boldsymbol{Q}) =$$
$$-\{\mathbb{E}_{\boldsymbol{x}\sim \boldsymbol{P}(\boldsymbol{X})}\big[\log\big(\boldsymbol{Q}(\boldsymbol{x})\big)\big] + \mathbb{E}_{\boldsymbol{x}\sim \bar{\boldsymbol{P}}(\boldsymbol{X})}\big[\log\big(\mathbf{1} - \boldsymbol{Q}(\boldsymbol{x})\big)\big]\}$$

Coming back: Discriminator

$$\mathrm{BCE}_{\boldsymbol{D}}(\boldsymbol{x};\boldsymbol{D}) = -\{\mathbb{E}_{\boldsymbol{x}\sim \boldsymbol{P_{Data}}}[\log(\boldsymbol{D}(\boldsymbol{x}))] + \mathbb{E}_{\boldsymbol{x}\sim \bar{\boldsymbol{P}}_{\boldsymbol{Data}}}[\log(\mathbf{1}-\boldsymbol{D}(\boldsymbol{x}))]\}$$

True

Fake

Discriminator

$$\mathrm{BCE}_{\boldsymbol{D}}(\boldsymbol{x}\,;\boldsymbol{D},\boldsymbol{G}) = -\{\mathbb{E}_{\boldsymbol{x}\sim\boldsymbol{P_{data}}}[\log(\boldsymbol{D}(\boldsymbol{x}))] + \mathbb{E}_{\boldsymbol{z}\sim\boldsymbol{P_z}}[\log(\mathbf{1}-\boldsymbol{D}(\boldsymbol{G}(\boldsymbol{z})))]\}$$

True **Generated**

Going forward we will use $\boldsymbol{P_X}$ for $\boldsymbol{P_{data}}$

Discriminator

$$\mathrm{BCE}_{\boldsymbol{D}}(\boldsymbol{x}\,;\boldsymbol{D},\boldsymbol{G}) = -\{\mathbb{E}_{\boldsymbol{x}\sim\boldsymbol{P_X}}\left[\log\left(\boldsymbol{D}(\boldsymbol{x})\right)\right] + \mathbb{E}_{\boldsymbol{x}\sim\boldsymbol{P_G}}[\log(\mathbf{1}-\boldsymbol{D}(\boldsymbol{x}))]\}$$

Implicit Model

Minimizing this is the same as maximizing

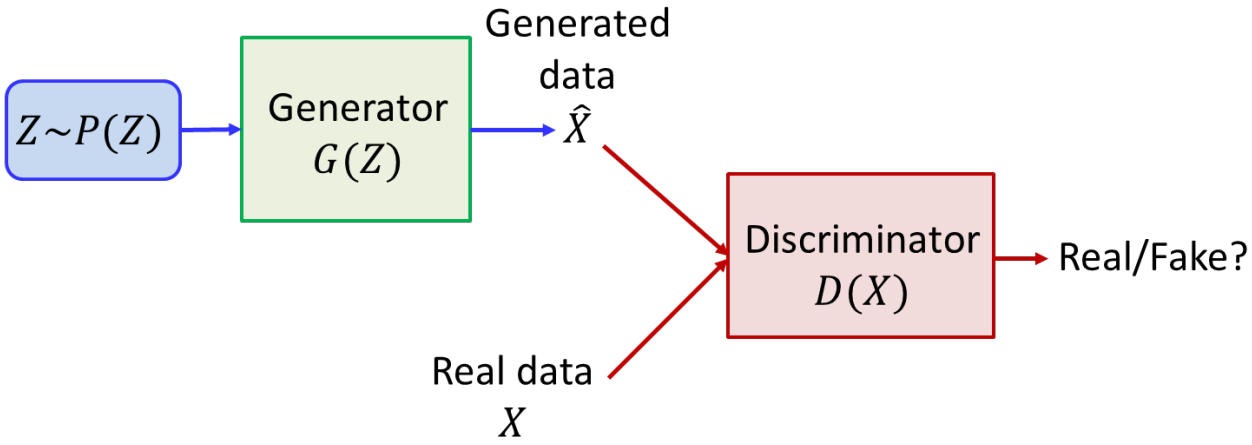
$$\mathbb{E}_{\boldsymbol{x}\sim\boldsymbol{P_X}}\left[\log\left(\boldsymbol{D}(\boldsymbol{x})\right)\right] + \mathbb{E}_{\boldsymbol{x}\sim\boldsymbol{P_G}}[\log(\mathbf{1}-\boldsymbol{D}(\boldsymbol{x}))]$$

Generator

But the generator wants to fool the Discriminator, so it wants to minimize

$$\mathbb{E}_{\boldsymbol{x} \sim \boldsymbol{P_X}}\left[\log\left(\boldsymbol{D}(\boldsymbol{x})\right)\right] + \mathbb{E}_{\boldsymbol{x} \sim \boldsymbol{P_G}}[\log(\mathbf{1} - \boldsymbol{D}(\boldsymbol{x}))]$$

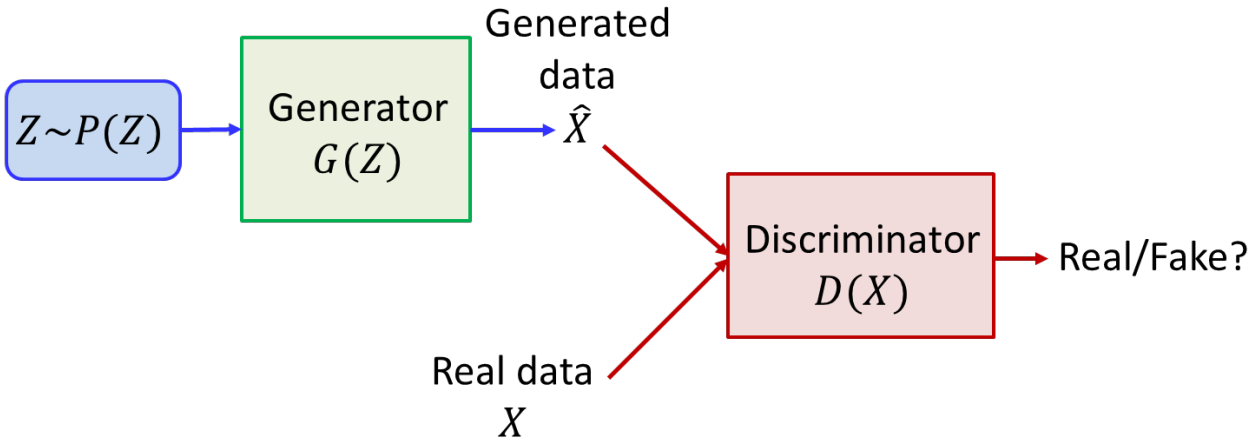
The GAN formulation



- Can be written as

$$\min_{G} \max_{D} \; E_{\boldsymbol{x} \sim \boldsymbol{P_X}} \log D(x) + E_{\boldsymbol{x} \sim \boldsymbol{P_G}} \log(1 - D(x))$$

The GAN formulation

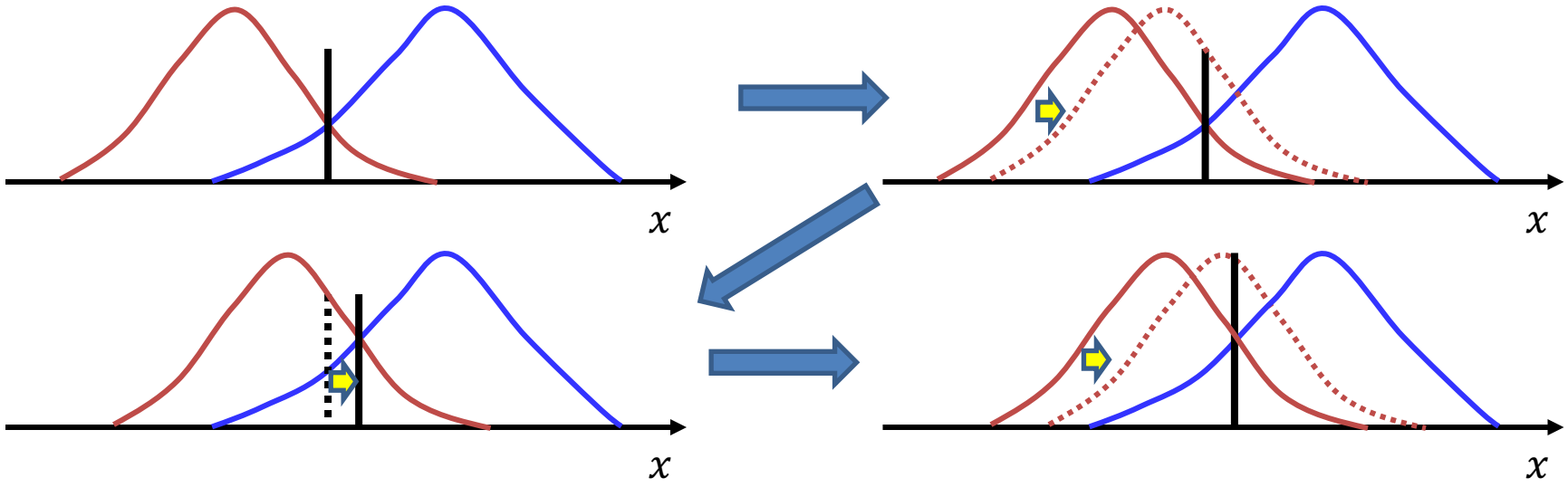


- The original GAN formulation is the following min-max optimization

$$\min_G \max_D \quad E_X \log D(x) + E_z \log(1 - D(G(z)))$$

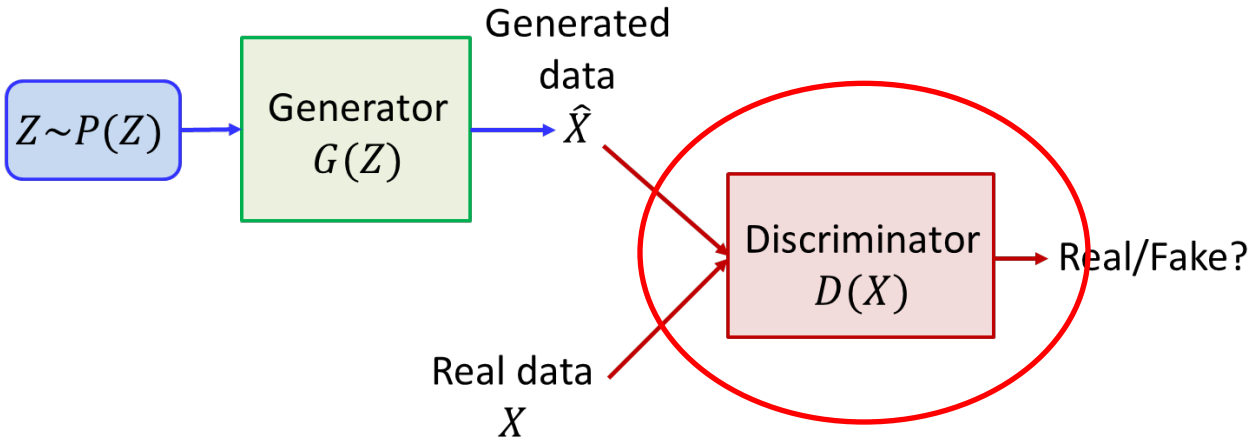
- Objective of D: $D(x) = 1$ and $D(G(z)) = 0$
- Objective of G: $D(G(z)) = 1$

The iterated learning



- Discriminator learns perfect boundary
- Generator moves its distribution past the boundary “into” the real distribution
- Discriminator relearns new “perfect” boundary
- Generator shifts distribution past new boundary
- …
- In the limit Generator’s distribution sits perfectly on “real” distribution and the perfect discriminator is still random

Analysis of optimal behavior: The optimal discriminator



- The **optimal discriminator** would be a Bayesian classifier

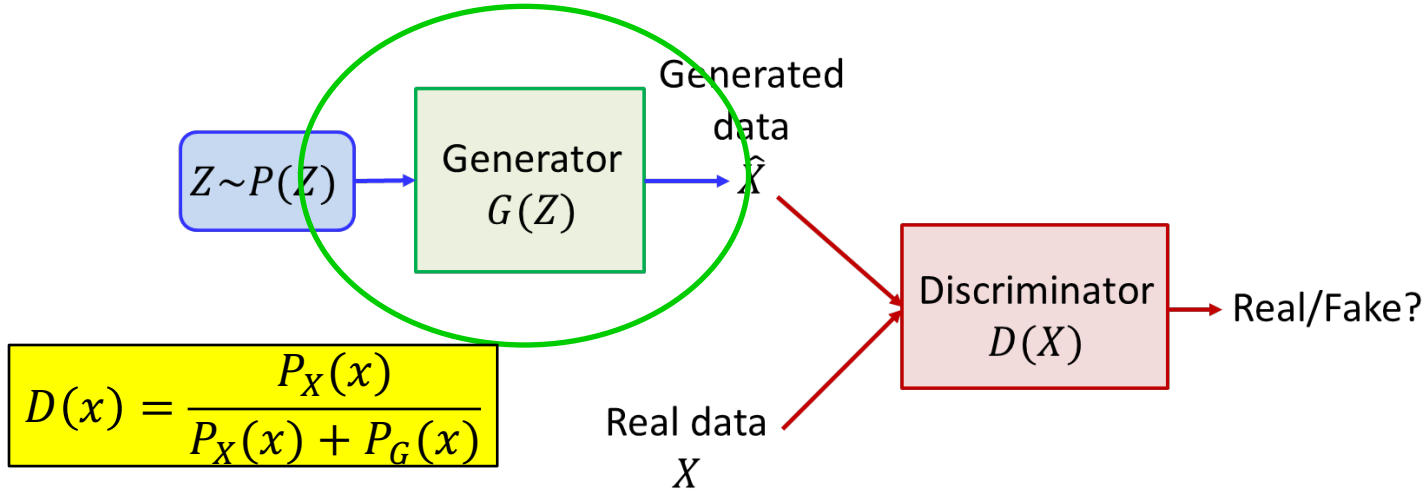
$$D(x) = \frac{P_X(x)}{P_X(x) + P_G(x)}$$

  - Assuming uniform prior

Black Board!

- Math at the end of the slide-deck

Analysis of optimal behavior: The optimal generator



$$\min_G \max_D \mathbb{E}_x \log D(x) + \mathbb{E}_z \log(1 - D(G(z)))$$

- **With a perfect discriminator**:

$$L = \mathbb{E}_{x \sim P_X(x)} \log D(x) + \mathbb{E}_{x \sim P_G(x)} \log(1 - D(x))$$
$$= \mathbb{E}_{x \sim P_X(x)} \left[\log \left(\frac{P_X(x)}{P_X(x) + P_G(x)} \right) \right] + \mathbb{E}_{X \sim P_G(X)} \left[\log \left(\frac{P_G(x)}{P_X(x) + P_G(x)} \right) \right]$$

- This is just the Jensen-Shannon divergence between $P_X(x)$ and $P_G(x)$ to within a scaling factor and a constant

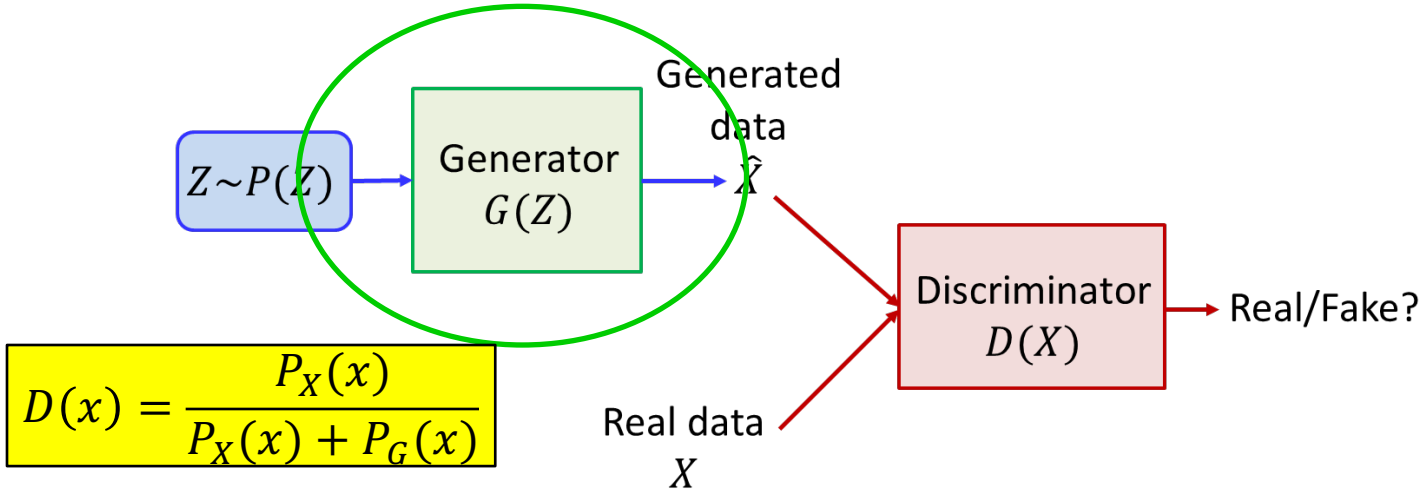
$$L = 2D_{JSD}\big(P_X(x), P_G(x)\big) - \log 4$$

The Jensen Shannon Divergence

$$D_{JSD}\,(P,Q) = \frac{D_{KL}\left(P,\frac{P+Q}{2}\right) + D_{KL}\left(Q,\frac{P+Q}{2}\right)}{2}$$

- A symmetric variant of KL that does not exaggerate instances to which one of the distributions assigns 0 probability
  - $D_{KL}(P,Q) = \sum_X P(x)\log(\frac{P(x)}{Q(x)})$ blows up the contributions of x with $Q(x) = 0$

Analysis of optimal behavior: The optimal generator



- The optimal generator:

$$\min_G 2D_{JSD}\left(P_X(x), P_G(x)\right) - \log 4$$

- The optimal generator minimizes the Jensen Shannon divergence between the distributions of the actual and synthetic data!
  - Tries to make the two distributions maximally similar

Min-Max Stationary Point

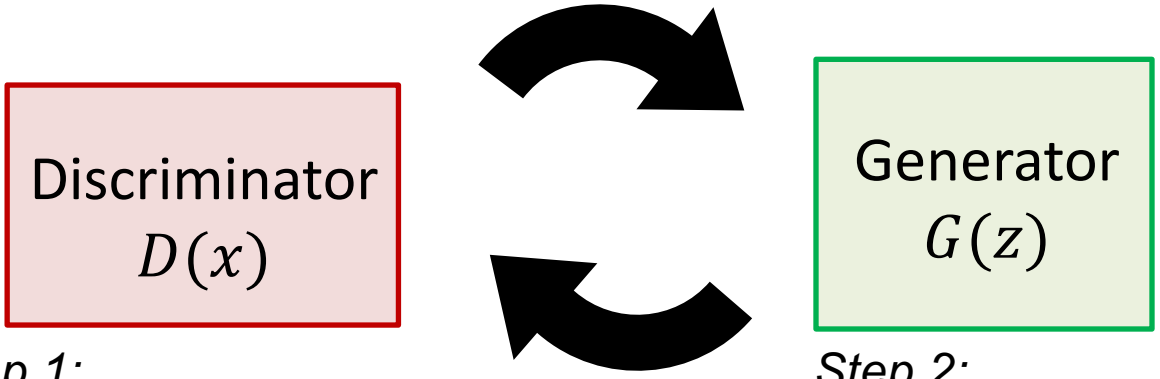
- There exists a stationary point:
  - If the generated data exactly matches the real data, the discriminator outputs 0.5 for all inputs
  - If discriminator outputs 0.5, the gradients for the generator is flat, so generator does not learn
  - Unfortunately, this is also true of a random discriminator

- Stationary points need not be stable (depends on the exact GANs formulation and other factors)
  - Generator may overshoot some values or oscillate around the optimum
  - A discriminator with unlimited capacity can still assign an arbitrarily large distance to 2 similar distributions

Min-Max Optimization

- Generator and the discriminator need to be trained simultaneously
  - If discriminator is undertrained, it provides sub-optimal feedback to the generator
  - If the discriminator is overtrained, there is no local feedback for marginal improvements

How to Train a GAN?



Step 1:
Train the Discriminator
using the current Generator

Step 2:
Train the Generator
to beat the Discriminator

Optimize: $\min_G \max_D \mathbb{E}_x \log D(x) + \mathbb{E}_z \log(1 - D(G(z)))$

The discriminator is not needed after convergence

Features and Challenges

- GANs can produce clear crisp results for many problems
- But they also have stability issues and are hard to train
  - Problems such as “mode collapse” are frequent
    - Producing outputs with very low variability

Variants and updates

- A number of variations have been proposed to improve the stability and outputs of GANs
  - LAPGAN
  - Wasserstein GAN
  - C-GAN
  - DCGAN
  - CycleGAN
  - StarGAN
  - …

Evaluate with Discriminative Network

- Inception Score
  - Use the Inception V3 image classifier to classify generated images
  - Inception should produce a variety of labels
    - **As measured by the entropy of the average label distribution**
  - Each label should have high confidence (low entropy)
    - **As measured by the average entropy of the Inception outputs for individual instances**
  - The two scores are combined into a single “inception” score

VAEs vs GANs

VAEs

- Minimizing the KL divergence between distributions of synthetic and true data
- Uses an encoder to predict latent distributions to optimize generator
- More complex formulation
- Simpler optimization. Trains faster and more reliably
- Results are blurry

GANs

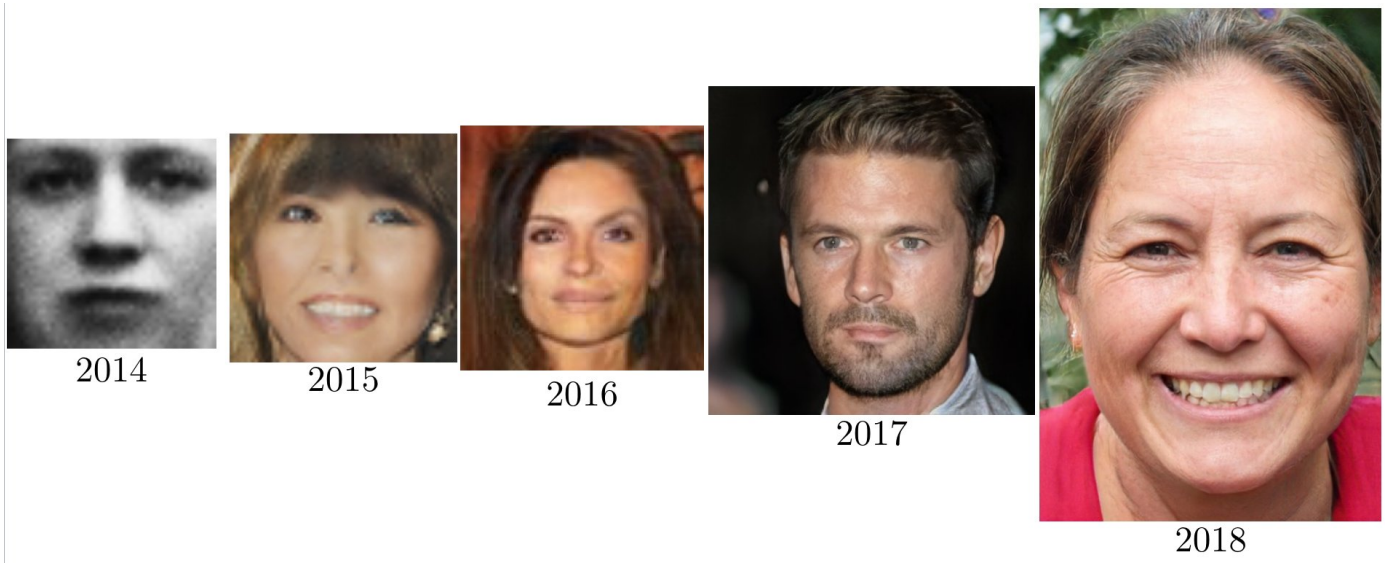
- Minimizing the Jenson-Shannon divergence between distributions of synthetic and true data
- Use a discriminator to optimize generator
- Simpler formulation
- Noisy and difficult optimization
- Sharper results

Original paper (GAN, 2014)

Output of original GAN paper, 2014 [GPM+14]

GANs with time

- Better quality
- High Resolution



https://twitter.com/goodfellow_ian/status/1084973596236144640?lang=en

StarGAN(2018)

Manipulating Celebrity Faces [CCK+17]

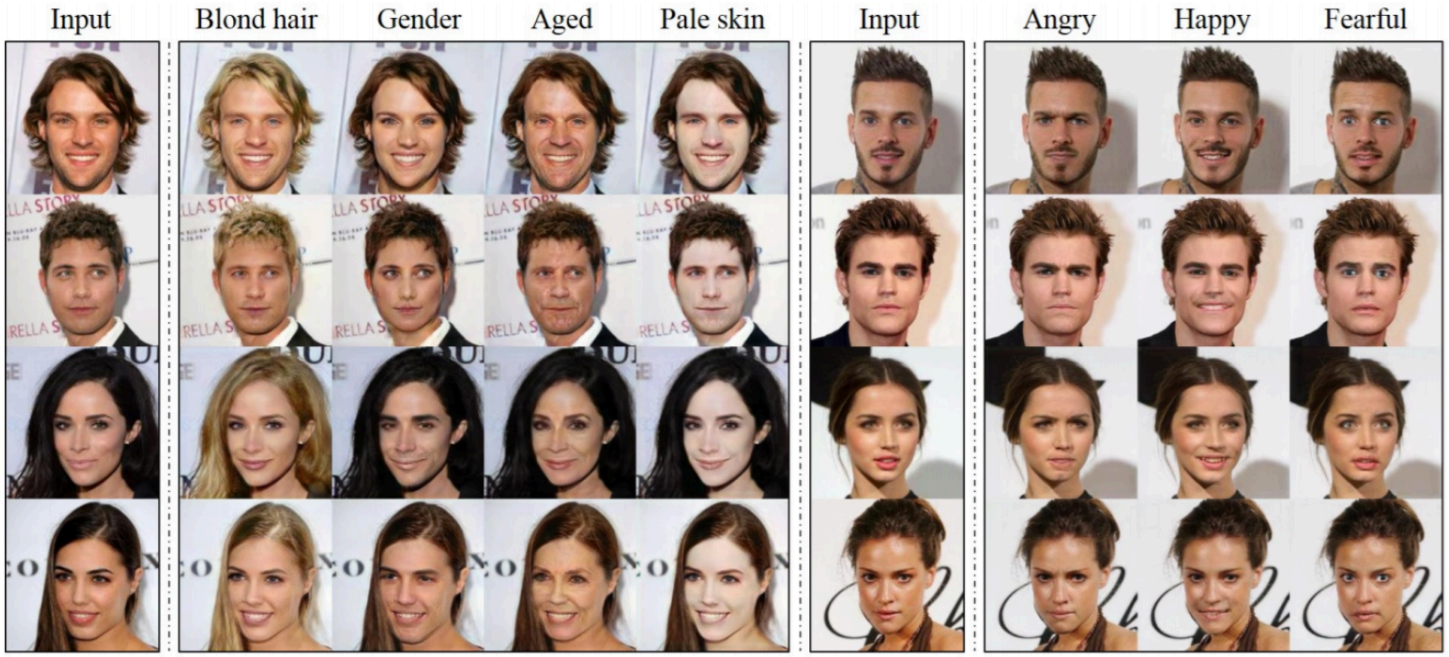


Figure 1. Multi-domain image-to-image translation results on the CelebA dataset via transferring knowledge learned from the RaFD dataset. The first and sixth columns show input images while the remaining columns are images generated by StarGAN. Note that the images are generated by a single generator network, and facial expression labels such as angry, happy, and fearful are from RaFD, not CelebA.

Progressive growing of GANs (2018)

Figure 5: 1024×1024 images generated using the CELEBA-HQ dataset. See Appendix F for a larger set of results, and the accompanying video for latent space interpolations.

High fidelity natural images (2019)

Generating High-Quality Images [BDS18]

Appendix

Math: Optimum Discriminator

$$\min_G \max_D \; \mathbb{E}_{x \sim P_x}[\log D(x)] + \mathbb{E}_{z \sim P_z}[\log(1 - D(G(z)))]$$

$$= \int_x P_x(x) \log D(x)\, dx + \int_z P_z(z) \log(1 - D(G(z)))\, dz$$

Use implicit model $P_G(x)$

$$= \int_x P_x(x) \log D(x)\, dx + \int_x P_G(x) \log(1 - D(x))\, dx$$

$$= \int_x \left(P_x(x) \log D(x) + P_G(x) \log(1 - D(x)) \right) dx$$

$$\frac{d}{dD(x)}(\ldots) = 0 \quad \# \text{ find optimum}$$

Math: Optimum Discriminator

$$\frac{d}{d\,D(x)}\left[P_x(x)\log(D(x)) + P_G(x)\log(1-D(x))\right] = 0$$

$$\Rightarrow \quad \frac{P_x(x)}{D(x)} - \frac{P_G(x)}{1-D(x)} = 0$$

$$\Rightarrow \quad \frac{P_x(x)}{D(x)} = \frac{P_G(x)}{1-D(x)} \Rightarrow P_x(x)\left(1-D(x)\right) = P_G(x)\,D(x)$$

$$D(x)_{\text{optimum}} = \frac{P_x(x)}{P_x(x) + P_G(x)} \Rightarrow \text{Bayes Optimum Classifier!}$$

Math: Optimum Generator

$$D(x) = \frac{P_X(x)}{P_X(x) + P_G(x)}$$

$\Rightarrow$ Plug this in for the generator for optimum generator!

$$\min_G \; \mathbb{E}_{x \sim P_X}\left[\log D(x)\right] + \mathbb{E}_{x \sim P_G}\left[\log(1 - D(x))\right]$$

$$\Rightarrow \mathbb{E}_{x \sim P_X}\left[\log \frac{P_X(x)}{P_X(x) + P_G(x)}\right] + \mathbb{E}_{x \sim P_G}\left[\log \frac{P_G(x)}{P_X(x) + P_G(x)}\right]$$

$$+\log 4 \quad -\log 4$$

$$\Rightarrow \mathbb{E}_{x \sim P_X}\left[\log \frac{2P_X(x)}{P_X(x) + P_G(x)}\right] + \mathbb{E}_{x \sim P_G}\left[\log \frac{2P_G(x)}{P_X(x) + P_G(x)}\right] - \log 4$$

Math: Optimum Generator

$$\Rightarrow \mathbb{E}_{x\sim P_x}\left[\log \frac{2P_x(x)}{P_x(x)+P_G(x)}\right] + \mathbb{E}_{x\sim P_G}\left[\log \frac{2P_G(x)}{P_x(x)+P_G(x)}\right] - \log 4$$

$$= \mathbb{E}_{x\sim P_x}\left[\log \frac{P_x(x)}{\frac{P_x(x)+P_G(x)}{2}}\right] + \mathbb{E}_{x\sim P_G}\left[\log \frac{P_G(x)}{\frac{P_x(x)+P_G(x)}{2}}\right] - \log 4$$

$$= D_{KL}\left(P_x(x), \frac{P_x(x)+P_G(x)}{2}\right) + D_{KL}\left(P_G(x), \frac{P_x(x)+P_G(x)}{2}\right) - \log 4$$

$$D_{JSD}(P,Q) = \frac{D_{KL}\left(P, \frac{P+Q}{2}\right) + D_{KL}\left(Q, \frac{P+Q}{2}\right)}{2}$$ JS divergence

$$D_{KL}\left(P, \frac{P+Q}{2}\right) = \mathbb{E}_P\left[\log \frac{P}{(P+Q)/2}\right]$$ KL Divergence

Math: Optimum Generator

$$= \frac{2}{2}\left(D_{KL}\left(P_x(x), \frac{P_x(x) + P_G(x)}{2} \right) + D_{KL}\left(P_G(x), \frac{P_x(x) + P_G(x)}{2} \right) \right) - \log 4$$

$$= 2\, D_{JSD}\left(P_x(x), P_G(x) \right) - \log 4$$

The optimum generator tries to minimize D_{JSD} !

$$\min_G \; 2\, D_{JSD}\left(P_x(x), P_G(x) \right) - \log 4$$