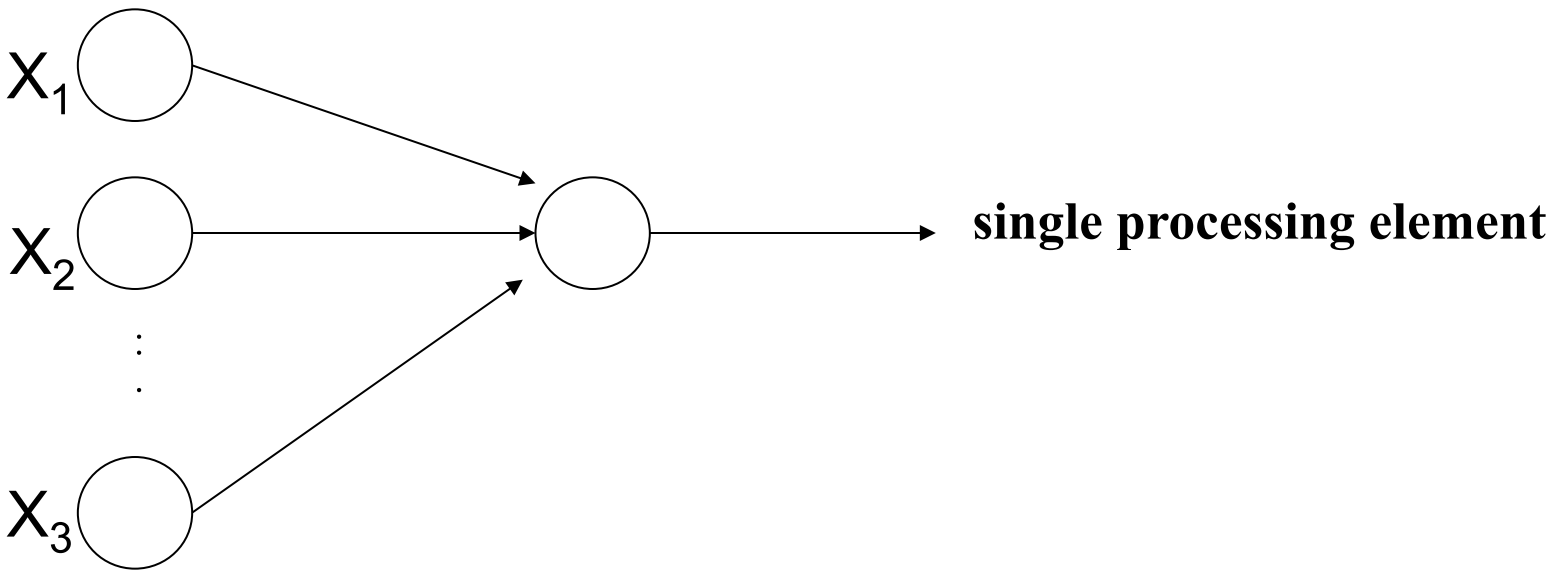
-Artificial Neural Network-

ADALINE and MADALINE

ADALINE

- ADALINE (Adaptive Linear Neuron) is a network model proposed by Bernard Widrow in 1959.



Training Rule

- The activation function used is
  y = 1 if y_in ≥ 0
  y = -1 if y_in < 0.
- The training rule is called the Widrow-Hoff rule or the Delta Rule
- It can be theoretically shown that the rule minimizes the root mean square error between the activation value and the target value.
- That's why it's called the the Least Mean Square (LMS) rule as well.

The δ Rule

The δ rule works also works for more than one output unit.

The δ Rule

Consider one single output unit.

The delta rule changes the weights of the neural connections so as to minimize the difference between the net input to the output unit *y_in* and the target value *t*.

The goal is to minimize the error over all training patterns.

However, this is accomplished by reducing the error to each pattern one at a time.

Weight corrections can also be accumulated over a number of training patterns (called *batch updating*) if desired.

The Training Algorithm

```
Initialize weights to small random values
Set learning rate α to a value between 0 and 1
while (the largest weight change ≤ threshold) do
        for each bipolar training pair s:t do
                {Set activation of input units i=1..n {xi = si}
                Compute net input to the output unit:
                        y_in = b + Σx_i w_i
                Update bias and weights:
                  for i=1..n {
                        b(new) = b(old) + α(t – y_in)
                        w_i(new) = w_i(old) + α (t – y_in)x_i}
                } //endfor
  }//end while
```

Setting Learning Parameter α

- Usually, just use a small value for α, something like 0.1.
- If the value is too large, the learning process will not converge.
- If the value of α is too small, learning will be extremely slow (Hecht-Nielsen 1990).
- For a single neuron, a practical range for α is $0.1 \leq n \times \alpha \leq 1.0$, where n is the number of input units (Widrow, Winger and Baxter 1988).

MADALINE

- When several ADALINE units are arranged in a single layer so that there are several output units, there is no change in how ADALINEs are trained from that of a single ADALINE.
- A MADALINE consists of many ADALINEs arranged in a multi-layer net.
- We can think of a MADALINE as having a hidden layer of ADALINEs.

MADALINE (Many Adalines)

- A Madaline is composed of several Adalines
- Each ADALINE unit has a bias. There are two hidden ADALINEs, z1 and z2. There is a single output ADALINE Y.
- Each ADALINE simply

applies a threshold function

to the unit's net input.

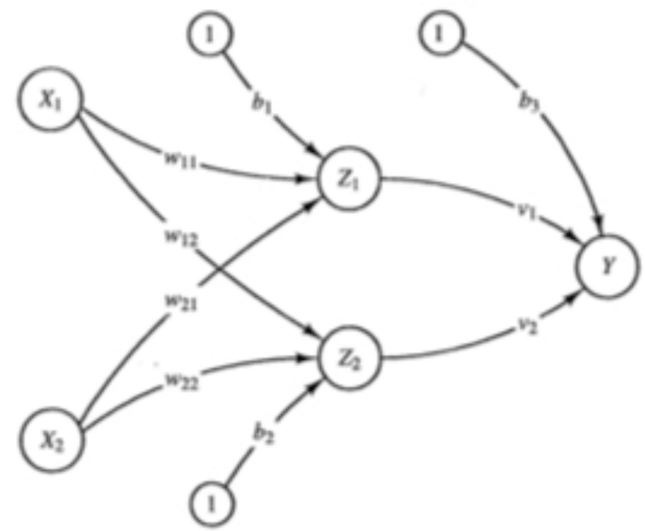
Y is a non-linear function of

the input vector (x1, x2).

The use of hidden units Z1 and Z2

gives the net additional power, but

makes training more complicated.



MADALINE Training

There are two training algorithms for a MADALINE with one hidden layer.

Algorithm *MR-I* is the original MADALINE training algorithm (Widrow and Hoff 1960).

MR-I changes the weights on to the hidden ADALINEs only. The weights for the output unit are fixed. It assumes that the output unit is an OR unit.

MR-II (Widrow, Winter and Baxter 1987) adjusts all weights in the net. It doesn't make the assumption that the output unit is an OR unit.

MR-I Training Algorithm

Determine the weights of units (here, v1, v2 and bias b3) such that the output unit Y behaves like an OR unit.

In other words, Y is 1 if the Z1 or Z2 (or both) is (are) 1; Y is -1 if both Z1 and Z2 are -1.

Here a weight of ½ on each of v1, v2 and v3 works.

The weights on the hidden ADALINEs are adjusted according to MR-I algorithm.

In this example, weights on the first ADALINE (w11 and w21) and weights on the second ADALINE (w12 and w22) are adjusted according to MR-I algorithm.

Remember the activation function is

$$f(x) = \begin{cases} 1 & \text{if } x \geq 0 \\ -1 & \text{if } x < 0 \end{cases}$$

MR-I Training Algorithm

Set learning parameter α   //Assume bipolar units and outputs. Only 1 hidden layer.

while stopping condition is false **do**

for **each** bipolar training pair **s**:**t do**

Set activation of input units i = 1 to n { xi = si }

Compute net input to hidden units, e.g., zin1 = b1 + x1 w11 + x2 w21

Determine output of each hidden ADALINE, e.g., z1 = f (z_in1)

Determine output of net: yin = b3 + z1 v1 + z2 v2; y = f(yin)

//Determine error and update weights

if t=y, **then** no updates are performed //no errror

if t=1, //error, the expected output is 1, the computed output is -1; at least one of the Z's should be 1

then update weights on Z_J, the unit whose net input is closest to 1 (or closest 0, both are the same)

b_J (new) = b_J (old) + α (1 – z_inJ)

w_iJ (new) = w_iJ (old) + α (1 – z_inJ) xi

if t=-1, **then** update weights on all units Z_k that have positive net input//error

endfor

endwhile

Stopping criterion: Weight changes have stopped or reached an acceptable level or after a certain number of iterations.

MR-I Training Algorithm

Motivation for performing updates: Update weights only if an error has occurred.

Update weights in such a way that it is more likely for the net to produce the desired response.

If t=1 and error has occurred (i.e., y=-1, or the OR unit is off when it should actually be on): It means that all Z units had value -1 and at least one Z unit needs to have value +1. Therefore, we consider Z_J to be the unit whose net input is closest to 0 and adjust its weights.

If t=-1 and error has occurred (i.e., y=1 or the OR unit is on when it should actually be off): It means that at least one Z unit had value +1 and all Z units must have value -1. Therefore, we adjust the weights on all of the Z units with positive net input.

Example of Use of MRI

- Solving the XOR problem using MRI
- The training patterns are:

x_1	x_2	t
1	1	−1
1	−1	1
−1	1	1
−1	−1	−1

Step 0.
The weights into Z_1 and into Z_2 are small random values; the weights into Y are those found in Example 2.19. The learning rate, α, is .5.

Weights into Z_1			Weights into Z_2			Weights into Y		
w_{11}	w_{21}	b_1	w_{12}	w_{22}	b_2	v_1	v_2	b_3
.05	.2	.3	.1	.2	.15	.5	.5	.5

Step 1. Begin training.

Step 2. For the first training pair, (1, 1): − 1

Step 3. $x_1 = 1, \quad x_2 = 1$

Step 4. $z\_in_1 = .3 + .05 + .2 = .55,$

$z\_in_2 = .15 + .1 + .2 = .45.$

Madaline Training for XOR Using MR1 Algorithm

Step5. $z_1 = 1,$
$z_2 = 1.$

Step 6. $y\_in = .5 + .5 + .5;$
$y = 1.$

Step 7. $t - y = -1 - 1 = -2 \neq 0$, so an error occurred.

Since $t = -1$, and both Z units have positive net input, update the weights on unit Z_1 as follows:

$$b_1(\text{new}) = b_1(\text{old}) + \alpha(-1 - z\_in_1)$$
$$= .3 + (.5)(-1.55)$$
$$= -.475$$
$$w_{11}(\text{new}) = w_{11}(\text{old}) + \alpha(-1 - z\_in_1)x_1$$
$$= .05 + (.5)(-1.55)$$
$$= -.725$$
$$w_{21}(\text{new}) = w_{21}(\text{old}) + \alpha(-1 - z\_in_1)x_2$$
$$= .2 + (.5)(-1.55)$$
$$= -.575$$

update the weights on unit Z_2 as follows:

$$b_2(\text{new}) = b_2(\text{old}) + \alpha(-1 - z\_in_2)$$
$$= .15 + (.5)(-1.45)$$
$$= -.575$$
$$w_{12}(\text{new}) = w_{12}(\text{old}) + \alpha(-1 - z\_in_2)x_1$$
$$= .1 + (.5)(-1.45)$$
$$= -.625$$
$$w_{22}(\text{new}) = w_{22}(\text{old}) + \alpha(-1 - z\_in_2)x_2$$
$$= .2 + (.5)(-1.45)$$
$$= -.525$$

After four epochs of training, the final weights are found to be:

$$w_{11} = -0.73 \qquad w_{12} = 1.27$$
$$w_{21} = 1.53 \qquad w_{22} = -1.33$$
$$b_1 = -0.99 \qquad b_2 = -1.09$$

Geometric Interpretation of Madaline MR1 weights

- The positive response region for the Madaline trained in the previous example is the union of the regions where each of the hidden units have a positive response.
- The decision boundary for each hidden unit can be calculated as described in Section 2.1.3 of Fausette's book.

For hidden unit Z_1, the boundary line is

$$x_2 = -\frac{w_{11}}{w_{21}} x_1 - \frac{b_1}{w_{21}}$$

$$= \frac{0.73}{1.53} x_1 + \frac{0.99}{1.53}$$

$$= 0.48\, x_1 + 0.65$$

For hidden unit Z_2, the boundary line is

$$x_2 = -\frac{w_{12}}{w_{22}} x_1 - \frac{b_2}{w_{22}}$$

$$= \frac{1.27}{1.33} x_1 + \frac{1.09}{1.33}$$

$$= 0.96\, x_1 - 0.82$$

Geometric Interpretation of Madaline MR1 weights

- We see the positive response regions for Z1 and Z2, and then the positive response region for the output Y unit which is the intersection of the two Z1 and Z2 regions.

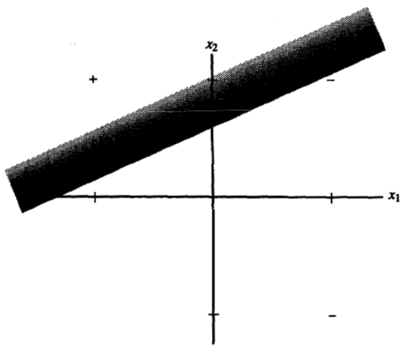


Figure 2.25 Positive response region for Z_1.

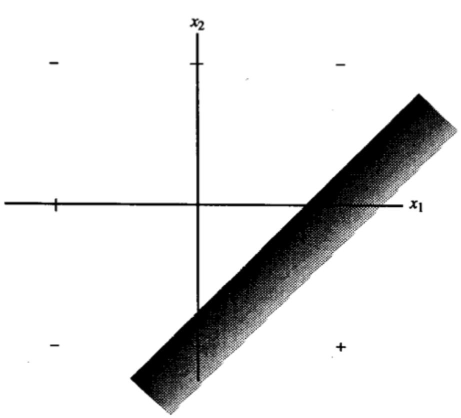


Figure 2.26 Positive response region for Z_2.

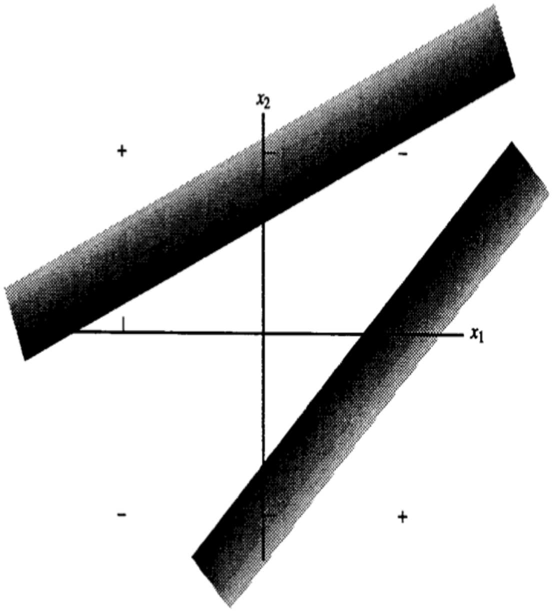


Figure 2.27 Positive response region for MADALINE for XOR function.

MR-II Training Algorithm

There is no assumption that the output unit acts as a logical OR.

The goal is to change weights in all layers of the net, i.e., in all hidden layers when we have several hidden layers + output layer.

But, we also want to cause *the least disturbance* in the net so that it remains stable from iteration to iteration.

This causes least "unlearning" of the patterns for which the net has been trained previously.

This is sometimes called the "don't rock the boat" principle.

Several output nodes may be used; the total error for any input pattern is the sum of the squares of the errors at each output unit.

MR-II Training Algorithm

The MR-II algorithm is considerably different from the back-propagation algorithm we will learn later.

The weights are initialized to small random values and training patterns are presented repeatedly *in epochs*.

The algorithm modifies the weights for the nodes in hidden layer=1, then layer=2, .. up to the output layer.

The training algorithm is a trial-and-error procedure following the minimum disturbance principle.

Nodes that can affect the output error incurring the least change in their weights have precedence in the learning process.

MR-II Training Algorithm

Set learning rate α
while stopping condition is false **do**
  for each bipolar training pair **s**:**t** **do**
    Compute output of the net based on current weights and activation function
      if t≠y, **then for each** unit whose net input is sufficiently close to 0
      (say, between -α and α, with α=0.25) **do**
        {Sort *all such* units in the network *at all levels* based on their net input values.
        Start with the unit whose net is closest to 0, then for the next closest, etc.
        Change the unit's output from +1 to -1, or vice versa
        If modifying the output of this node improves network performance
            (i.e., reduces error on test set)
        then //if the error is not reduced, undo the reversal
          adjust the weights on this unit to achieve the output reversal} //how to do is not given
  endfor
endwhile

Stopping criterion: Weight changes have stopped or reached an acceptable level or after a certain number of iterations.

MR-II Training Algorithm

```
Algorithm MRII;
  repeat
    Present a training pattern i to the network;
    Compute outputs of all hidden nodes and the output node;
      Let h = 1;
      while the pattern is misclassified
                and h ≤ the number of hidden layers, do
          Sort the Adalines in layer h, in the order of
            increasing net input magnitude (|∑_j w_j i_j|),
            but omitting nodes for which |∑_j w_j i_j| > θ,
            where θ is a predetermined threshold;
          Let S = (A_1, ..., A_k) be the sorted sequence;
          while network output differs from desired output,
                  and S contains nodes not yet examined
                  in this iteration, do
              if reversing output of the next element A_j ∈ S
                  can improve network performance,
              then Modify connection weights leading into A_j
                  to accomplish the output reversal;
              end-if;
          end-while
          h := h + 1;
      end-while
  until performance is considered satisfactory or the upper
    bound on the number of iterations has been reached.
```