GANs (Generative Adversarial Networks)

By Yash Belhe, Hao Liang

Agenda

- Generative models
- Revisiting GANs
- WGAN
- WGAN-Gradient penalty (WGANGP)
    - Code walk through GANS, WGAN, WGANGP
- Cycle GAN
    - Code walk through Cycle GAN
- STAR GAN
    - Code walk through STAR GAN

Generative Models

Basic idea is to learn the underlying distribution of the data and generate more samples for the distribution.

Some examples of generative models

- Probabilistic Graphical Models
- Bayesian Networks
- Variational Autoencoder
- Generative Adversarial Networks

Generative Models

- Unknown distribution P_r (r for real)
- Known distribution P_θ
- Two approaches
  - Optimise P_θ to estimate P_r
  - Learn a function $g_\theta(Z)$ which transforms Z into P_θ

Approach 1: Optimise P_θ to estimate P_r

- Maximum Likelihood Estimation (MLE) : $max_{\theta \epsilon R^d} \frac{1}{m} \sum_{i=1}^{m} log P_\theta(x^{(i)})$
  - This is same as minimizing the KL divergence
- Kullback-Leibler (KL) divergence: $KL(P||Q) = \int_x log(\frac{P(x)}{Q(x)}) P(x) dx$
- Issue: Exploding of KL-divergence for zero values of P_θ
  - Add random noise to P_θ

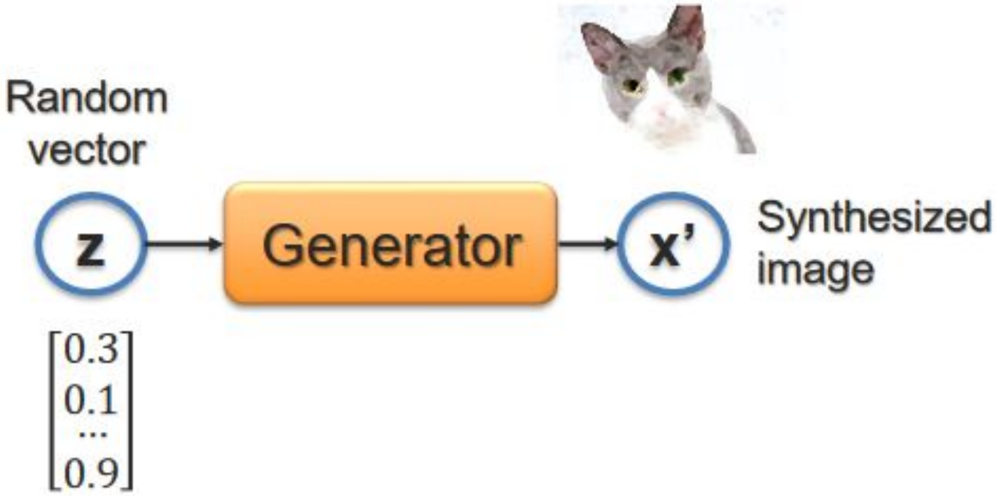
Approach 2: Learn a function $g_{\theta}(z)$

- We learn a function $g_{\theta}(z)$ that transforms z into P_{θ}
  - Z is a known distribution like Uniform or Gaussian
- We train g_{θ} by minimizing the distance between g_{θ} and P_r
- Any of the distance metrics like KL divergence, JS divergence or Earth Mover (EM) distance can be used.

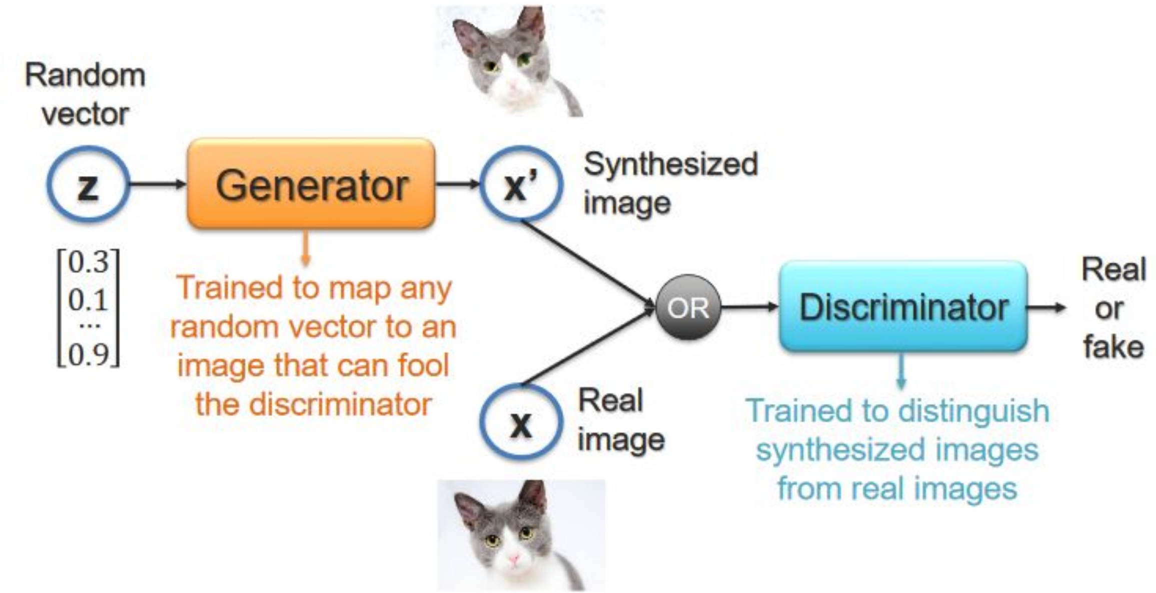
Revisiting GANs

- GANs are generative models which try to understand underlying distribution to generate more sample.
- GANs typically have 2 networks trained in an adversarial fashion.
    - Generator
    - Discriminator

Revisiting GANs- Generative Network



Revisiting GANs- Generator + Discriminator

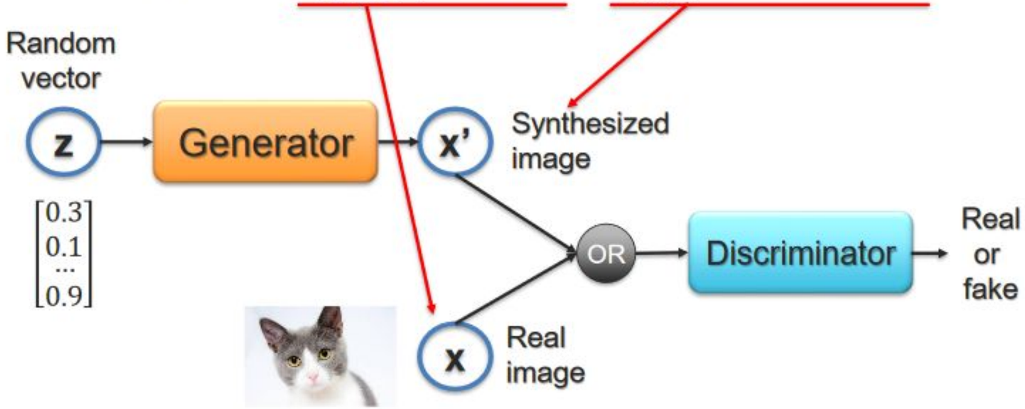


Revisiting GANs - training

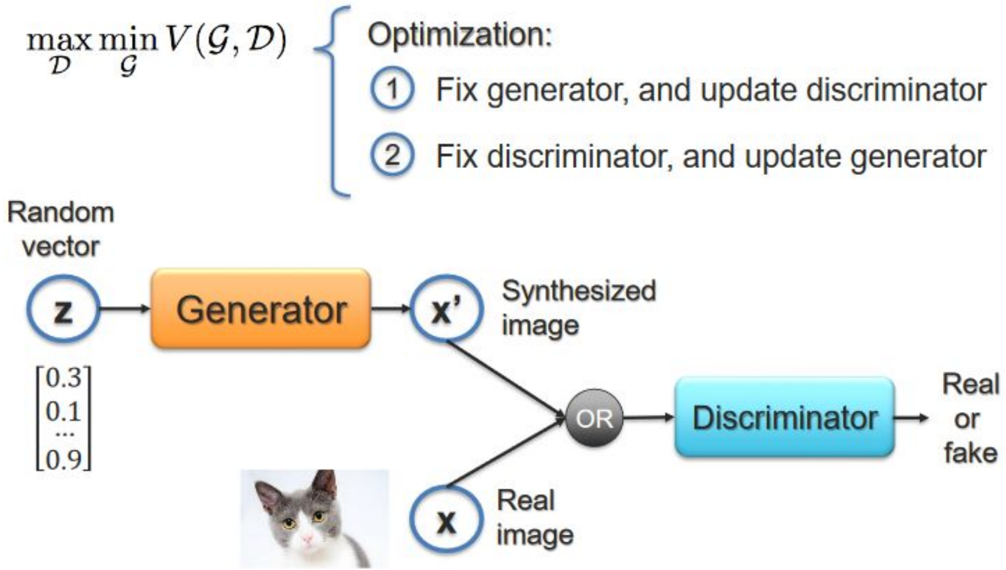
$$\max_{\mathcal{D}} \min_{\mathcal{G}} V(\mathcal{G}, \mathcal{D})$$

How do we optimize this objective function?

$$V(\mathcal{G}, \mathcal{D}) = \mathbb{E}_{p_{data}(\mathbf{x})} \log \mathcal{D}(\mathbf{x}) + \mathbb{E}_{p_g(\mathbf{x})} \log(1 - \mathcal{D}(\mathbf{x}))$$



Revisiting GANs - training

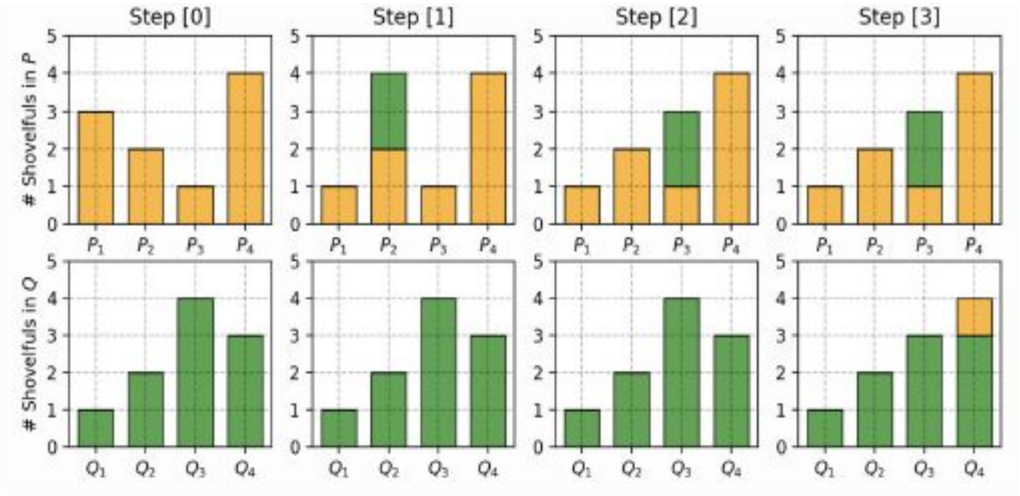


WGANs-Earth Mover Distance

Wasserstein distance: the minimum energy cost of moving and transforming a pile of dirt in the shape of one probability distribution to the the shape of other distribution.

P and Q: 4 piles of dirt made up of 10 shovelfuls of dirt present.

- P1 = 3, P2 = 2, P3 = 1, P4 = 4
- Q1 = 1, Q2 = 2, Q3 = 4, Q4 = 3
- W = 5



WGANs-Objective function

- We train GANs using this wasserstein distance.
- Discriminative is no more a direct critic. It is trained to estimate the wasserstein distance between real and generated data.

$$L_D = E_X D(X) - E_Z D(G(Z))$$

- Lipschitz is clipped to 1 i.e. $|f(x) - f(y)|/(x-y) <= 1$
    - This bound on discriminator is not good, instead we clip the gradients.

WGAN-Gradient Penalty

- Bound on discriminator is not great and leads to poor discriminator.
- We can add the gradient penalty in the loss function making sure that the lipschitz is almost 1 everywhere.

$$L_D = E_X D(X) - E_Z D(G(Z)) + \lambda E_{X'}(\|\nabla D(X')\|_2 - 1)^2$$

- *We do not constraint the gradients everywhere.*
  - *We penalize where there is linear interpolation between real and fake data.*

Code Walkthrough

GANs, WGAN-GP

Image translation

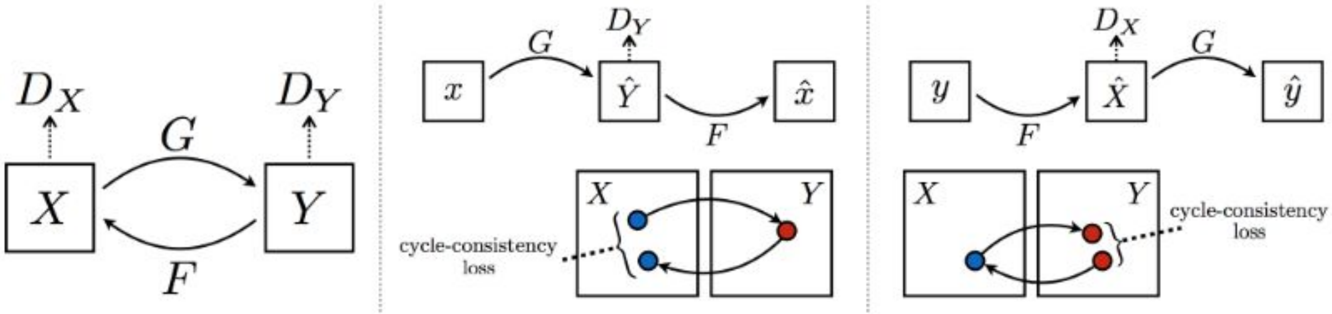
- Image-to-image translation involves generating a new synthetic version of a given image.
- Example: Changing a summer landscape --> winter landscape, blonde --> black hair, image --> painting.
- Data for such image translation is very limited or sometimes difficult to generate.
- 2 variants of GANs are used for this specific task.
  - Cycle GAN
  - STAR GAN

Cycle GANs

- Instead of a single Generator-Discriminator we have two Generators and discriminators.
    - One generator takes images from the first domain and outputs images from the second domain.
    - Discriminator models are used to determine how plausible generated images are and update the generator accordingly.
- The overall loss function for the cycle GAN is given below apart from the standard objective we have an added cycle-consistency loss.

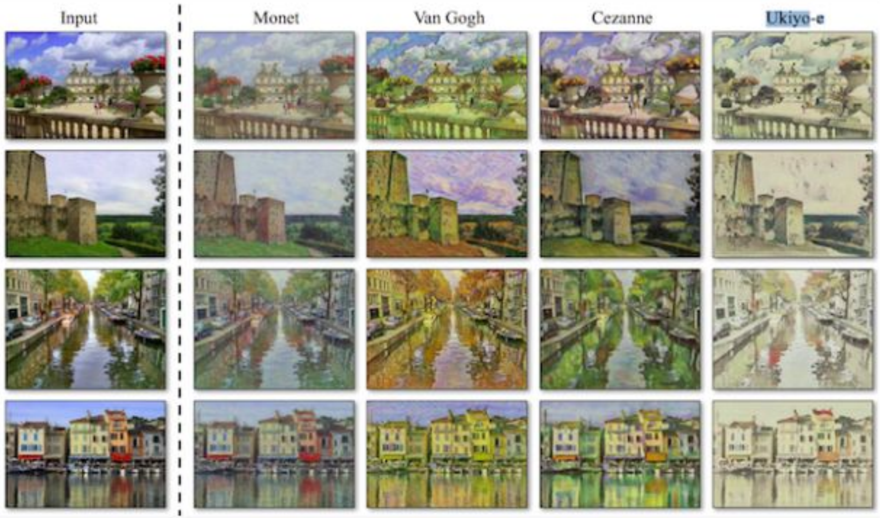
$$\begin{aligned}\mathcal{L}(G, F, D_X, D_Y) =& \mathcal{L}_{\text{GAN}}(G, D_Y, X, Y) \\ &+ \mathcal{L}_{\text{GAN}}(F, D_X, Y, X) \\ &+ \lambda \mathcal{L}_{\text{cyc}}(G, F),\end{aligned}$$

Cycle GAN



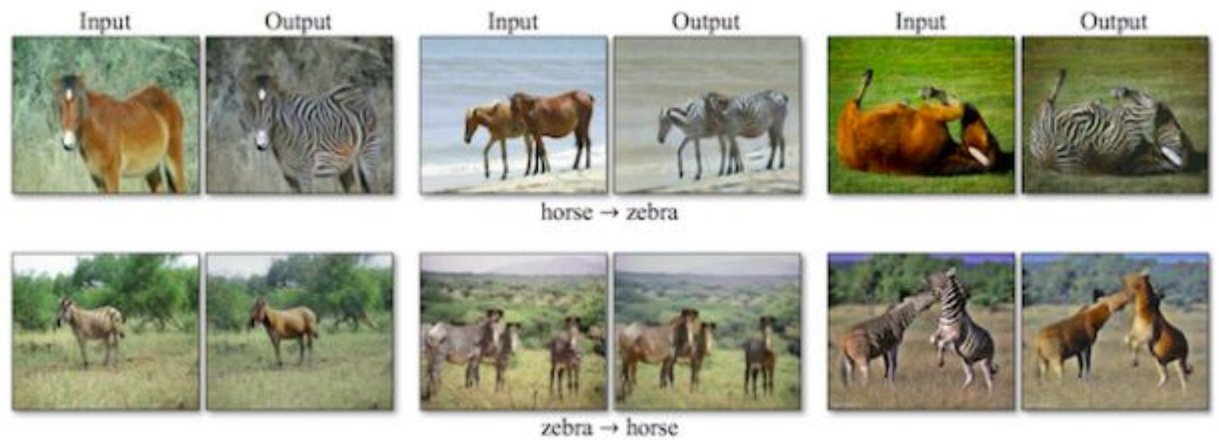
Cycle-consistency loss: $\mathcal{L}_{\text{cyc}}(G, F) = \mathbb{E}_{x \sim p_{\text{data}}(x)}[\|F(G(x)) - x\|_1] + \mathbb{E}_{y \sim p_{\text{data}}(y)}[\|G(F(y)) - y\|_1].$

Application: Style Transfer



Example of Style Transfer from Famous Painters to Photographs of Landscapes.
Taken from: Unpaired Image-to-Image Translation Using Cycle-Consistent Adversarial Networks.

Application: Object Transfiguration

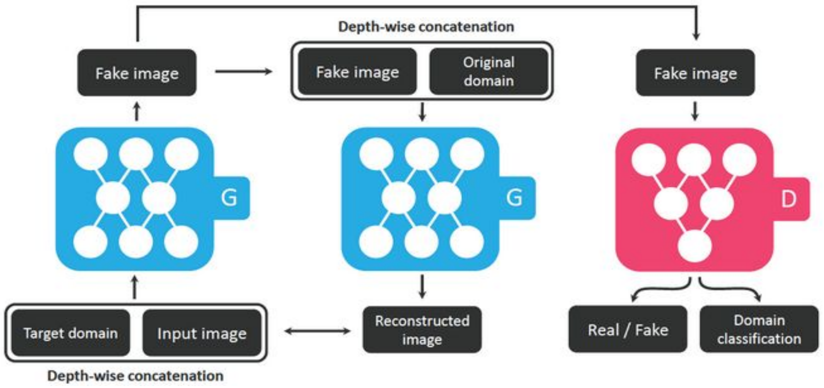


Example of Object Transfiguration from Horses to Zebra and Zebra to Horses.
Taken from: Unpaired Image-to-Image Translation Using Cycle-Consistent Adversarial Networks.

Star GAN (Unified GAN for Multi-Domain I2I translation

- Star GAN helps us to generate images in target domain given an input and target domain.
    - Image of a man and target domain is gender.
    - Image of a person and target domain is age.
- We train the generator-discriminator in adversarial fashion with an added auxiliary classifier.
- Along with normal adversarial loss this loss is added while training the generator and discriminator.

Star GAN - Generator



- Generator have 3 objectives:
  - Tries to generate realistic images
  - The weights of generator are adjusted so that the generated images are classified as target domain by the discriminator.
  - Construct original image from the fake image given the original label domain label.

Objective function:

$$\mathcal{L}_G = \mathcal{L}_{adv} + \lambda_{cls}\,\mathcal{L}_{cls}^{f} + \lambda_{rec}\,\mathcal{L}_{rec}$$

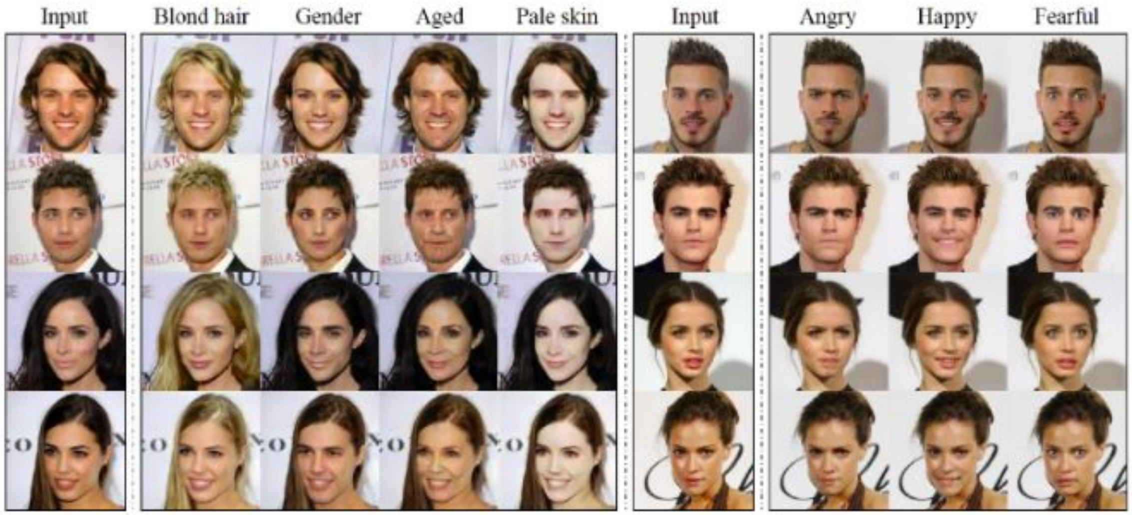
Star GAN - Discriminator

- Discriminator has 2 objectives:
  - Whether the image is fake or real
  - What is the domain in which the image belongs.
- If the generator is able to generate fool the discriminator then discriminator would predict the target domain and we stop training.

Objective function:

$$\mathcal{L}_D = -\mathcal{L}_{adv} + \lambda_{cls}\, \mathcal{L}^r_{cls}$$

Applications



Thank You!

Thank You!

Slow and steady wins the race is a lie, so pace up: Amit

Code Walkthrough

Cycle GAN and STAR GAN

References

- https://arxiv.org/abs/1701.07875 (Wasserstein GAN)
- https://arxiv.org/abs/1703.10593 (Cycle GAN)
- https://arxiv.org/abs/1711.09020 (Star GAN)
- https://machinelearningmastery.com/what-is-cyclegan/
- https://towardsdatascience.com/stargan-image-to-image-translation-44d4230fbb48
- Lecture notes of 11-777

GANs - Code Walkthrough

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GAN Loss Function

Some Notation:

$p(x)$ – **The distribution over all possible real images that we want to model**

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Real Image Label - 1

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$$\mathscr{L}_{GAN} = \min_{G} \max_{D} \mathbb{E}_{x \sim p(x)}[\log(D(x))] + \mathbb{E}_{z \sim p(z)}[\log(1 - D(G(z)))]$$

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Let $\mathscr{X}$ be a minibatch of samples drawn from $p(x), |\mathscr{X}| = N$

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Generator Loss

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Rough Code Implementation
(full code link)

```
G = generator()
D = discriminator()

bce_loss = nn.BCELoss()
D_optimizer = optim.Adam(D.parameters())
G_optimizer = optim.Adam(G.parameters())

z = get_noise()
x = get_real()

D_real_loss = bce_loss(D(x), torch.ones(batch_size))
D_fake_loss = bce_loss(D(G(z)), torch.zeros(batch_size))

D_loss = D_real_loss + D_fake_loss
D_loss.backward()
D_optimizer.step()

G_loss = bce_loss(D(G(z)), torch.ones(batch_size))
G_loss.backward()
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W-GAN

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Where $\|D\|_L \leq K,$ **i.e D is K-Lipschitz Continuous**

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- Measures the Wasserstein/ Earth Mover Distance between two distributions

How To Enforce K-Lipschitz Continuity for the Discriminator?

- Heuristic: Clip each weight w of the discriminator s.t $|w| < c$

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- Heuristic: Clip each weight w of the discriminator s.t $|w| < c$
- Is this a good way of maintaining Lipschitz Continuity - No
- Does it work? Somewhat

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$$\mathscr{L}_D = \max_D \mathbb{E}_{x\sim p_r}[D(x)] - \mathbb{E}_{z\sim p_r(z)}[D(G(z))]$$

$$\mathscr{L}_D = \max_D \frac{1}{N}\sum_{x\in\mathscr{X}} D(x) - \frac{1}{N}\sum_{z\in Z} D(G(z))$$

$$\mathscr{L}_D = \min_D \left[-\frac{1}{N}\sum_{x\in\mathscr{X}} D(x) + \frac{1}{N}\sum_{z\in Z} D(G(z))\right]$$

W-GAN Discriminator Loss

$$\mathscr{L}_D = \max_D \mathbb{E}_{x \sim p_r}[D(x)] - \mathbb{E}_{z \sim p_r(z)}[D(G(z))]$$

$$\mathscr{L}_D = \max_D \frac{1}{N} \sum_{x \in \mathscr{X}} D(x) - \frac{1}{N} \sum_{z \in Z} D(G(z))$$

$$\mathscr{L}_D = \min_D \left[-\frac{1}{N} \sum_{x \in \mathscr{X}} D(x) + \frac{1}{N} \sum_{z \in Z} D(G(z)) \right]$$

```
D_loss =  -D(x).mean() + D(G(z)).mean()
```

W-GAN Discriminator Loss

$$\mathscr{L}_D = \max_D \mathbb{E}_{x \sim p_r}[D(x)] - \mathbb{E}_{z \sim p_r(z)}[D(G(z))]$$

$$\mathscr{L}_D = \max_D \frac{1}{N} \sum_{x \in \mathcal{X}} D(x) - \frac{1}{N} \sum_{z \in Z} D(G(z))$$

$$\mathscr{L}_D = \min_D \left[-\frac{1}{N} \sum_{x \in \mathcal{X}} D(x) + \frac{1}{N} \sum_{z \in Z} D(G(z)) \right]$$

```
D_loss =  -D(x).mean() + D(G(z)).mean()
```

For Lipschitz Continuity:

W-GAN Discriminator Loss

$$\mathscr{L}_D = \max_D \mathbb{E}_{x \sim p_r}[D(x)] - \mathbb{E}_{z \sim p_r(z)}[D(G(z))]$$

$$\mathscr{L}_D = \max_D \frac{1}{N} \sum_{x \in \mathscr{X}} D(x) - \frac{1}{N} \sum_{z \in Z} D(G(z))$$

$$\mathscr{L}_D = \min_D \left[-\frac{1}{N} \sum_{x \in \mathscr{X}} D(x) + \frac{1}{N} \sum_{z \in Z} D(G(z)) \right]$$

```
D_loss =  -D(x).mean() + D(G(z)).mean()
```

For Lipschitz Continuity:

```
for p in D.parameters():
    p.data.clamp_(-c, c)
```

W-GAN Generator Loss

$$\mathscr{L}_D = \min_{G} \mathbb{E}_{x \sim p_r}[D(x)] - \mathbb{E}_{z \sim p_r(z)}[D(G(z))]$$

W-GAN Generator Loss

$$\mathscr{L}_D = \min_G \mathbb{E}_{x \sim p_r}[D(x)] - \mathbb{E}_{z \sim p_r(z)}[D(G(z))]$$

$$\mathscr{L}_D = \min_G - \frac{1}{N} \sum_{z \in Z} D(G(z))$$

W-GAN Generator Loss

$$\mathscr{L}_D = \min_G \mathbb{E}_{x \sim p_r}[D(x)] - \mathbb{E}_{z \sim p_r(z)}[D(G(z))]$$

$$\mathscr{L}_D = \min_G -\frac{1}{N} \sum_{z \in Z} D(G(z))$$

```
G_loss = -D(G(z)).mean()
```

Rough Code Implementation

(full code link)

```
G = generator()
D = discriminator()

c = 0.01 #Some small number

D_optimizer = optim.Adam(D.parameters())
G_optimizer = optim.Adam(G.parameters())

z = get_noise()
x = get_real()

D_loss =  -D(x).mean() + D(G(z)).mean()
D_loss.backward()
D_optimizer.step()

for p in D.parameters():
    p.data.clamp_(-c, c)

G_loss = -D(G(z)).mean()
G_loss.backward()
G_optimizer.step()
```