Recitation 8

CTC Decoding & Beam Search

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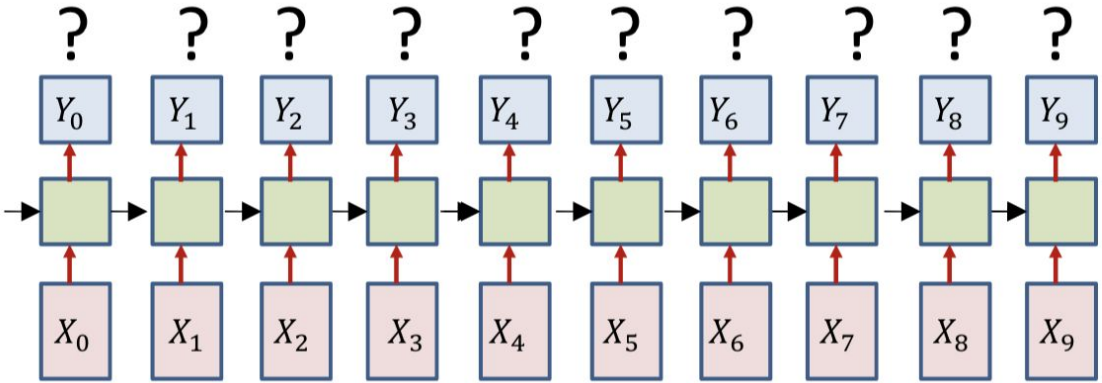
Sequence to Sequence Modeling

Order-Synchronous, Not Time-Synchronous Output

1. Training -> we already know how to do that
2. Testing -> “Decoding” or “obtaining an output from a sequence-to-sequence network”



A key decoding problem

- Consider a problem where the output symbols are characters
- We have a decode: RRROOOOOD
- Is this the merged symbol sequence ROD or ROOD?

How to distinguish between an extended symbol and repetitions of a symbol?

A key decoding problem

Solution: Introduce an explicit extra symbol which serves to separate discrete versions of a symbol

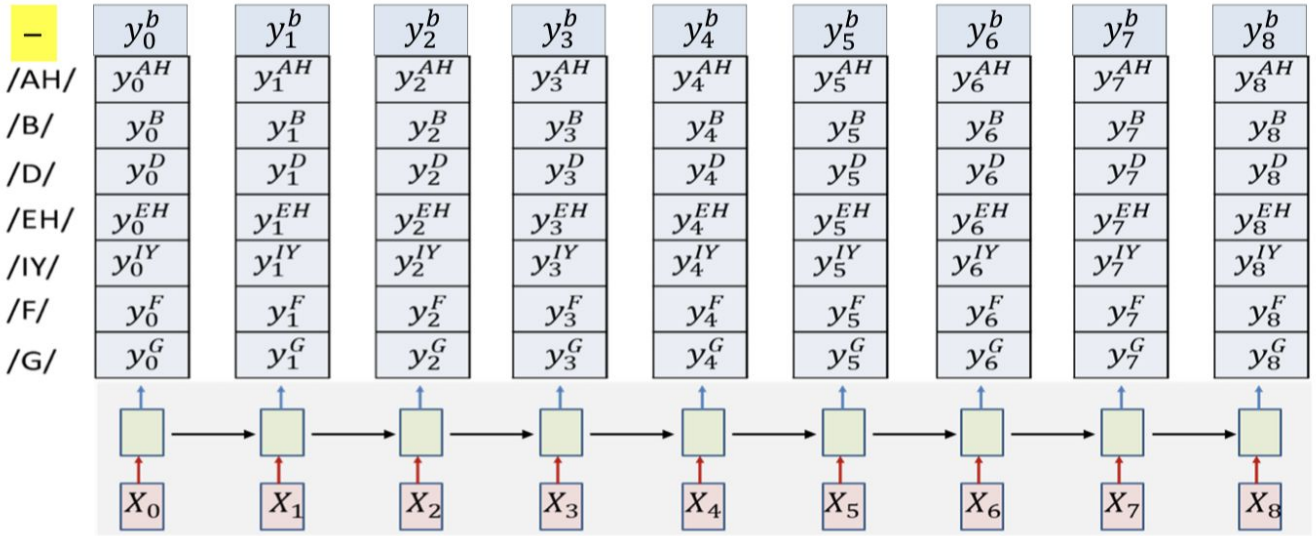
- A “blank” (represented by “-”)
- RRR---OO---DDD = ROD
- RR-R---OO---D-DD = RRODD
- R-R-R---O-ODD-DDDD-D = RRROODDD

The symbol set recognized by the network must now include the extra blank symbol

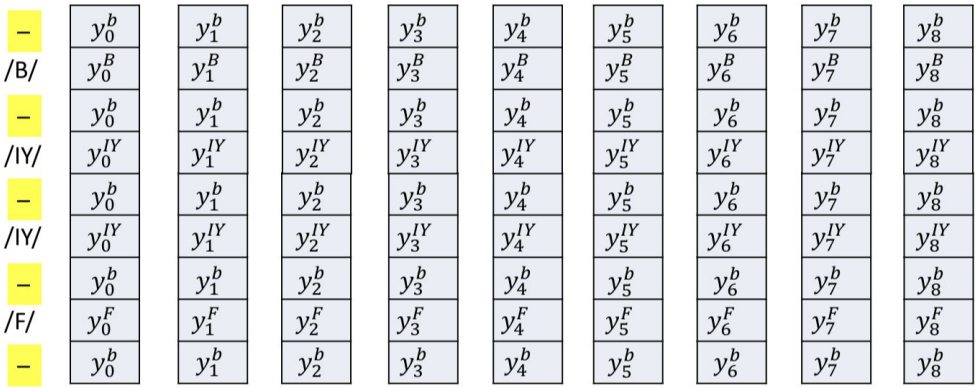
- Which too must be trained

The modified forward output

Note the extra “blank”at the output



Composing graph for training



- With blanks
- Note: a row of blanks between any two symbols
- Also blanks at the very beginning and the very end

Train as before!

CTC: Connectionist Temporal Classification

- The overall framework we saw is referred to as CTC

  – Applies when “duplicating” labels at the output is considered acceptable, and when output sequence length < input sequence length

Returning to the decoding problem

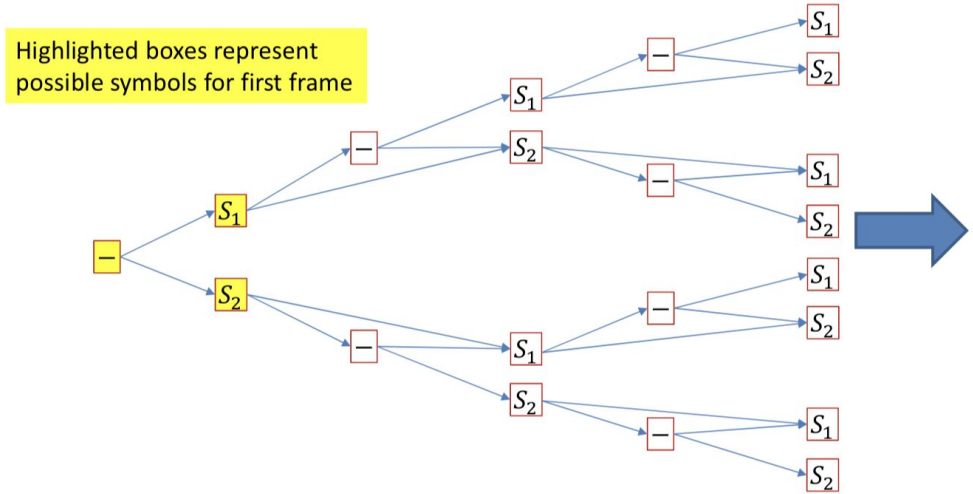
How to decode at test time?

- Greedy decode -> choose symbol with highest probability at each time step and merge
  - Sub-optimal decode which finds most likely synchronous output sequence

- Objective of decoding -> Most likely asynchronous symbol sequence
  - Find all decodings and pick the most likely decode!
  - Unfortunately, explicit computation of this will require evaluate of an exponential number of symbol sequences
  - Solution: Organize all possible symbol sequences as a (semi)tree

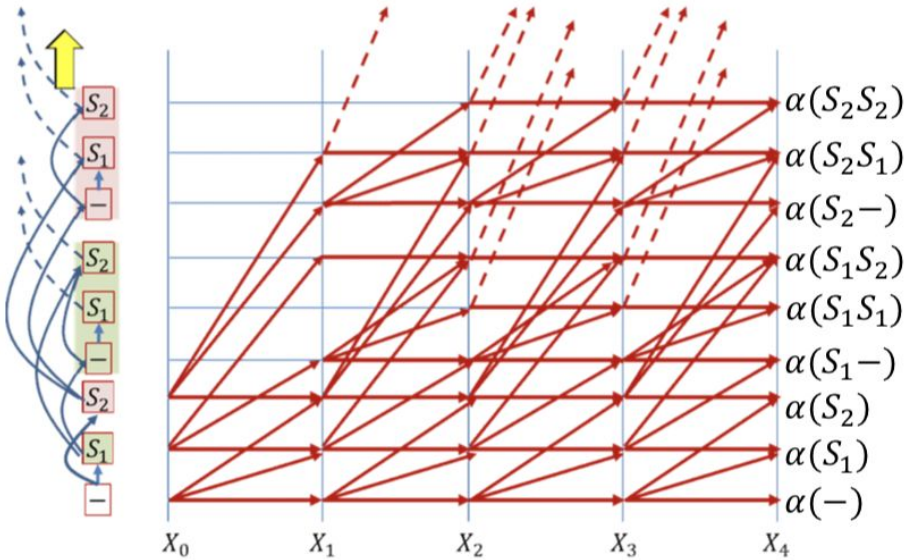
Hypothesis semi-tree

- The semi tree of hypotheses (assuming only 3 symbols in the vocabulary)
- Every symbol connects to every symbol other than itself
- It also connects to a blank, which connects to every symbol including itself
- The simple structure repeats recursively
- Each node represents a unique symbol sequence!



Decoding graph for the tree

- The figure to the left is the tree, drawn in a vertical line
- The graph is just the tree unrolled over time
- The alpha at final time represents the full forward score for a unique symbol sequence
- Select the symbol sequence with the largest alpha



Pruning

- This is the “theoretically correct” CTC decoder
- In practice, the graph gets exponentially large very quickly
- To prevent this pruning strategies are employed to keep the graph (and computation) manageable

Beam Search

- **`PathScore`** : array of scores for paths ending with symbols
- **`BlankPathScore`** : array of scores for paths ending with blanks
- **`SymbolSet`** : A list of symbols *not* including the blank

BEAM SEARCH

```
Global PathScore = [], BlankPathScore = []

# First time instant: Initialize paths with each of the symbols,
# including blank, using score at time t=1
NewPathsWithTerminalBlank, NewPathsWithTerminalSymbol, NewBlankPathScore, NewPathScore =
                           InitializePaths(SymbolSet, y[:,0])

# Subsequent time steps
for t = 1:T
    # Prune the collection down to the BeamWidth
    PathsWithTerminalBlank, PathsWithTerminalSymbol, PathScore, BlankPathScore =
                  Prune(NewPathsWithTerminalBlank, NewPathsWithTerminalSymbol,
                                                   NewBlankPathScore, NewPathScore, BeamWidth)
    # First extend paths by a blank
    NewPathsWithTerminalBlank, NewBlankPathScore = ExtendWithBlank(PathsWithTerminalBlank,
                                                                   PathsWithTerminalSymbol, y[:,t])

    # Next extend paths by a symbol
    NewPathsWithTerminalSymbol, NewPathScore = ExtendWithSymbol(PathsWithTerminalBlank,
                                                        PathsWithTerminalSymbol, SymbolSet, y[:,t])

end

# Merge identical paths differing only by the final blank
MergedPaths, FinalPathScore = MergeIdenticalPaths(NewPathsWithTerminalBlank, NewBlankPathScore
                                                  NewPathsWithTerminalSymbol, NewPathScore)

# Pick best path
BestPath = argmax(FinalPathScore) # Find the path with the best score
```

BEAM SEARCH

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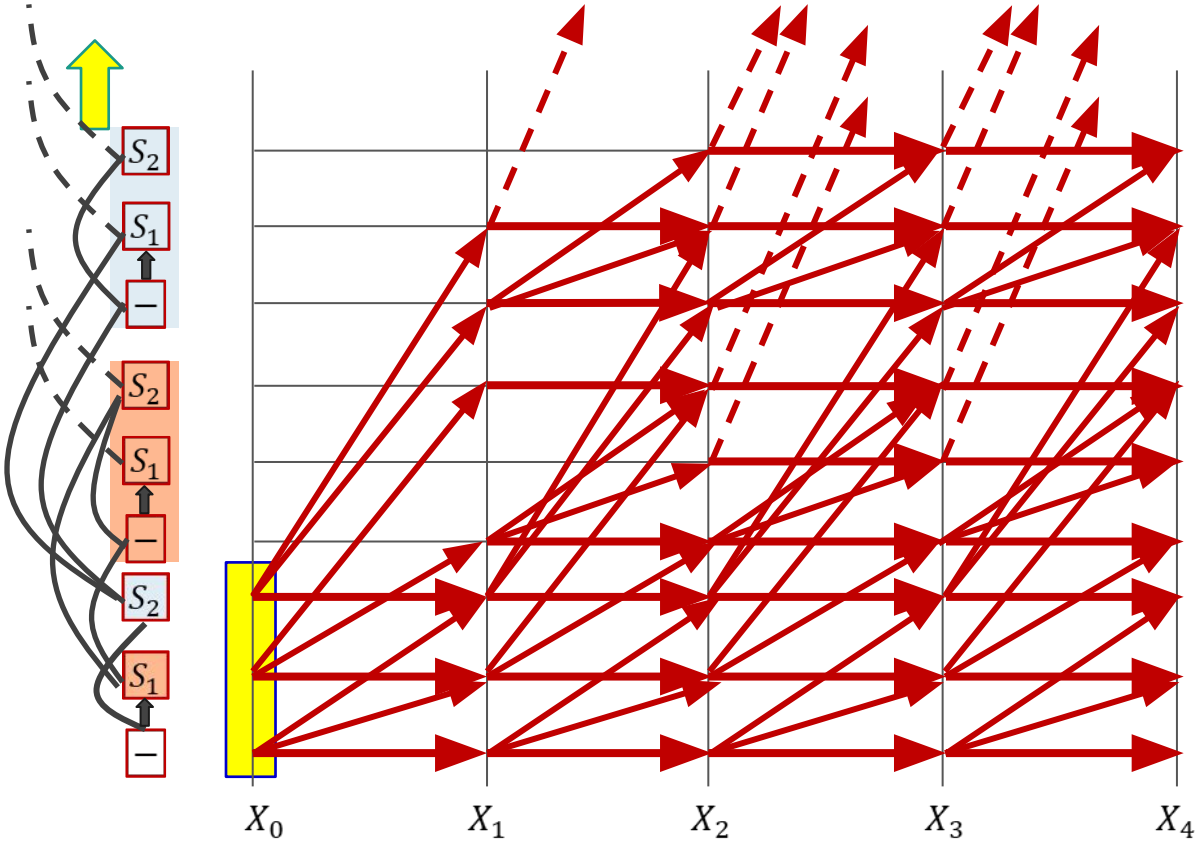

S_2
S_1
–
S_2
S_1
–
S_2
S_1
–
X_0
X_1
X_2
X_3
X_4

BEAM SEARCH InitializePaths: FIRST TIME INSTANT

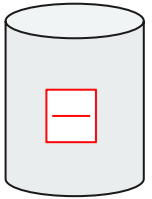
```
function InitializePaths(SymbolSet, y)

InitialBlankPathScore = [],  InitialPathScore = []
# First push the blank into a path-ending-with-blank stack. No symbol has been invoked yet
path = null
InitialBlankPathScore[path] = y[blank]  # Score of blank at t=1
InitialPathsWithFinalBlank = {path}

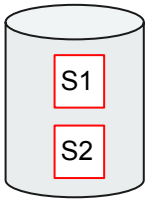
# Push rest of the symbols into a path-ending-with-symbol stack
InitialPathsWithFinalSymbol = {}
for c in SymbolSet  # This is the entire symbol set, without the blank
    path = c
    InitialPathScore[path] = y[c]  # Score of symbol c at t=1
    InitialPathsWithFinalSymbol += path # Set addition
end

return InitialPathsWithFinalBlank, InitialPathsWithFinalSymbol,
       InitialBlankPathScore, InitialPathScore
```

InitialPathWithFinalBlank



InitialPathWithFinalSymbols



BEAM SEARCH

```
Global PathScore = [], BlankPathScore = []

# First time instant: Initialize paths with each of the symbols,
# including blank, using score at time t=1
NewPathsWithTerminalBlank, NewPathsWithTerminalSymbol, NewBlankPathSc
                          InitializePaths(SymbolSet, y[:,0])

# Subsequent time steps
for t = 1:T
    # Prune the collection down to the BeamWidth
    PathsWithTerminalBlank, PathsWithTerminalSymbol, PathScore, BlankPathScore =
                  Prune(NewPathsWithTerminalBlank, NewPathsWithTerminalSymbol,
                                        NewBlankPathScore, NewPathScore, BeamWidth)
    # First extend paths by a blank
    NewPathsWithTerminalBlank, NewBlankPathScore = ExtendWithBlank(PathsWithTerminalBlank,
                                                        PathsWithTerminalSymbol, y[:,t])

    # Next extend paths by a symbol
    NewPathsWithTerminalSymbol, NewPathScore = ExtendWithSymbol(PathsWithTerminalBlank,
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end

# Merge identical paths differing only by the final blank
MergedPaths, FinalPathScore = MergeIdenticalPaths(NewPathsWithTerminalBlank, NewBlankPathScore
                                                  NewPathsWithTerminalSymbol, NewPathScore)

# Pick best path
BestPath = argmax(FinalPathScore) # Find the path with the best score
```

We will visit this routine after discussing the rest of the loop
(to avoid confusion)

BEAM SEARCH

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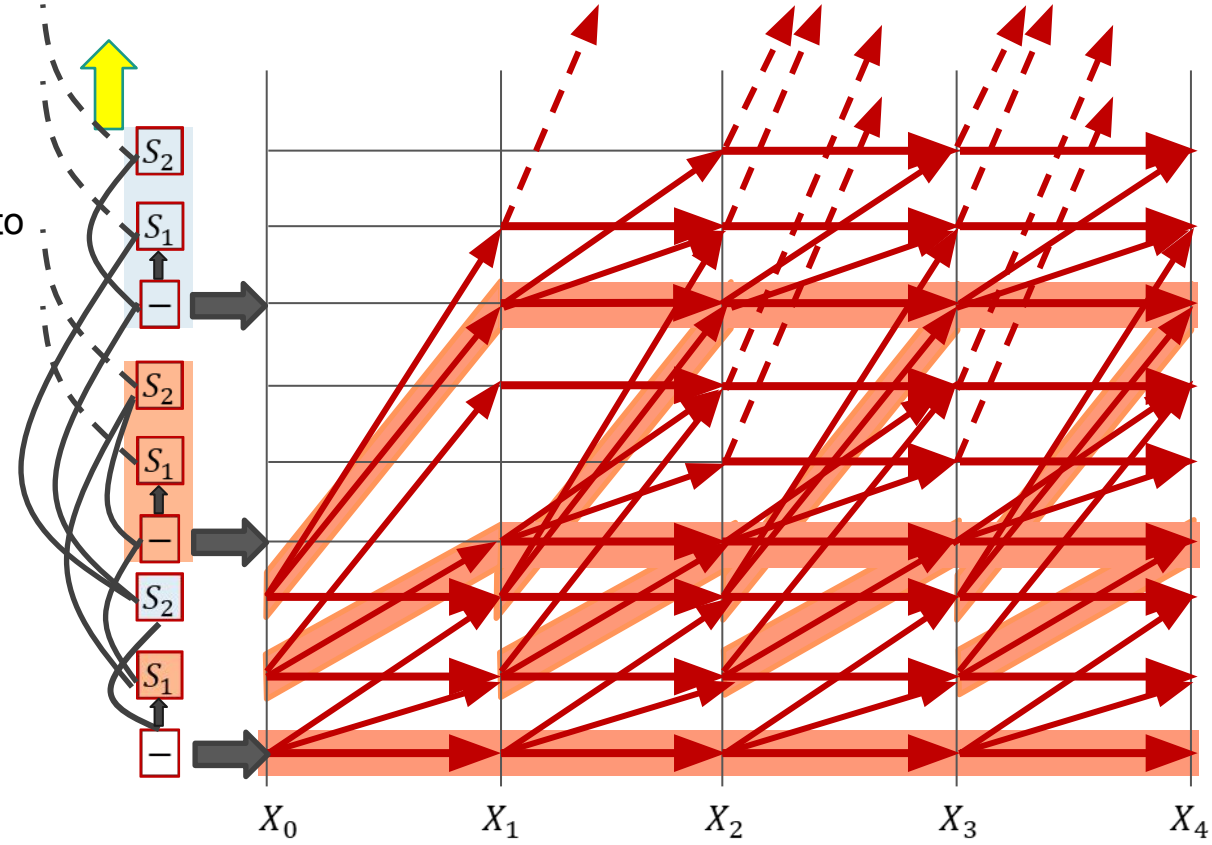
BestPath = argmax(FinalPathScore) # Find the path with the best score
```

Only transitions into nodes on the rows corresponding to blanks



BEAM SEARCH: Extending with blanks

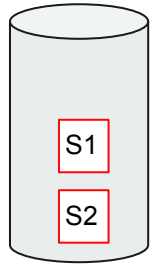
```
Global PathScore, BlankPathScore

function ExtendWithBlank(PathsWithTerminalBlank, PathsWithTerminalSymbol, y)
    UpdatedPathsWithTerminalBlank = {}
    UpdatedBlankPathScore = []
    # First work on paths with terminal blanks
    #(This represents transitions along horizontal trellis edges for blanks)
    for path in PathsWithTerminalBlank:
        # Repeating a blank doesn't change the symbol sequence
        UpdatedPathsWithTerminalBlank += path    # Set addition
        UpdatedBlankPathScore[path] = BlankPathScore[path]*y[blank]
    end

    # Then extend paths with terminal symbols by blanks
    for path in PathsWithTerminalSymbol:
        # If there is already an equivalent string in UpdatesPathsWithTerminalBlank
        # simply add the score. If not create a new entry
        if path in UpdatedPathsWithTerminalBlank
            UpdatedBlankPathScore[path] += Pathscore[path]* y[blank]
        else
            UpdatedPathsWithTerminalBlank += path  # Set addition
            UpdatedBlankPathScore[path] = PathScore[path] * y[blank]
        end
    end

    return UpdatedPathsWithTerminalBlank,
           UpdatedBlankPathScore
```

(only at t=1)
UpdatedPathsWIthTerminalBlank



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    end

    # Then extend paths with terminal symbols by blanks
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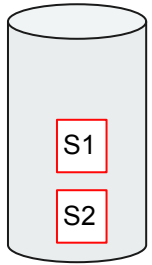
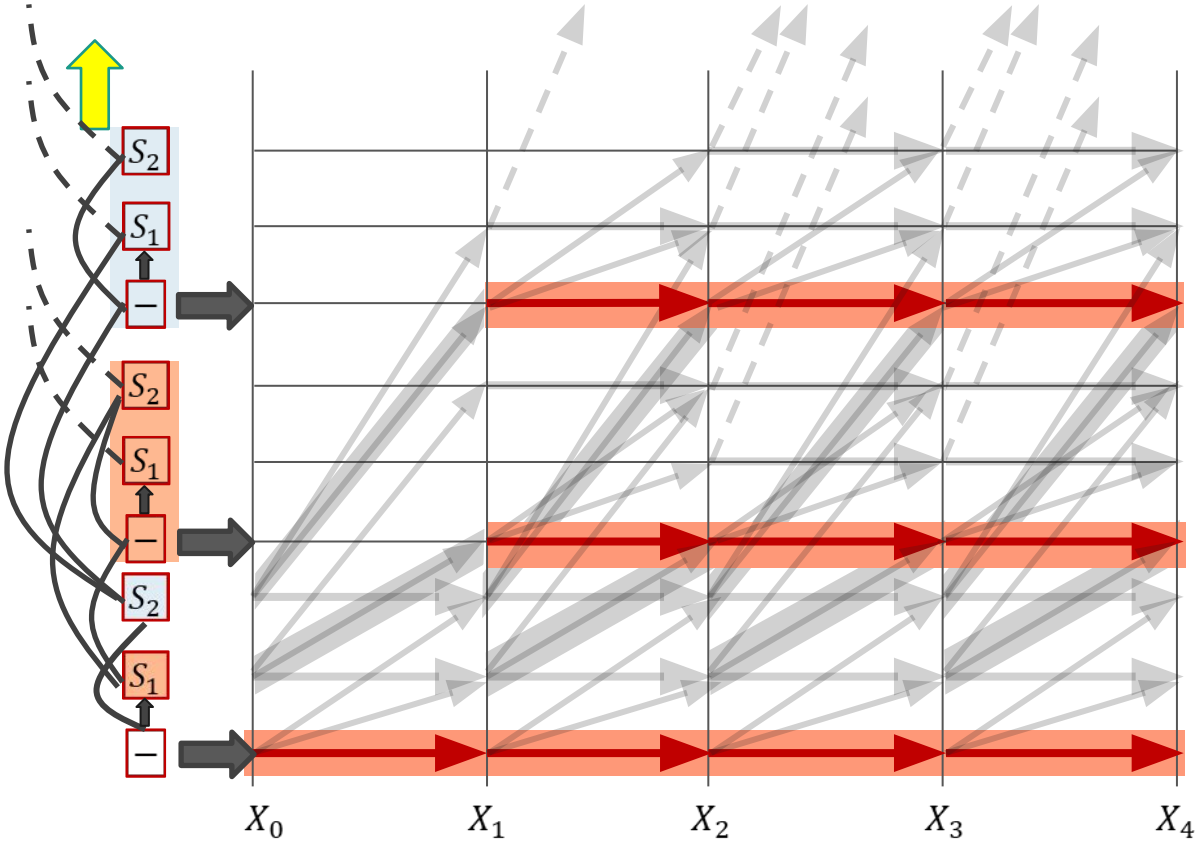
        # If there is already an equivalent string in UpdatesPathsWithTerminalBlank
        # simply add the score. If not create a new entry
        if path in UpdatedPathsWithTerminalBlank
            UpdatedBlankPathScore[path] += Pathscore[path]* y[blank]
        else
            UpdatedPathsWithTerminalBlank += path  # Set addition
            UpdatedBlankPathScore[path] = PathScore[path] * y[blank]
        end
    end

    return UpdatedPathsWithTerminalBlank,
           UpdatedBlankPathScore
```

(only at t=1)
UpdatedPathsWIthTerminalBlank





Transitions *from* "blank" lines *to* "blank" lines (which will all be horizontal edges)

BEAM SEARCH: Extending with blanks

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    UpdatedPathsWithTerminalBlank = {}
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    # First work on paths with terminal blanks
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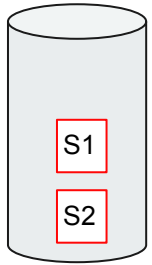
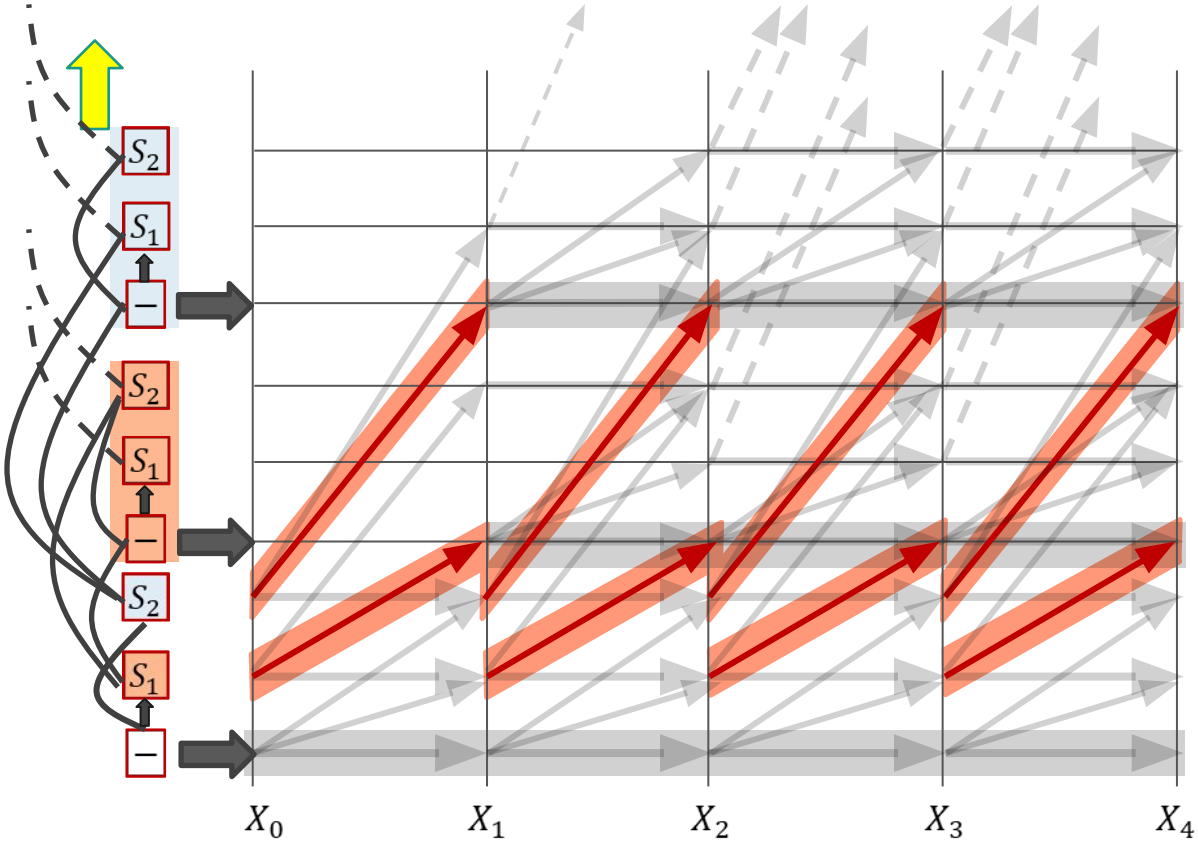
    for path in PathsWithTerminalBlank:
        # Repeating a blank doesn't change the symbol sequence
        UpdatedPathsWithTerminalBlank += path    # Set addition
        UpdatedBlankPathScore[path] = BlankPathScore[path]*y[blank]
    end

    # Then extend paths with terminal symbols by blanks
    for path in PathsWithTerminalSymbol:
        # If there is already an equivalent string in UpdatesPathsWithTerminalBlank
        # simply add the score. If not create a new entry
        if path in UpdatedPathsWithTerminalBlank
            UpdatedBlankPathScore[path] += Pathscore[path]* y[blank]
        else
            UpdatedPathsWithTerminalBlank += path  # Set addition
            UpdatedBlankPathScore[path] = PathScore[path] * y[blank]
        end
    end

    return UpdatedPathsWithTerminalBlank,
           UpdatedBlankPathScore
```

(only at t=1)
UpdatedPathsWIthTerminalBlank





Transitions *from* "symbol" lines *to* "blank" lines

BEAM SEARCH

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Global PathScore = [], BlankPathScore = []

# First time instant: Initialize paths with each of the symbols,
# including blank, using score at time t=1
NewPathsWithTerminalBlank, NewPathsWithTerminalSymbol, NewBlankPathScore, NewPathScore =
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# Subsequent time steps
for t = 1:T
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    NewPathsWithTerminalSymbol, NewPathScore = ExtendWithSymbol(PathsWithTerminalBlank,
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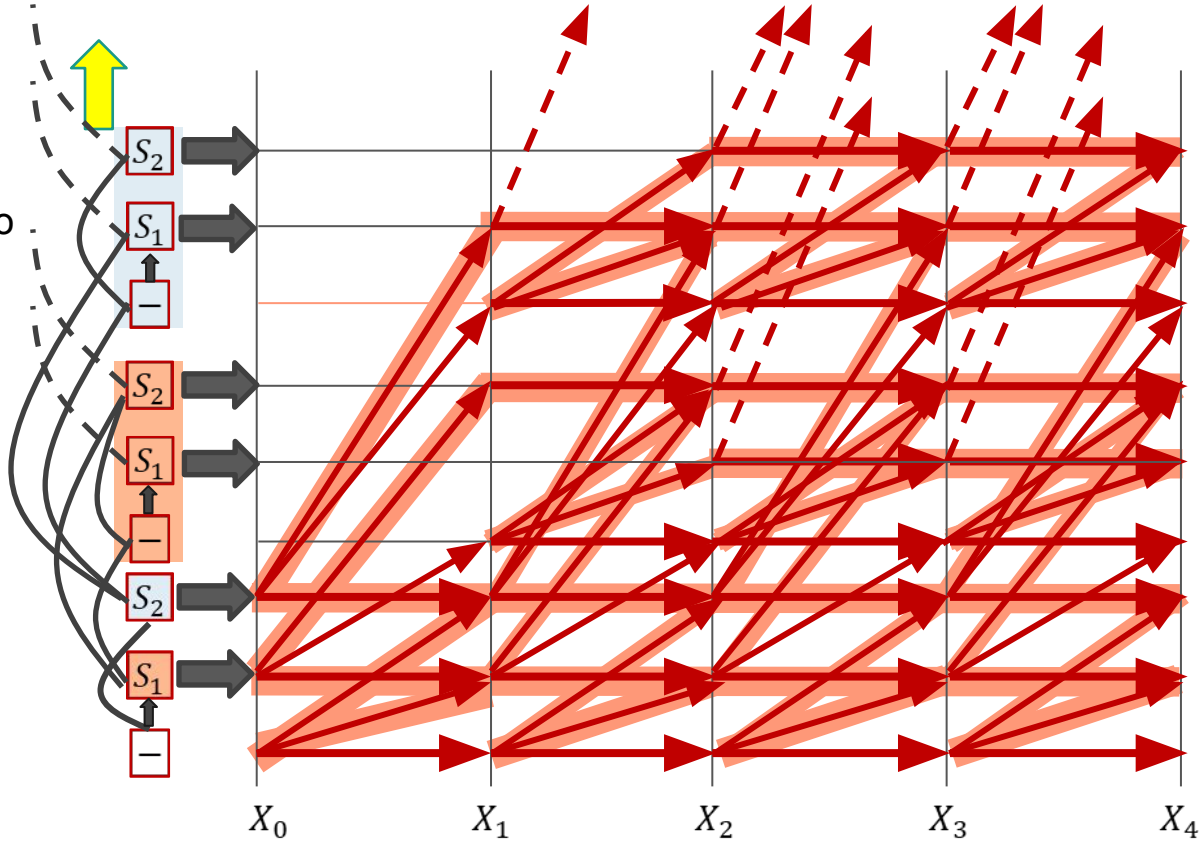
end

# Merge identical paths differing only by the final blank
MergedPaths, FinalPathScore = MergeIdenticalPaths(NewPathsWithTerminalBlank, NewBlankPathScore
                                                  NewPathsWithTerminalSymbol, NewPathScore)

# Pick best path
BestPath = argmax(FinalPathScore) # Find the path with the best score
```

Only transitions into nodes on the rows corresponding to non-blank symbols



(figure shows path extensions for only 2 time steps)

BEAM SEARCH: Extending with symbols

```
Global PathScore, BlankPathScore

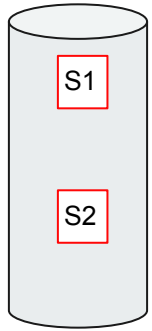
function ExtendWithSymbol(PathsWithTerminalBlank, PathsWithTerminalSymbol, SymbolSet, y)
    UpdatedPathsWithTerminalSymbol = {}
    UpdatedPathScore = []

    # First extend the paths terminating in blanks. This will always create a new sequence
    for path in PathsWithTerminalBlank:
        for c in SymbolSet:  # SymbolSet does not include blanks
            newpath = path + c  # Concatenation
            UpdatedPathsWithTerminalSymbol += newpath # Set addition
            UpdatedPathScore[newpath] = BlankPathScore[path] * y(c)
        end
    end

    # Next work on paths with terminal symbols
    for path in PathsWithTerminalSymbol:
        # Extend the path with every symbol other than blank
        for c in SymbolSet:  # SymbolSet does not include blanks
            newpath = (c == path[end]) ? path : path + c  # Horizontal transitions don't extend the sequence
            if newpath in UpdatedPathsWithTerminalSymbol: # Already in list, merge paths
                UpdatedPathScore[newpath] += PathScore[path] * y[c]
            else # Create new path
                UpdatedPathsWithTerminalSymbol += newpath  # Set addition
                UpdatedPathScore[newpath] = PathScore[path] * y[c]
            end
        end
    end

    return UpdatedPathsWithTerminalSymbol, UpdatedPathScore
```

(only at t=1)
UpdatedPathsWIthTerminalSymbol



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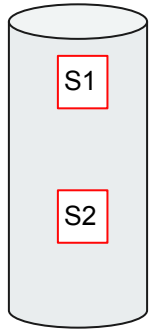
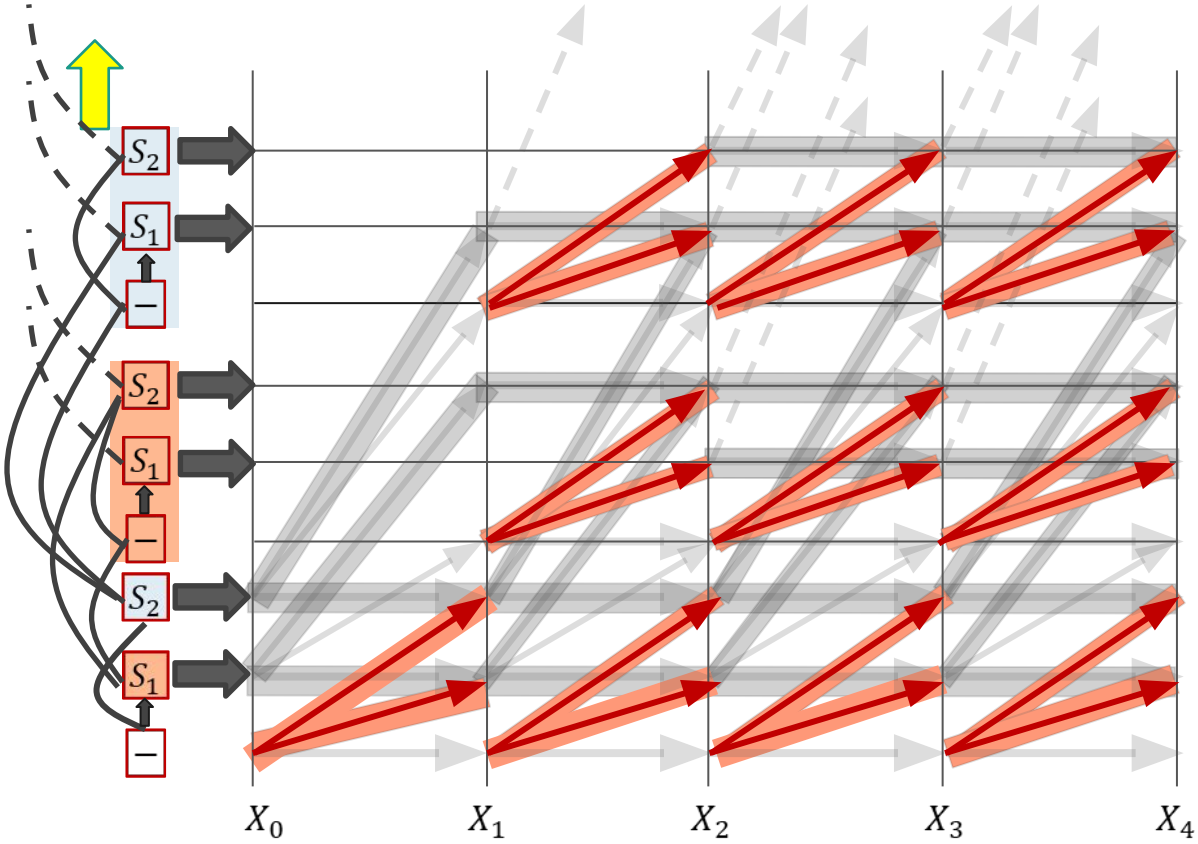
            if newpath in UpdatedPathsWithTerminalSymbol: # Already in list, merge paths
                UpdatedPathScore[newpath] += PathScore[path] * y[c]
            else # Create new path
                UpdatedPathsWithTerminalSymbol += newpath  # Set addition
                UpdatedPathScore[newpath] = PathScore[path] * y[c]
            end
        end
    end

    return UpdatedPathsWithTerminalSymbol, UpdatedPathScore
```

(only at t=1)
UpdatedPathsWIthTerminalSymbol





Transitions *from* "blank" lines *to* "symbol" lines

(figure shows path extensions for only 2 time steps)

BEAM SEARCH: Extending with symbols

```
Global PathScore, BlankPathScore

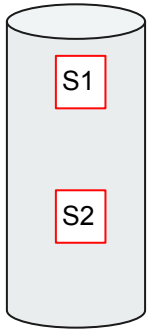
function ExtendWithSymbol(PathsWithTerminalBlank, PathsWithTerminalSymbol, SymbolSet, y)
    UpdatedPathsWithTerminalSymbol = {}
    UpdatedPathScore = []

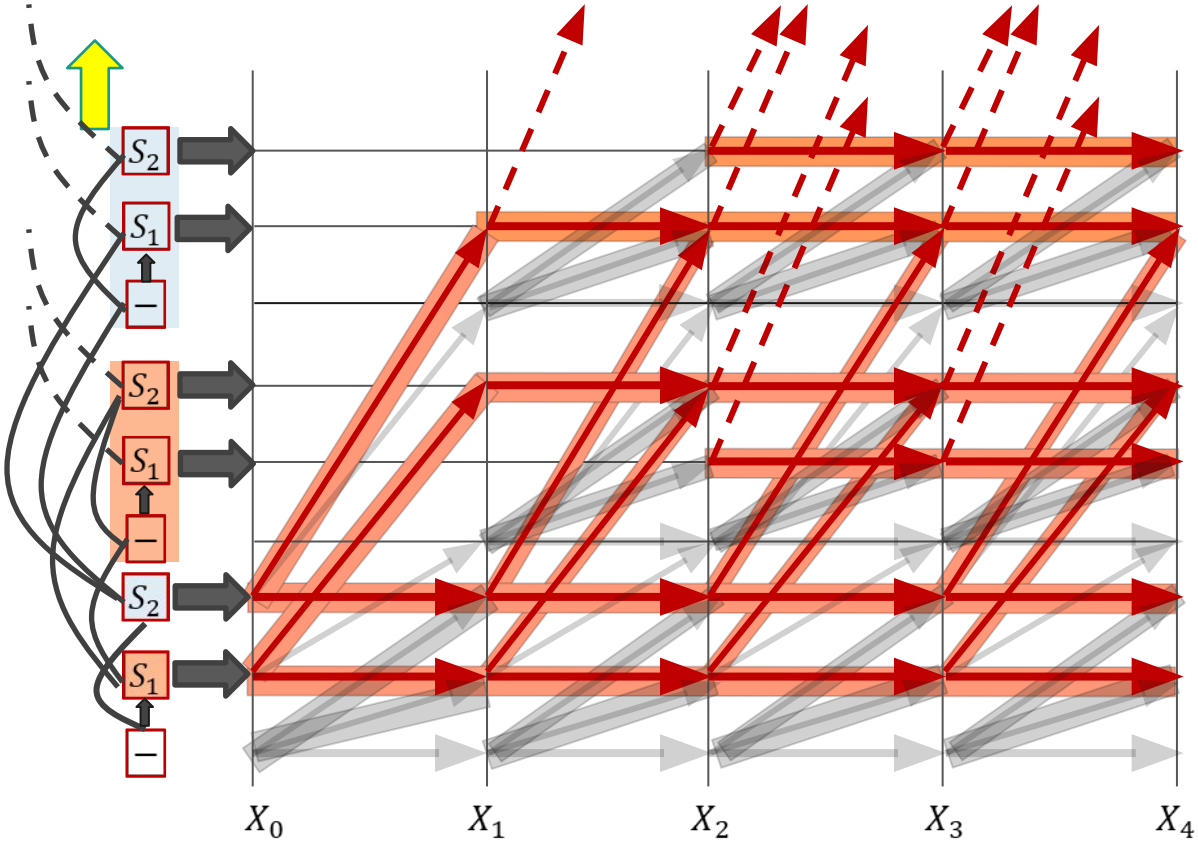
    # First extend the paths terminating in blanks. This will always create a new sequence
    for path in PathsWithTerminalBlank:
        for c in SymbolSet:  # SymbolSet does not include blanks
            newpath = path + c  # Concatenation
            UpdatedPathsWithTerminalSymbol += newpath # Set addition
            UpdatedPathScore[newpath] = BlankPathScore[path] * y(c)
        end
    end

    # Next work on paths with terminal symbols
    for path in PathsWithTerminalSymbol:
        # Extend the path with every symbol other than blank
        for c in SymbolSet:  # SymbolSet does not include blanks
            newpath = (c == path[end]) ? path : path + c  # Horizontal transitions don't extend the sequence
            if newpath in UpdatedPathsWithTerminalSymbol: # Already in list, merge paths
                UpdatedPathScore[newpath] += PathScore[path] * y[c]
            else # Create new path
                UpdatedPathsWithTerminalSymbol += newpath  # Set addition
                UpdatedPathScore[newpath] = PathScore[path] * y[c]
            end
        end
    end

    return UpdatedPathsWithTerminalSymbol, UpdatedPathScore
```

(only at t=1)
UpdatedPathsWIthTerminalSymbol





Transitions *from* "symbol" lines *to* "symbol" lines (including horizontal transitions)

(figure shows path extensions for only 2 time steps)

BEAM SEARCH

```
Global PathScore = [], BlankPathScore = []

# First time instant: Initialize paths with each of the symbols,
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# Subsequent time steps
for t = 1:T
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    PathsWithTerminalBlank, PathsWithTerminalSymbol, PathScore, BlankPathScore =
                  Prune(NewPathsWithTerminalBlank, NewPathsWithTerminalSymbol,
                                        NewBlankPathScore, NewPathScore, BeamWidth)
    # First extend paths by a blank
    NewPathsWithTerminalBlank, NewBlankPathScore = ExtendWithBlank(Pa
                                                                   Pat

    # Next extend paths by a symbol
    NewPathsWithTerminalSymbol, NewPathScore = ExtendWithSymbol(Paths
                                                    PathsWithTerminalSymbol, SymbolSet, y[:,t])

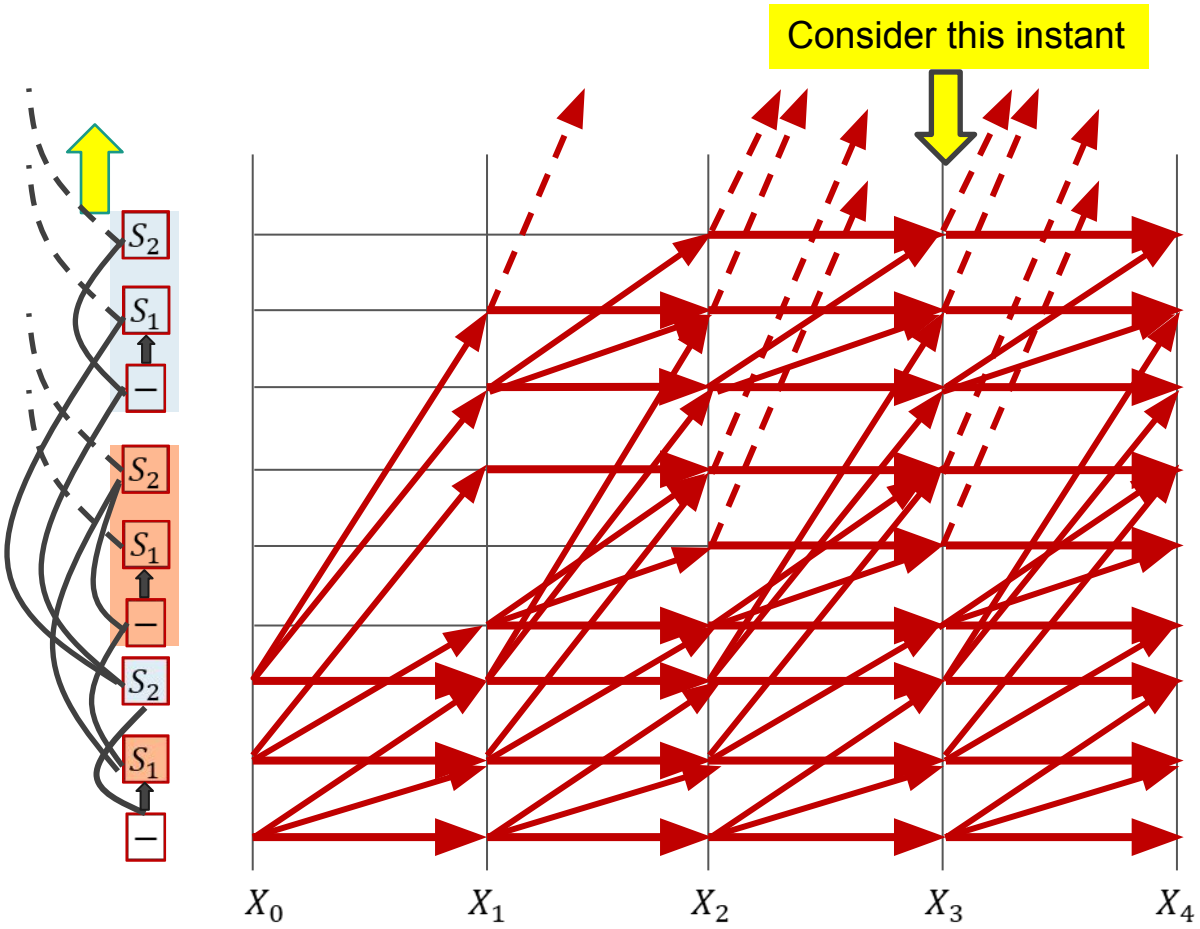
end

# Merge identical paths differing only by the final blank
MergedPaths, FinalPathScore = MergeIdenticalPaths(NewPathsWithTerminalBlank, NewBlankPathScore
                                                  NewPathsWithTerminalSymbol, NewPathScore)

# Pick best path
BestPath = argmax(FinalPathScore) # Find the path with the best score
```

Returning to this routine

Pruning deletes unpromising paths from contention, to reduce computation


Consider this instant
S_2
S_1
–
S_2
S_1
–
S_2
S_1
–
X_0
X_1
X_2
X_3
X_4

BEAM SEARCH: Pruning low-scoring entries

```
Global PathScore, BlankPathScore

function Prune(PathsWithTerminalBlank, PathsWithTerminalSymbol, BlankPathScore, PathScore, BeamWidth)
    PrunedBlankPathScore = []
    PrunedPathScore = []
    # First gather all the relevant scores
    i = 1
    for p in PathsWithTerminalBlank
        scorelist[i] = BlankPathScore[p]
        i++
    end
    for p in PathsWithTerminalSymbol
        scorelist[i] = PathScore[p]
        i++
    end

    # Sort and find cutoff score that retains exactly BeamWidth paths
    sort(scorelist)  # In decreasing order
    cutoff = BeamWidth < length(scorelist) ?  scorelist[BeamWidth] : scorelist[end]

    PrunedPathsWithTerminalBlank = {}
    for p in PathsWithTerminalBlank
        if BlankPathScore[p] >= cutoff
            PrunedPathsWithTerminalBlank += p # Set addition
            PrunedBlankPathScore[p] = BlankPathScore[p]
        end
    end

    PrunedPathsWithTerminalSymbol = {}
    for p in PathsWithTerminalSymbol
        if PathScore[p] >= cutoff
            PrunedPathsWithTerminalSymbol += p # Set addition
            PrunedPathScore[p] = PathScore[p]
        end
    end

    return PrunedPathsWithTerminalBlank, PrunedPathsWithTerminalSymbol, PrunedBlankPathScore, PrunedPathScore
```

BEAM SEARCH: Pruning low-scoring entries

```
Global PathScore, BlankPathScore

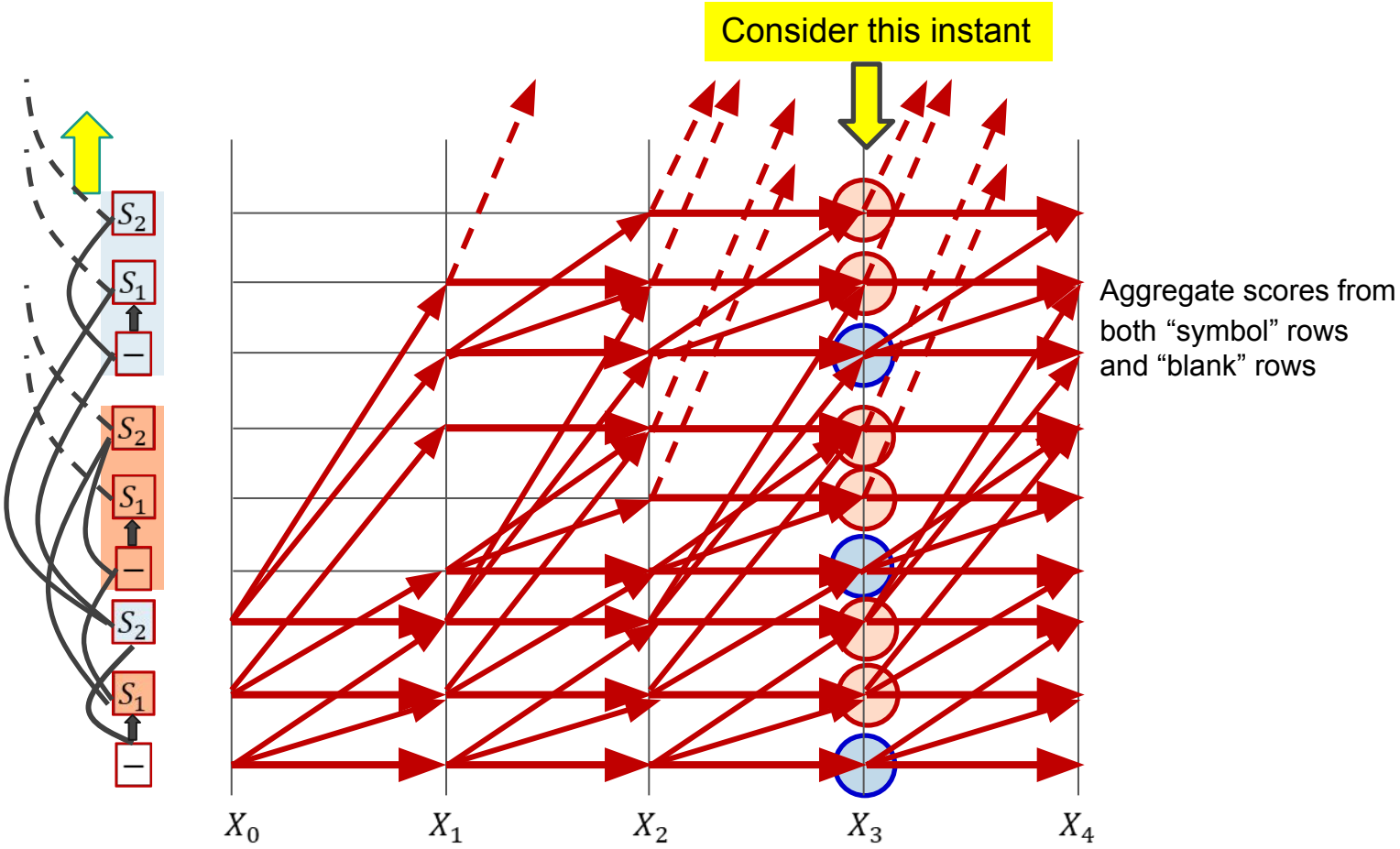
function Prune(PathsWithTerminalBlank, PathsWithTerminalSymbol, BlankPathScore, PathScore, BeamWidth)
    PrunedBlankPathScore = []
    PrunedPathScore = []
    # First gather all the relevant scores
    i = 1
    for p in PathsWithTerminalBlank
        scorelist[i] = BlankPathScore[p]
        i++
    end
    for p in PathsWithTerminalSymbol
        scorelist[i] = PathScore[p]
        i++
    end

    # Sort and find cutoff score that retains exactly BeamWidth paths
    sort(scorelist)  # In decreasing order
    cutoff = BeamWidth < length(scorelist) ?  scorelist[BeamWidth] : scorelist[end]

    PrunedPathsWithTerminalBlank = {}
    for p in PathsWithTerminalBlank
        if BlankPathScore[p] >= cutoff
            PrunedPathsWithTerminalBlank += p # Set addition
            PrunedBlankPathScore[p] = BlankPathScore[p]
        end
    end

    PrunedPathsWithTerminalSymbol = {}
    for p in PathsWithTerminalSymbol
        if PathScore[p] >= cutoff
            PrunedPathsWithTerminalSymbol += p # Set addition
            PrunedPathScore[p] = PathScore[p]
        end
    end

    return PrunedPathsWithTerminalBlank, PrunedPathsWithTerminalSymbol, PrunedBlankPathScore, PrunedPathScore
```


Consider this instant
S_2
S_1
–
S_2
S_1
–
S_2
S_1
–
X_0
X_1
X_2
X_3
X_4
Aggregate scores from both “symbol” rows and “blank” rows

BEAM SEARCH: Pruning low-scoring entries

```
Global PathScore, BlankPathScore

function Prune(PathsWithTerminalBlank, PathsWithTerminalSymbol, BlankPathScore, PathScore, BeamWidth)
    PrunedBlankPathScore = []
    PrunedPathScore = []
    # First gather all the relevant scores
    i = 1
    for p in PathsWithTerminalBlank
        scorelist[i] = BlankPathScore[p]
        i++
    end
    for p in PathsWithTerminalSymbol
        scorelist[i] = PathScore[p]
        i++
    end

    # Sort and find cutoff score that retains exactly BeamWidth paths
    sort(scorelist)  # In decreasing order
    cutoff = BeamWidth < length(scorelist) ?  scorelist[BeamWidth] : scorelist[end]

    PrunedPathsWithTerminalBlank = {}
    for p in PathsWithTerminalBlank
        if BlankPathScore[p] >= cutoff
            PrunedPathsWithTerminalBlank += p # Set addition
            PrunedBlankPathScore[p] = BlankPathScore[p]
        end
    end

    PrunedPathsWithTerminalSymbol = {}
    for p in PathsWithTerminalSymbol
        if PathScore[p] >= cutoff
            PrunedPathsWithTerminalSymbol += p # Set addition
            PrunedPathScore[p] = PathScore[p]
        end
    end

    return PrunedPathsWithTerminalBlank, PrunedPathsWithTerminalSymbol, PrunedBlankPathScore, PrunedPathScore
```

Sort the scores
Find the largest score
Find the cutoff score (the Kth largest score)

BEAM SEARCH: Pruning low-scoring entries

```
Global PathScore, BlankPathScore

function Prune(PathsWithTerminalBlank, PathsWithTerminalSymbol, BlankPathScore, PathScore, BeamWidth)
    PrunedBlankPathScore = []
    PrunedPathScore = []
    # First gather all the relevant scores
    i = 1
    for p in PathsWithTerminalBlank
        scorelist[i] = BlankPathScore[p]
        i++
    end
    for p in PathsWithTerminalSymbol
        scorelist[i] = PathScore[p]
        i++
    end

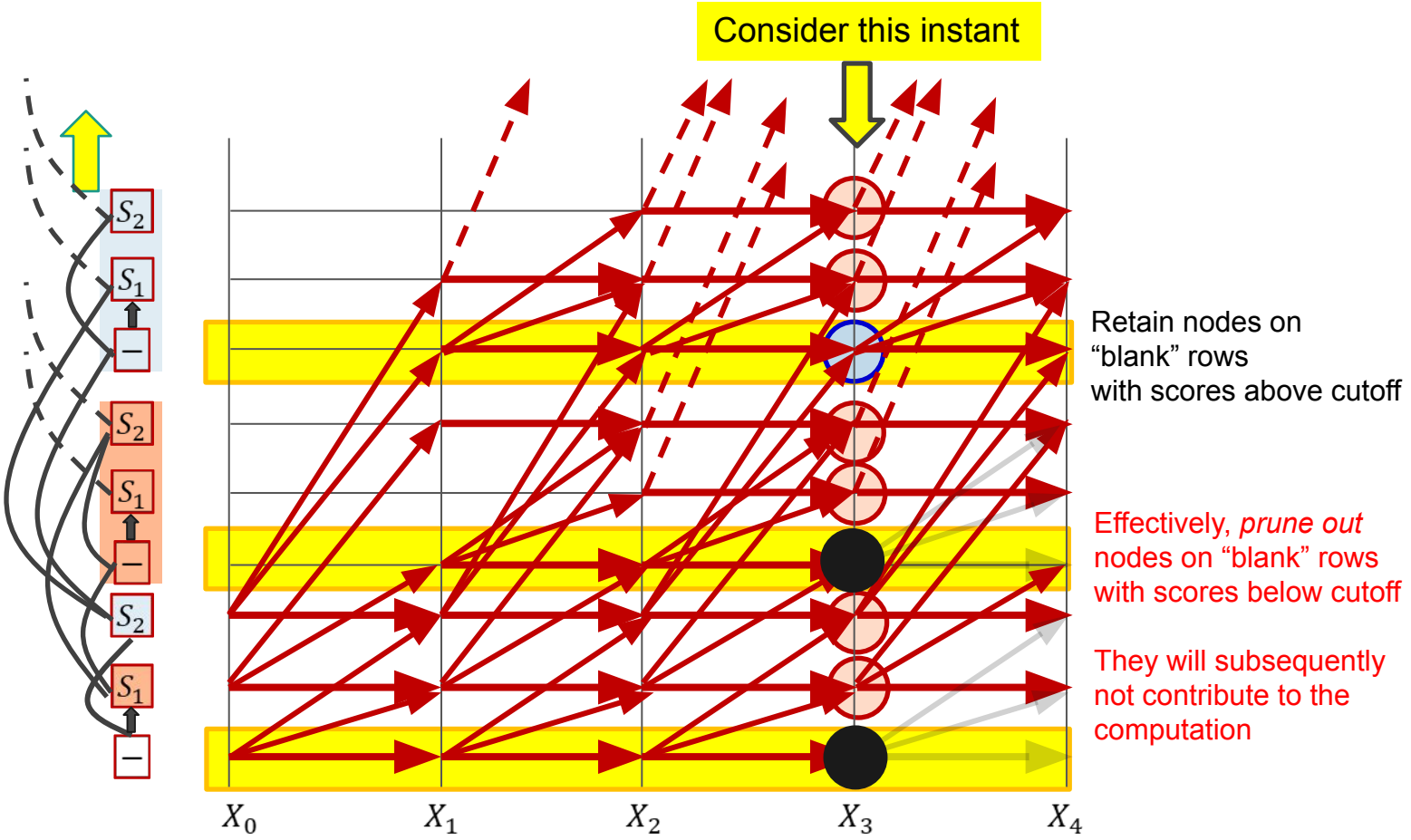
    # Sort and find cutoff score that retains exactly BeamWidth paths
    sort(scorelist)  # In decreasing order
    cutoff = BeamWidth < length(scorelist) ?  scorelist[BeamWidth] : scorelist[end]

    PrunedPathsWithTerminalBlank = {}
    for p in PathsWithTerminalBlank
        if BlankPathScore[p] >= cutoff
            PrunedPathsWithTerminalBlank += p # Set addition
            PrunedBlankPathScore[p] = BlankPathScore[p]
        end
    end

    PrunedPathsWithTerminalSymbol = {}
    for p in PathsWithTerminalSymbol
        if PathScore[p] >= cutoff
            PrunedPathsWithTerminalSymbol += p # Set addition
            PrunedPathScore[p] = PathScore[p]
        end
    end

    return PrunedPathsWithTerminalBlank, PrunedPathsWithTerminalSymbol, PrunedBlankPathScore, PrunedPathScore
```

Find nodes on
“blank” rows
with scores above cutoff
and add them to the
“active” list


Consider this instant
S_2
S_1
–
S_2
S_1
–
S_2
S_1
–
X_0
X_1
X_2
X_3
X_4
Retain nodes on
“blank” rows
with scores above cutoff
Effectively, *prune out*
nodes on “blank” rows
with scores below cutoff
They will subsequently
not contribute to the
computation

BEAM SEARCH: Pruning low-scoring entries

```
Global PathScore, BlankPathScore

function Prune(PathsWithTerminalBlank, PathsWithTerminalSymbol, BlankPathScore, PathScore, BeamWidth)
    PrunedBlankPathScore = []
    PrunedPathScore = []
    # First gather all the relevant scores
    i = 1
    for p in PathsWithTerminalBlank
        scorelist[i] = BlankPathScore[p]
        i++
    end
    for p in PathsWithTerminalSymbol
        scorelist[i] = PathScore[p]
        i++
    end

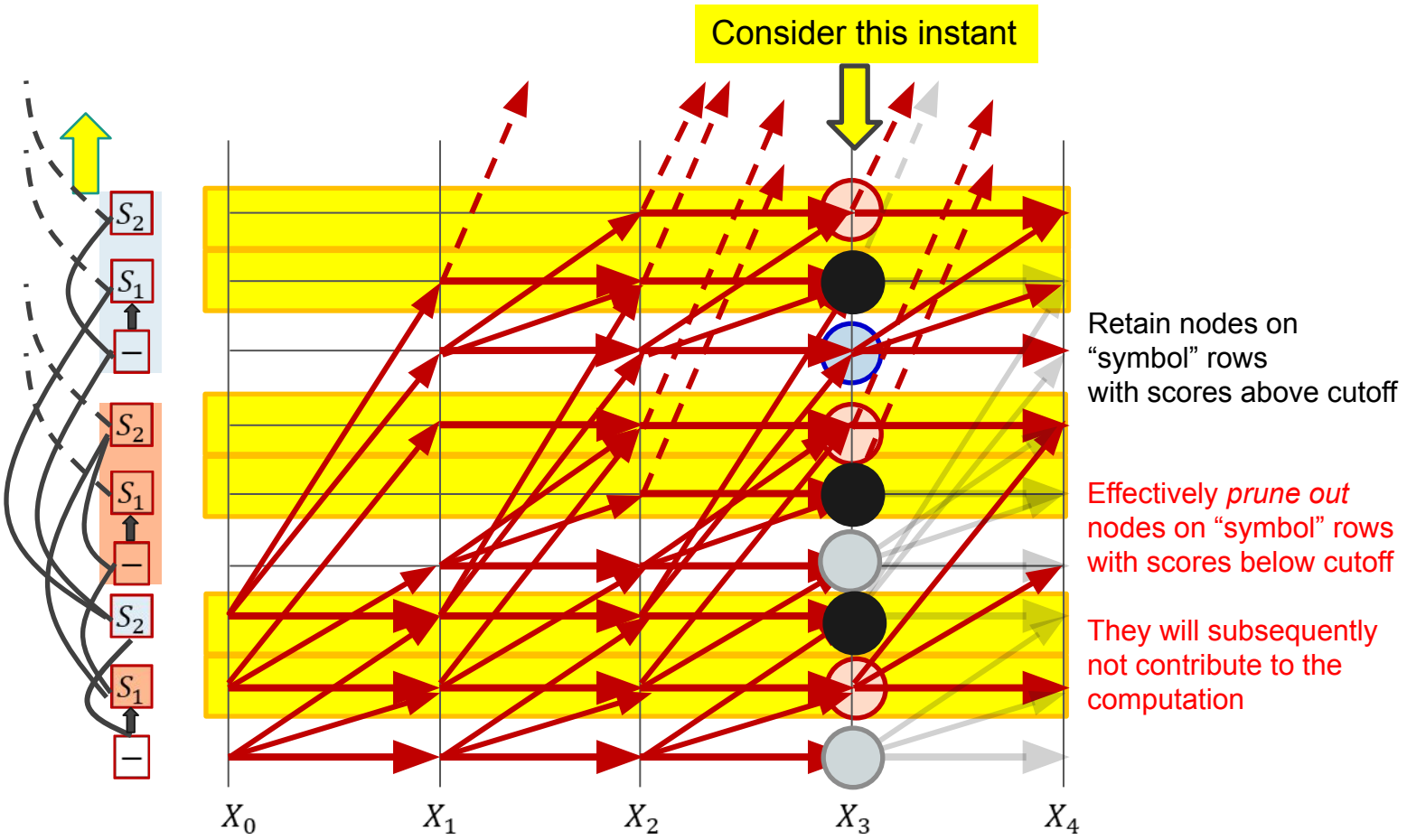
    # Sort and find cutoff score that retains exactly BeamWidth paths
    sort(scorelist)  # In decreasing order
    cutoff = BeamWidth < length(scorelist) ?  scorelist[BeamWidth] : scorelist[end]

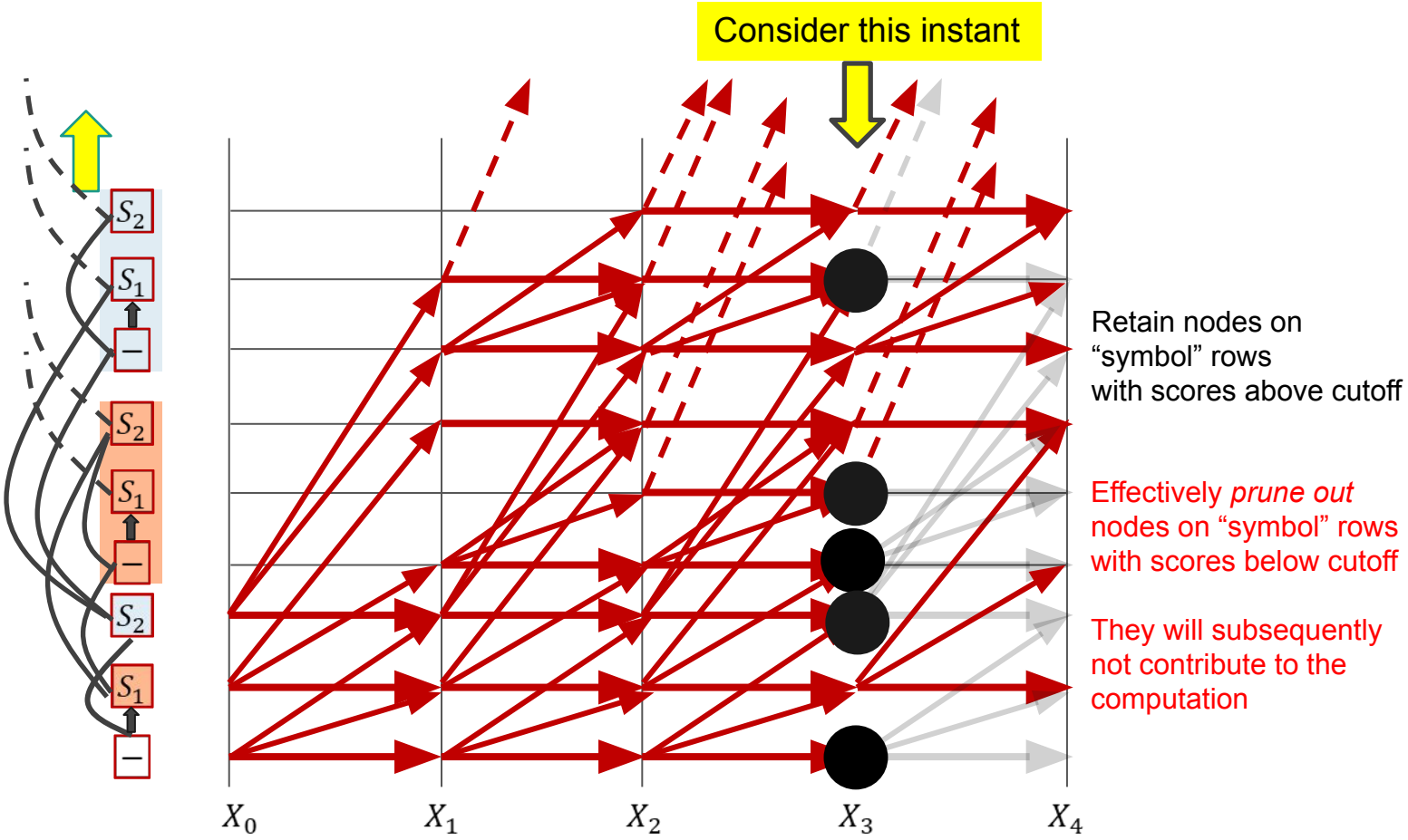
    PrunedPathsWithTerminalBlank = {}
    for p in PathsWithTerminalBlank
        if BlankPathScore[p] >= cutoff
            PrunedPathsWithTerminalBlank += p # Set addition
            PrunedBlankPathScore[p] = BlankPathScore[p]
        end
    end

    PrunedPathsWithTerminalSymbol = {}
    for p in PathsWithTerminalSymbol
        if PathScore[p] >= cutoff
            PrunedPathsWithTerminalSymbol += p # Set addition
            PrunedPathScore[p] = PathScore[p]
        end
    end

    return PrunedPathsWithTerminalBlank, PrunedPathsWithTerminalSymbol, PrunedBlankPathScore, PrunedPathScore
```

Find nodes on “symbol” rows with scores above cutoff and add them to the “active” list


Consider this instant
S_2
S_1
–
S_2
S_1
–
S_2
S_1
–
X_0
X_1
X_2
X_3
X_4
Retain nodes on "symbol" rows with scores above cutoff
Effectively *prune out* nodes on "symbol" rows with scores below cutoff
They will subsequently not contribute to the computation


Consider this instant
S_2
S_1
–
S_2
S_1
–
S_2
S_1
–
X_0
X_1
X_2
X_3
X_4
Retain nodes on "symbol" rows with scores above cutoff
Effectively *prune out* nodes on "symbol" rows with scores below cutoff
They will subsequently not contribute to the computation

BEAM SEARCH: Pruning low-scoring entries

```
Global PathScore, BlankPathScore

function Prune(PathsWithTerminalBlank, PathsWithTerminalSymbol, BlankPathScore, PathScore, BeamWidth)
    PrunedBlankPathScore = []
    PrunedPathScore = []
    # First gather all the relevant scores
    i = 1
    for p in PathsWithTerminalBlank
        scorelist[i] = BlankPathScore[p]
        i++
    end
    for p in PathsWithTerminalSymbol
        scorelist[i] = PathScore[p]
        i++
    end

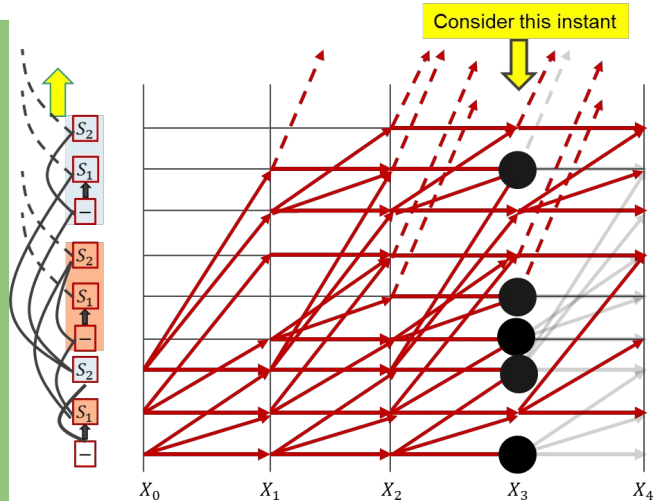
    # Sort and find cutoff score that retains exactly BeamWidth paths
    sort(scorelist)  # In decreasing order
    cutoff = BeamWidth < length(scorelist) ?  scorelist[BeamWidth] : scorelist[end]

    PrunedPathsWithTerminalBlank = {}
    for p in PathsWithTerminalBlank
        if BlankPathScore[p] >= cutoff
            PrunedPathsWithTerminalBlank += p # Set addition
            PrunedBlankPathScore[p] = BlankPathScore[p]
        end
    end

    PrunedPathsWithTerminalSymbol = {}
    for p in PathsWithTerminalSymbol
        if PathScore[p] >= cutoff
            PrunedPathsWithTerminalSymbol += p # Set addition
            PrunedPathScore[p] = PathScore[p]
        end
    end

    return PrunedPathsWithTerminalBlank, PrunedPathsWithTerminalSymbol, PrunedBlankPathScore, PrunedPathScore
```

The overall effect of these steps:



BEAM SEARCH

```
Global PathScore = [], BlankPathScore = []

# First time instant: Initialize paths with each of the symbols,
# including blank, using score at time t=1
NewPathsWithTerminalBlank, NewPathsWithTerminalSymbol, NewBlankPathScore, NewPathScore =
                          InitializePaths(SymbolSet, y[:,0])

# Subsequent time steps
for t = 1:T
    # Prune the collection down to the BeamWidth
    PathsWithTerminalBlank, PathsWithTerminalSymbol, PathScore, BlankPathScore =
                  Prune(NewPathsWithTerminalBlank, NewPathsWithTerminalSymbol,
                                        NewBlankPathScore, NewPathScore, BeamWidth)
    # First extend paths by a blank
    NewPathsWithTerminalBlank, NewBlankPathScore = ExtendWithBlank
                                                                    )

    # Next extend paths by a symbol
    NewPathsWithTerminalSymbol, NewPathScore = ExtendWithSymbol(Pa
                                                        PathsWithT

end

# Merge identical paths differing only by the final blank
MergedPaths, FinalPathScore = MergeIdenticalPaths(NewPathsWithTerminalBlank, NewBlankPathScore
                                                  NewPathsWithTerminalSymbol, NewPathScore)

# Pick best path
BestPath = argmax(FinalPathScore) # Find the path with the best score
```

Why is the pruning here and not at the *end* of the loop?

Because we *don't* want to prune paths at the final time. This loses information.
Instead at the final time we will *merge* paths that represent the same symbol sequence

BEAM SEARCH

```
Global PathScore = [], BlankPathScore = []

# First time instant: Initialize paths with each of the symbols,
# including blank, using score at time t=1
NewPathsWithTerminalBlank, NewPathsWithTerminalSymbol, NewBlankPathScore, NewPathScore =
                           InitializePaths(SymbolSet, y[:,0])

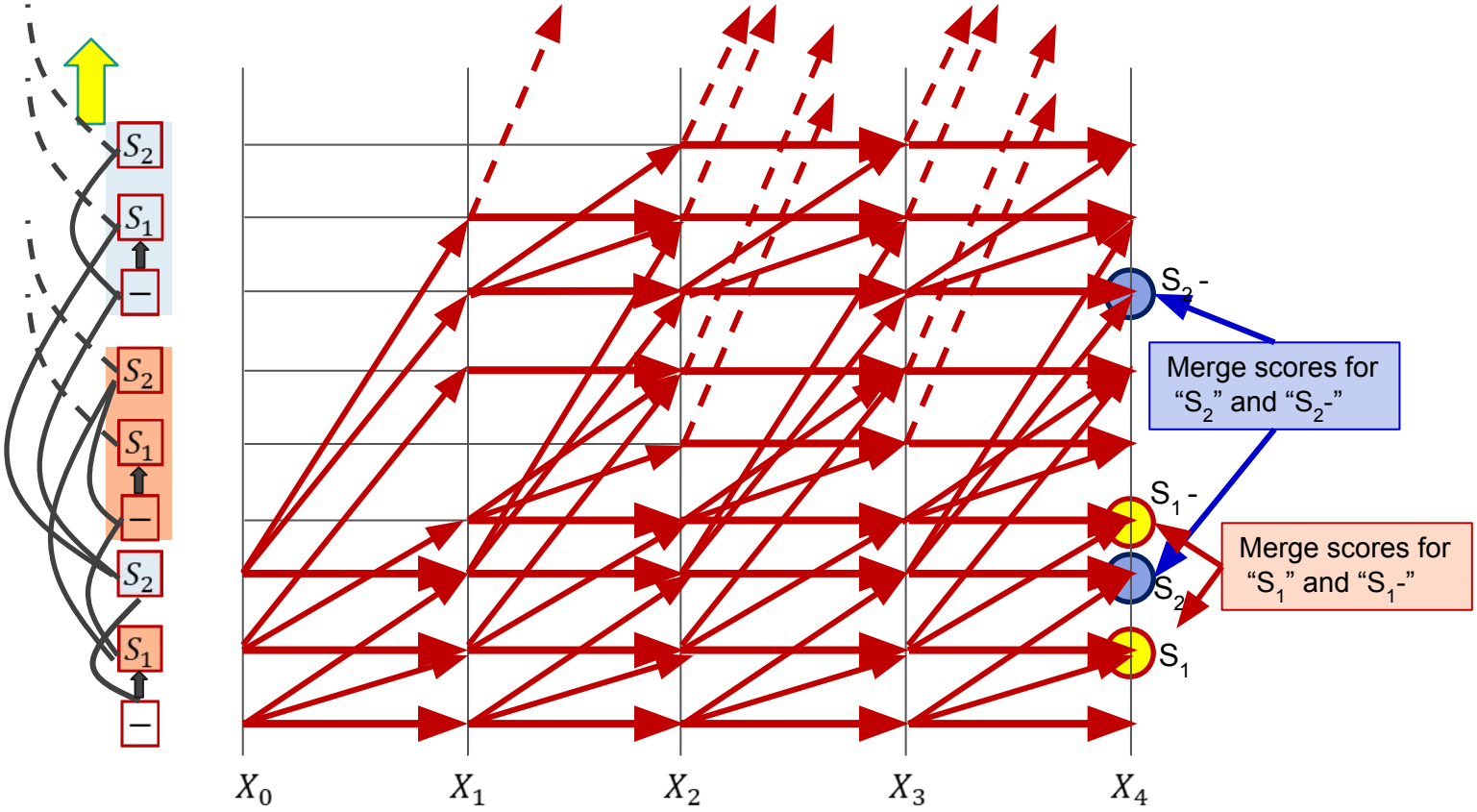
# Subsequent time steps
for t = 1:T
    # Prune the collection down to the BeamWidth
    PathsWithTerminalBlank, PathsWithTerminalSymbol, PathScore, BlankPathScore =
                  Prune(NewPathsWithTerminalBlank, NewPathsWithTerminalSymbol,
                                                   NewBlankPathScore, NewPathScore, BeamWidth)
    # First extend paths by a blank
    NewPathsWithTerminalBlank, NewBlankPathScore = ExtendWithBlank(PathsWithTerminalBlank,
                                                                   PathsWithTerminalSymbol, y[:,t])

    # Next extend paths by a symbol
    NewPathsWithTerminalSymbol, NewPathScore = ExtendWithSymbol(PathsWithTerminalBlank,
                                                        PathsWithTerminalSymbol, SymbolSet, y[:,t])

end

# Merge identical paths differing only by the final blank
MergedPaths, FinalPathScore = MergeIdenticalPaths(NewPathsWithTerminalBlank, NewBlankPathScore
                                                  NewPathsWithTerminalSymbol, NewPathScore)

# Pick best path
BestPath = argmax(FinalPathScore) # Find the path with the best score
```


S_2
S_1
–
S_2
S_1
–
S_2
S_1
–
X_0
X_1
X_2
X_3
X_4
S_2 -
Merge scores for "S_2" and "S_2-"
S_1 -
Merge scores for "S_1" and "S_1-"
S_2
S_1

BEAM SEARCH: Merging final paths

```
Global PathScore, BlankPathScore

function MergeIdenticalPaths(PathsWithTerminalBlank, PathsWithTerminalSymbol)

    # All paths with terminal symbols will remain
    MergedPaths = PathsWithTerminalSymbol
    FinalPathScore = PathScore

    # Paths with terminal blanks will contribute scores to existing identical paths from
    # PathsWithTerminalSymbol if present, or be included in the final set, otherwise
    for p in PathsWithTerminalBlank
        if p in MergedPaths
            FinalPathScore[p] += BlankPathScore[p]
        else
            MergedPaths += p# Set addition
            FinalPathScore[p] = BlankPathScore[p]
        end
    end

    return MergedPaths, FinalPathScore
```

BEAM SEARCH

```
Global PathScore = [], BlankPathScore = []

# First time instant: Initialize paths with each of the symbols,
# including blank, using score at time t=1
NewPathsWithTerminalBlank, NewPathsWithTerminalSymbol, NewBlankPathScore, NewPathScore =
                           InitializePaths(SymbolSet, y[:,0])

# Subsequent time steps
for t = 1:T
    # Prune the collection down to the BeamWidth
    PathsWithTerminalBlank, PathsWithTerminalSymbol, PathScore, BlankPathScore =
                  Prune(NewPathsWithTerminalBlank, NewPathsWithTerminalSymbol,
                                                   NewBlankPathScore, NewPathScore, BeamWidth)
    # First extend paths by a blank
    NewPathsWithTerminalBlank, NewBlankPathScore = ExtendWithBlank(PathsWithTerminalBlank,
                                                                   PathsWithTerminalSymbol, y[:,t])

    # Next extend paths by a symbol
    NewPathsWithTerminalSymbol, NewPathScore = ExtendWithSymbol(PathsWithTerminalBlank,
                                                        PathsWithTerminalSymbol, SymbolSet, y[:,t])

end

# Merge identical paths differing only by the final blank
MergedPaths, FinalPathScore = MergeIdenticalPaths(NewPathsWithTerminalBlank, NewBlankPathScore
                                                  NewPathsWithTerminalSymbol, NewPathScore)

# Pick best path
BestPath = argmax(FinalPathScore) # Find the path with the best score
```