Deep Learning
Recurrent Networks : 1
Spring 2020

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Which open source project?

```c
/* Increment the size file of the new incorrect UI_FILTER group information */
/* of the size generatively. */

static int indicate_policy(void)
{
    int error;
    if (fd == MAREN_EPT) {
        /*
         * The kernel blank will coed it to userspace.
         */
        if (ss->segment < mem_total)
            unblock_graph_and_set_blocked();
        else
            ret = 1;
        goto bail;
    }
    segaddr = in_SB(in.addr);
    selector = seg / 16;
    setup_works = true;
    for (i = 0; i < blocks; i++) {
        seq = buf[i++];
        bpf = bd->bd.next + i * search;
        if (fd) {
            current = blocked;
        }
    }
    rw->name = "Getjbbregs";
    bprm_self_clear(&iv->version);
    regs->new = blocks[(BPF_STATS << info->historidac)] | PFMR_CLOBATHINC_SECON
    return segtable;
}
Related math. What is it talking about?

Proof. Omitted.

Lemma 0.1. Let \( \mathcal{C} \) be a set of the construction.
Let \( \mathcal{C} \) be a gerber covering. Let \( \mathcal{F} \) be a quasi-coherent sheaves of \( \mathcal{O} \)-modules. We have to show that

\[
\mathcal{O}_{\mathcal{O}_X} = \mathcal{O}_X(\mathcal{L})
\]

Proof. This is an algebraic space with the composition of sheaves \( \mathcal{F} \) on \( X_{\text{etale}} \) we have

\[
\mathcal{O}_X(\mathcal{F}) = \{ \text{morph} \times \mathcal{O}_X (\mathcal{G}, \mathcal{F}) \}
\]

where \( \mathcal{G} \) defines an isomorphism \( \mathcal{F} \to \mathcal{F} \) of \( \mathcal{O} \)-modules.

Lemma 0.2. This is an integer \( \mathcal{Z} \) is injective.

Proof. See Spaces, Lemma ??.

Lemma 0.3. Let \( \mathcal{S} \) be a scheme. Let \( \mathcal{X} \) be a scheme and \( \mathcal{X} \) is an affine open covering. Let \( \mathcal{U} \subset \mathcal{X} \) be a canonical and locally of finite type. Let \( \mathcal{X} \) be a scheme. Let \( \mathcal{X} \) be a scheme which is equal to the formal complex.

The following to the construction of the lemma follows.
Let \( \mathcal{X} \) be a scheme. Let \( \mathcal{X} \) be a scheme covering. Let

\[
b : \mathcal{X} \to \mathcal{Y} \to \mathcal{Y} \to \mathcal{Y} \to \mathcal{X}
\]

be a morphism of algebraic spaces over \( \mathcal{S} \) and \( \mathcal{Y} \).

Proof. Let \( \mathcal{X} \) be a nonzero scheme of \( \mathcal{X} \). Let \( \mathcal{X} \) be an algebraic space. Let \( \mathcal{F} \) be a quasi-coherent sheaf of \( \mathcal{O}_X \)-modules. The following are equivalent

1. \( \mathcal{F} \) is an algebraic space over \( \mathcal{S} \).
2. If \( \mathcal{X} \) is an affine open covering.

Consider a common structure on \( \mathcal{X} \) and \( \mathcal{X} \) the functor \( \mathcal{O}_X(U) \) which is locally of finite type.

This since \( \mathcal{F} \in \mathcal{F} \) and \( x \in \mathcal{G} \) the diagram

\[
\begin{array}{ccc}
S & \to & \mathcal{O}_X \\downarrow \\
\mathcal{G} & \to & \mathcal{O}_X \\
\mathcal{G} & \to & \mathcal{O}_X \\
\end{array}
\]

is a limit. Then \( \mathcal{G} \) is a finite type and assume \( \mathcal{S} \) is a flat and \( \mathcal{F} \) and \( \mathcal{G} \) is a finite type \( f_\bullet \). This is of finite type diagrams, and

- the composition of \( \mathcal{G} \) is a regular sequence,
- \( \mathcal{G}_X \) is a sheaf of rings.

Proof. We have see that \( \mathcal{X} \to \text{Spec}(\mathcal{R}) \) and \( \mathcal{F} \) is a finite type representable by algebraic space. The property \( \mathcal{F} \) is a finite morphism of algebraic stacks. Then the cohomology of \( \mathcal{X} \) is an open neighbourhood of \( \mathcal{U} \).

Proof. This is clear that \( \mathcal{G} \) is a finite presentation, see Lemmas ??.

A reduced above we conclude that \( \mathcal{U} \) is an open covering of \( \mathcal{C} \). The functor \( \mathcal{F} \) is a \( \mathcal{O}_X \)-field

\[
\mathcal{O}_{\mathcal{O}_X} \to \mathcal{F} \to \mathcal{O}_{\mathcal{O}_X}
\]

is an isomorphism of covering of \( \mathcal{O}_X \). If \( \mathcal{F} \) is the unique element of \( \mathcal{F} \) such that \( \mathcal{X} \) is an isomorphism.

The property \( \mathcal{F} \) is a disjoint union of Proposition ?? and we can filtered set of presentations of a scheme \( \mathcal{O}_X \)-algebra with \( \mathcal{F} \) are opens of finite type over \( \mathcal{S} \).

If \( \mathcal{F} \) is a scheme theoretic image points.

If \( \mathcal{F} \) is a finite direct sum \( \mathcal{O}_{\mathcal{X}_s} \) is a closed immersion, see Lemma ??, This is a sequence of \( \mathcal{F} \) is a similar morphism.
And a Wikipedia page explaining it all

Naturalism and decision for the majority of Arab countries' capitalide was grounded by the Irish language by [[John Clair]], [[An Imperial Japanese Revolt]], associated with Guangzham's sovereignty. His generals were the powerful ruler of the Portugal in the [[Protestant Immineners]], which could be said to be directly in Cantonese Communication, which followed a ceremony and set inspired prison, training. The emperor travelled back to [[Antioch, Perth, October 25|21]] to note, the Kingdom of Costa Rica, unsuccessful fashioned the [[Thrales]], [[Cynth's Dajoard]], known in western [[Scotland]], near Italy to the conquest of India with the conflict. Copyright was the succession of independence in the slop of Syrian influence that was a famous German movement based on a more popular servicious, non-doctrinal and sexual power post. Many governments recognize the military housing of the [[Civil Liberalization and Infantry Resolution 265 National Party in Hungary]], that is sympathetic to be the [[Punjab Resolution]] (PJS)[http://www.humah.yahoo.com/guardian.cfm/7754800786d17551963s89.htm Official economics Adjoint for the Nazism, Montgomery was swear to advance to the resources for those Socialism's rule, was starting to signing a major tripad of aid exile.]]
The unreasonable effectiveness of recurrent neural networks..

• All previous examples were *generated* blindly by a *recurrent* neural network..
  – With simple architectures

• [http://karpathy.github.io/2015/05/21/rnn-effectiveness/](http://karpathy.github.io/2015/05/21/rnn-effectiveness/)
Modern text generation is a lot more sophisticated than that.

- One of the many sages of the time, the Bodhisattva Bodhisattva Sakyamuni (1575-1611) was a popular religious figure in India and around the world. This Bodhisattva Buddha was said to have passed his life peacefully and joyfully, without passion and anger. For over twenty years he lived as a lay man and dedicated himself toward the welfare, prosperity, and welfare of others. Among the many spiritual and philosophical teachings he wrote, three are most important; the first, titled the "Three Treatises of Avalokiteśvara"; the second, the teachings of the "Ten Questions;" and the third, "The Eightfold Path of Discipline."
  - Entirely randomly generated
Modelling Series

• In many situations one must consider a *series* of inputs to produce an output
  – Outputs too may be a series

• Examples: ..
What did I say?

“To be” or not “to be”??

• Speech Recognition
  – Analyze a series of spectral vectors, determine what was said
• Note: Inputs are sequences of vectors. Output is a classification result
The Steelers, meanwhile, continue to struggle to make stops on defense. They've allowed, on average, 30 points a game, and have shown no signs of improving anytime soon.

• Text analysis
  – E.g. analyze document, identify topic
    • Input series of words, output classification output
  – E.g. read English, output French
    • Input series of words, output series of words
Should I invest..

To invest or not to invest?

• Note: Inputs are sequences of vectors. Output may be scalar or vector
  – Should I invest, vs. should I not invest in X?
  – Decision must be taken considering how things have fared over time
These are classification and prediction problems

• Consider a sequence of inputs
  – Input vectors
• Produce one or more outputs
• This can be done with neural networks
  – Obviously
Representational shortcut

- Input at each time is a vector
- Each layer has many neurons
  - Output layer too may have many neurons
- But will represent everything by simple boxes
  - Each box actually represents an entire layer with many units
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The stock prediction problem...

To invest or not to invest?

- Stock market
  - Must consider the series of stock values in the past several days to decide if it is wise to invest today
    - Ideally consider all of history
The stock predictor network

• The sliding predictor
  – Look at the last few days
  – This is just a convolutional neural net applied to series data
    • Also called a *Time-Delay neural network*
The stock predictor network

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Finite-response model

• This is a finite response system
  – Something that happens today only affects the output of the system for $N$ days into the future
  • $N$ is the width of the system

\[ Y_t = f(X_t, X_{t-1}, \ldots, X_{t-N}) \]
The stock predictor

• This is a *finite response* system
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  • *N* is the *width* of the system
    \[ Y_t = f(X_t, X_{t-1}, \ldots, X_{t-N}) \]
Finite-response model

- Something that happens *today* only affects the output of the system for $N$ days into the future
  - Predictions consider $N$ days of history
- To consider more of the past to make predictions, you must increase the “history” considered by the system
• Problem: Increasing the “history” makes the network more complex
  – No worries, we have the CPU and memory
    • Or do we?
Systems often have long-term dependencies

- Longer-term trends –
  - Weekly trends in the market
  - Monthly trends in the market
  - Annual trends
  - Though longer historic tends to affect us less than more recent events.
We want *infinite* memory

- Required: *Infinite* response systems
  - What happens today can continue to affect the output forever
    - Possibly with weaker and weaker influence

\[
Y_t = f(X_t, X_{t-1}, \ldots, X_{t-\infty})
\]
Examples of infinite response systems

\[ Y_t = f(X_t, Y_{t-1}) \]

- Required: Define initial state: \( Y_{-1} \) for \( t = 0 \)
- An input at \( X_0 \) at \( t = 0 \) produces \( Y_0 \)
- \( Y_0 \) produces \( Y_1 \) which produces \( Y_2 \) and so on until \( Y_\infty \) even if \( X_1 \ldots X_\infty \) are 0
  - i.e. even if there are no further inputs!
  - A single input influences the output for the rest of time!

- This is an instance of a NARX network
  - “nonlinear autoregressive network with exogenous inputs”
  - \( Y_t = f(X_{0:t}, Y_{0:t-1}) \)
- Output contains information about the entire past
A one-tap NARX network

- A NARX net with recursion from the output
A one-tap NARX network

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• A NARX net with recursion from the output
A more complete representation

- A NARX net with recursion from the output
- Showing all computations
- All columns are identical
- An input at $t=0$ affects outputs forever

Brown boxes show output nodes
Yellow boxes are outputs
A NARX net with recursion from the output

Showing all computations

All columns are identical

An input at \( t=0 \) affects outputs forever

Brown boxes show output nodes
All outgoing arrows are the same output
A more generic NARX network

The output $Y_t$ at time $t$ is computed from the past $K$ outputs $Y_{t-1}, \ldots, Y_{t-K}$ and the current and past $L$ inputs $X_t, \ldots, X_{t-L}$
A “complete” NARX network

The output $Y_t$ at time $t$ is computed from all past outputs and all inputs until time $t$

— Not really a practical model
NARX Networks

• Very popular for time-series prediction
  – Weather
  – Stock markets
  – As alternate system models in tracking systems
• Any phenomena with distinct “innovations” that “drive” an output
• Note: here the “memory” of the past is in the output itself, and not in the network
Lets make memory more explicit

• Task is to “remember” the past
• Introduce an explicit memory variable whose job it is to remember

\[ m_t = r(y_{t-1}, h_{t-1}, m_{t-1}) \]
\[ h_t = f(x_t, m_t) \]
\[ y_t = g(h_t) \]

• \( m_t \) is a “memory” variable
  – Generally stored in a “memory” unit
  – Used to “remember” the past
• Memory unit simply retains a running average of past outputs
    • Input is constant (called a “plan”)
    • Objective is to train net to produce a specific output, given an input plan
  – Memory has fixed structure; does not “learn” to remember
    • The running average of outputs considers entire past, rather than immediate past
Elman Networks

- Separate memory state from output
  - “Context” units that carry historical state
    - For the purpose of training, this was approximated as a set of T independent 1-step history nets
- Only the weight from the memory unit to the hidden unit is learned
  - But during training no gradient is backpropagated over the “1” link
In time series analysis, models must look at past inputs along with current input
  - Looking at a finite horizon of past inputs gives us a convolutional network

Looking into the infinite past requires recursion

NARX networks recurse by feeding back the output to the input
  - May feed back a finite horizon of outputs

“Simple” recurrent networks:
  - Jordan networks maintain a running average of outputs in a “memory” unit
  - Elman networks store hidden unit values for one time instant in a “context” unit
  - “Simple” (or partially recurrent) because during learning current error does not actually propagate to the past
    - “Blocked” at the memory units in Jordan networks
    - “Blocked” at the “context” unit in Elman networks
An alternate model for infinite response systems: the state-space model

\[ h_t = f(x_t, h_{t-1}) \]
\[ y_t = g(h_t) \]

- \( h_t \) is the state of the network
  - Model directly embeds the memory in the state
- Need to define initial state \( h_{-1} \)

- This is a fully recurrent neural network
  - Or simply a recurrent neural network
- \textit{State} summarizes information about the entire past
The simple state-space model

- The state (green) at any time is determined by the input at that time, and the state at the previous time
- An input at $t=0$ affects outputs forever
- Also known as a recurrent neural net
An alternate model for infinite response systems: the state-space model

\[ h_t = f(x_t, h_{t-1}) \]
\[ y_t = g(h_t) \]

- \( h_t \) is the state of the network
- Need to define initial state \( h_{-1} \)
- The state can be arbitrarily complex
Single hidden layer RNN

- Recurrent neural network
- All columns are identical
- *An input at $t=0$ affects outputs forever*
Multiple recurrent layer RNN

- Recurrent neural network
- All columns are identical
- An input at $t=0$ affects outputs forever
Multiple recurrent layer RNN

- We can also have skips..
A more complex state

• All columns are identical

• An input at $t=0$ affects outputs forever
Or the network may be even more complicated

• Shades of NARX
• All columns are identical
• An input at $t=0$ affects outputs forever
Generalization with other recurrences

• All columns (including incoming edges) are identical
The simplest structures are most popular

- Recurrent neural network
- All columns are identical
- An input at $t=0$ affects outputs forever
A Recurrent Neural Network

- Simplified models often drawn
- The loops imply recurrence
The detailed version of the simplified representation
Multiple recurrent layer RNN
Multiple recurrent layer RNN
Note superscript in indexing, which indicates layer of network from which inputs are obtained

Assuming vector function at output, e.g. softmax

The state node activation, $f_1()$ is typically tanh()

Every neuron also has a bias input

$$h_{i}^{(1)}(-1) = \text{part of network parameters}$$

$$h_{i}^{(1)}(t) = f_1\left(\sum_j w_{ji}^{(1)} X_j(t) + \sum_j w_{ji}^{(11)} h_{i}^{(1)}(t-1) + b_{i}^{(1)}\right)$$

$$Y(t) = f_2\left(\sum_j w_{jk}^{(2)} h_{j}^{(1)}(t) + b_{k}^{(2)}, k = 1 \ldots M\right)$$
**Equations**

\[ h_i^{(1)}(-1) = \text{part of network parameters} \]
\[ h_i^{(2)}(-1) = \text{part of network parameters} \]

\[ h_i^{(1)}(t) = f_1 \left( \sum_j w_{ji}^{(1)} X_j(t) + \sum_j w_{ji}^{(11)} h_i^{(1)}(t-1) + b_i^{(1)} \right) \]

\[ h_i^{(2)}(t) = f_2 \left( \sum_j w_{ji}^{(2)} h_j^{(1)}(t) + \sum_j w_{ji}^{(22)} h_i^{(2)}(t-1) + b_i^{(2)} \right) \]

\[ Y(t) = f_3 \left( \sum_j w_{jk}^{(3)} h_j^{(2)}(t) + b_k^{(3)}, k = 1..M \right) \]

- Assuming vector function at output, e.g. softmax \( f_3() \)
- The *state* node activations, \( f_k() \) are typically \( \tanh() \)
- Every neuron also has a *bias* input
Equations

\[ h_i^{(1)}(-1) = \text{part of network parameters} \]
\[ h_i^{(2)}(-1) = \text{part of network parameters} \]

\[ h_i^{(1)}(t) = f_1 \left( \sum_j w_{ji}^{(0,1)} X_j(t) + \sum_i w_{ii}^{(1,1)} h_i^{(1)}(t - 1) + b_i^{(1)} \right) \]

\[ h_i^{(2)}(t) = f_2 \left( \sum_j w_{ji}^{(1,2)} h_j^{(1)}(t) + \sum_j w_{ji}^{(0,2)} X_j(t) + \sum_i w_{ii}^{(2,2)} h_i^{(2)}(t - 1) + b_i^{(2)} \right) \]

\[ Y_i(t) = f_3 \left( \sum_j w_{jk}^{(2)} h_j^{(2)}(t) + \sum_j w_{jk}^{(1,3)} h_j^{(1)}(t) + b_k^{(3)}, k = 1..M \right) \]
Variants on recurrent nets

- 1: Conventional MLP
- 2: Sequence generation, e.g. image to caption
- 3: Sequence based prediction or classification, e.g. Speech recognition, text classification

Images from Karpathy

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Variants

• 1: *Delayed* sequence to sequence, e.g. machine translation
• 2: Sequence to sequence, e.g. stock problem, label prediction
• Etc...

Images from Karpathy
Story so far

• Time series analysis must consider past inputs along with current input
• Looking into the infinite past requires recursion

• NARX networks achieve this by feeding back the output to the input

• “Simple” recurrent networks maintain separate “memory” or “context” units to retain some information about the past
  – But during learning the current error does not influence the past

• State-space models retain information about the past through recurrent hidden states
  – These are “fully recurrent” networks
  – The initial values of the hidden states are generally learnable parameters as well

• State-space models enable current error to update parameters in the past
How do we *train* the network

- Back propagation through time (BPTT)

- Given a collection of *sequence* inputs
  - \((X_i, D_i)\), where
  - \(X_i = X_{i,0}, \ldots, X_{i,T}\)
  - \(D_i = D_{i,0}, \ldots, D_{i,T}\)

- Train network parameters to minimize the error between the output of the network \(Y_i = Y_{i,0}, \ldots, Y_{i,T}\) and the desired outputs
  - This is the most generic setting. In other settings we just “remove” some of the input or output entries
Training: Forward pass

- For each training input:
- Forward pass: pass the entire data sequence through the network, generate outputs
Recurrent Neural Net
Assuming time-synchronous output

# Assuming h(-1,*) is known
# Assuming L hidden-state layers and an output layer
# W_c(*) and W_r(*) are matrices, b(*) are vectors
# W_c are weights for inputs from current time
# W_r is recurrent weight applied to the previous time
# W_o are output layer weights


definitions:

for t = 0:T-1  # Including both ends of the index
    h(t,0) = x(t)  # Vectors. Initialize h(0) to input
for l = 1:L  # hidden layers operate at time t
    z(t,l) = W_c(l)h(t,l-1) + W_r(l)h(t-1,l) + b(l)
    h(t,l) = tanh(z(t,l))  # Assuming tanh activ.
    z_o(t) = W_o h(t,L) + b_o
    Y(t) = softmax(z_o(t))

Subscript “c” - current
Subscript “r” - recurrent
Training: Computing gradients

For each training input:
- Backward pass: Compute gradients via backpropagation
  - Back Propagation Through Time
Back Propagation Through Time

Will only focus on one training instance

All subscripts represent components and not training instance index
Back Propagation Through Time

- The divergence computed is between the sequence of outputs by the network and the desired sequence of outputs:
  - $\text{DIV}$ is a scalar function of a series of vectors!
- This is not just the sum of the divergences at individual times:
  - Unless we explicitly define it that way
• $Y(t)$ is the output at time $t$
  – $Y_i(t)$ is the $i$th output
• $Z^{(2)}(t)$ is the pre-activation value of the neurons at the output layer at time $t$
• $h(t)$ is the output of the hidden layer at time $t$
  – Assuming only one hidden layer in this example
• $Z^{(1)}(t)$ is the pre-activation value of the hidden layer at time $t$
**Back Propagation Through Time**

First step of backprop: Compute \( \frac{d\text{DIV}}{dY_i(T)} \) for all \( i \)

Note: DIV is a function of all outputs \( Y(0) \) ... \( Y(T) \)

In general we will be required to compute \( \frac{d\text{DIV}}{dY_i(t)} \) for all \( i \) and \( t \) as we will see. This can be a source of significant difficulty in many scenarios.
Special case, when the overall divergence is a simple sum of local divergences at each time:

Must compute

\[ \frac{dDIV}{dY_i(t)} \text{ for all } i \text{ for all } T \]

Will usually get

\[ \frac{dDIV}{dY_i(t)} = \frac{dDiv(t)}{dY_i(t)} \]
Back Propagation Through Time

First step of backprop: Compute $\frac{dDIV}{dY_i(T)}$ for all $i$

$$\nabla_{Z^{(2)}_{(T)}} DIV = \nabla_{Y(T)} DIV \nabla_{Z^{(2)}_{(T)}} Y(T)$$

Vector output activation

$$\frac{dDIV}{dZ^{(2)}_{i}(T)} = \frac{dDIV}{dY_i(T)} \frac{dY_i(T)}{dZ^{(2)}_{i}(T)}$$

OR

$$\frac{dDIV}{dZ^{(2)}_{i}(T)} = \sum_j \frac{dDIV}{dY_j(T)} \frac{dY_j(T)}{dZ^{(2)}_{i}(T)}$$
Back Propagation Through Time

$$h^{-1}$$

$$X(0)$$  $$X(1)$$  $$X(2)$$  $$X(T-2)$$  $$X(T-1)$$  $$X(T)$$

$$Y(0)$$  $$Y(1)$$  $$Y(2)$$  $$Y(T-2)$$  $$Y(T-1)$$  $$Y(T)$$

$$DIV$$

$$D(1..T)$$

$$\frac{dDIV}{dh_i(T)}$$ for all i

$$\frac{dDIV}{dZ_i^{(2)}(T)} = \frac{dDiv(T)}{dY_i(T)} \frac{dY_i(T)}{dZ_i^{(2)}(T)}$$

$$\nabla_{h(T)} DIV = \nabla_{Z^{(2)}(T)} DIV W^{(2)}$$
Back Propagation Through Time

\[
\frac{dDIV}{dZ_i^{(2)}(T)} = \frac{dDiv(T)}{dY_i(T)} \frac{dY_i(T)}{dZ_i^{(2)}(T)}
\]

\[
\frac{dDIV}{dh_i(T)} = \sum_j w_{ij}^{(2)} \frac{dDIV}{dZ_j^{(2)}(T)}
\]

\[
\nabla_{W^{(2)}} DIV = h(T) \nabla_{Z^{(2)}(T)} DIV
\]

\[
\frac{dDIV}{dw_{ij}^{(2)}} = \frac{dDIV}{dZ_j^{(2)}(T)} h_i(T)
\]
**Back Propagation Through Time**

\[ \nabla_{Z^{(1)}(T)} DIV = \nabla_{h(T)} DIV \nabla_{Z^{(1)}(T)} h(T) \]

\[ \frac{dDIV}{dZ^{(2)}_i(T)} = \frac{dDIV}{dY_i(T)} \frac{dY_i(T)}{dZ^{(2)}_i(T)} \]

\[ \frac{dDIV}{dh_i(T)} = \sum_j w^{(2)}_{ij} \frac{dDIV}{dZ^{(2)}_j(T)} \]

\[ \frac{dDIV}{dw^{(2)}_{ij}} = \frac{dDIV}{dZ^{(2)}_j(T)} h_i(T) \]
Back Propagation Through Time

\[ Y(0) \rightarrow Y(1) \rightarrow Y(2) \rightarrow \cdots \rightarrow Y(T) \]

\[ X(0) \rightarrow X(1) \rightarrow X(2) \rightarrow \cdots \rightarrow X(T) \]

\[ D(1..T) \]

\[ \nabla_{W(1)} DIV = X(T) \nabla_{Z(1)(T)} DIV \]

\[ \frac{dDIV}{dw_{ij}^{(1)}} = \frac{dDIV}{dZ_{j}^{(1)}(T)} X_i(T) \]
Back Propagation Through Time

\[
Y(0) \quad Y(1) \quad Y(2) \quad Y(T-2) \quad Y(T-1) \quad Y(T)
\]

\[
X(0) \quad X(1) \quad X(2) \quad X(T-2) \quad X(T-1) \quad X(T)
\]

\[DIV\]

\[
\nabla_{W^{(1)}} DIV = h(T-1) \nabla_{Z^{(1)}(T)} DIV
\]

\[
\frac{dDIV}{dw_{ij}^{(1)}} = \frac{dDIV}{dZ_{j}^{(1)}(T)} X_i(T)
\]

\[
\frac{dDIV}{dw_{ij}^{(11)}} = \frac{dDIV}{dZ_{j}^{(1)}(T)} h_i(T-1)
\]
Back Propagation Through Time

\[ \nabla_{Z^{(2)}(T-1)} DIV = \nabla_{Y(T-1)} DIV \nabla_{Z^{(2)}(T)} Y(T-1) \]

\[
\frac{dDIV}{dZ_i^{(2)}(T-1)} = \frac{dDIV}{dY_i(T-1)} \frac{dY_i(T-1)}{dZ_i^{(2)}(T-1)}
\]

OR

\[
\frac{dDIV}{dZ_i^{(2)}(T-1)} = \sum_j \frac{dDIV}{dY_j(T-1)} \frac{dY_j(T-1)}{dZ_i^{(2)}(T-1)}
\]
Back Propagation Through Time

\[
\frac{dDIV}{dh_i(T - 1)} = \sum_j w_{ij}^{(2)} \frac{dDIV}{dZ_j^{(2)}(T - 1)} + \sum_j w_{ij}^{(11)} \frac{dDIV}{dZ_j^{(1)}(T)}
\]

\[
\nabla_{h(T-1)} DIV = \nabla_{Z^{(2)}(T-1)} DIV W^{(2)} + \nabla_{Z^{(1)}(T)} DIV W^{(11)}
\]
Back Propagation Through Time

\[
\frac{dDIV}{dh_i(T-1)} = \sum_j w_{ij}^{(2)} \frac{dDIV}{dZ_j^{(2)}(T-1)} + \sum_j w_{ij}^{(11)} \frac{dDIV}{dZ_j^{(1)}(T)}
\]

Note the addition

\[
\frac{dDIV}{dW_{ij}^{(2)}} = \frac{dDIV}{dZ_j^{(2)}(T-1)} h_i(T-1)
\]

\[
\nabla_{W^{(2)}} DIV += h(T-1) \nabla_{Z^{(2)}(T-1)} DIV
\]
Back Propagation Through Time

\[
\frac{dDIV}{dZ_i^{(1)}(T-1)} = \frac{dDIV}{dh_i(T-1)} \frac{dh_i(T-1)}{dZ_i^{(1)}(T-1)}
\]

\[
\nabla_{Z^{(1)}(T-1)}DIV = \nabla_{h(T-1)}DIV \nabla_{Z^{(1)}(T-1)}h(T-1)
\]
Back Propagation Through Time

\[
\frac{dDIV}{dZ_i^{(1)}(T-1)} = \frac{dDIV}{dh_i(T-1)} \frac{dh_i(T-1)}{dZ_i^{(1)}(T-1)}
\]

\[
\frac{dDIV}{dw_{ij}^{(1)}} + \frac{dDIV}{dZ_j^{(1)}(T-1)} = X_i(T-1)
\]

Note the addition

\[
\nabla W^{(1)} DIV \quad +\quad X(T-1) \nabla Z^{(1)}_{(T-1)} DIV
\]
Back Propagation Through Time

Note the addition

$$\nabla_{W_{(11)}} DIV + = h(T - 2) \nabla_{Z_{(1)(T-1)}} DIV^0$$
Back Propagation Through Time

Continue computing derivatives going backward through time until...

\[
\frac{dDIV}{dh_{t}(-1)} = \sum_{j} w_{ij}^{(11)} \frac{dDIV}{dz_{j}^{(1)}(0)}
\]

\[
\nabla_{h_{t-1}} DIV = \nabla_{z^{(1)}(0)} DIV W^{(11)}
\]
Back Propagation Through Time

\[
\frac{dDIV}{dh_i^{(k)}(t)} = \sum_j w_{i,j}^{(k+1)} \frac{dDIV}{dZ_j^{(k+1)}(t)} + \sum_j w_{i,j}^{(k,k)} \frac{dDIV}{dZ_j^{(k)}(t + 1)}
\]

Not showing derivatives at output neurons

\[
\frac{dDIV}{dZ_i^{(k)}(t)} = \frac{dDIV}{dh_i^{(k)}(t)} f'_k \left( Z_i^{(k)}(t) \right)
\]
Back Propagation Through Time

\[
\frac{dDIV}{dh_i(-1)} = \sum_j w_{ij}^{(11)} \frac{dDIV}{dZ_j^{(1)}(0)}
\]

\[
\frac{dDIV}{dw_{ij}^{(1)}} = \sum_t \frac{dDIV}{dZ_j^{(1)}(t)} X_i(t)
\]

\[
\frac{dDIV}{dw_{ij}^{(11)}} = \sum_t \frac{dDIV}{dZ_j^{(1)}(t)} h_i(t - 1)
\]
# Assuming forward pass has been completed
# Jacobian(x,y) is the jacobian of x w.r.t. y
# Assuming \(dY(t) = \text{gradient}(\text{div}, Y(t))\) available for all \(t\)
# Assuming all \(dz, dh, dW\) and \(db\) are initialized to 0

for \(t = T-1\):downto:0  # Backward through time
    \(dz_0(t) = dY(t) \cdot \text{Jacobian}(Y(t), z_0(t))\)
    \(dW_0 += h(t,L) \cdot dz_0(t)\)
    \(db_0 += dz_0(t)\)
    \(dh(t,L) += dz_0(t)W_0\)

for \(l = L:1\)  # Reverse through layers
    \(dz(t,l) = dh(t,l) \cdot \text{Jacobian}(h(t,l), z(t,l))\)
    \(dh(t,l-1) += dz(t,l) \cdot W_c(l)\)
    \(dh(t-1,l) = dz(t,l) \cdot W_r(l)\)
    \(dW_c(l) += h(t,l-1) \cdot dz(t,l)\)
    \(dW_r(l) += h(t-1,l) \cdot dz(t,l)\)
    \(db(l) += dz(t,l)\)
• Can be generalized to any architecture
Extensions to the RNN: **Bidirectional RNN**

- RNN with both forward and backward recursion
  - Explicitly models the fact that just as the future can be predicted from the past, the past can be deduced from the future

Proposed by Schuster and Paliwal 1997
• A forward net process the data from $t=0$ to $t=T$
• A backward net processes it backward from $t=T$ down to $t=0$
Bidirectional RNN: Processing an input string

- The forward net processes the data from $t=0$ to $t=T$
  - Only computing the hidden states, initially
The backward nets processes the input data in reverse time, end to beginning:

- Initially only the hidden state values are computed
  - Clearly, this is not an online process and requires the entire input data
  - Note: This is not the backward pass of backprop.
Bidirectional RNN: Processing an input string

- The computed states of both networks are used to compute the final output at each time.
• Need to talk in terms of bidirectional *layers* for both forward and backward.

• Introduce it as a variant?

• Simple modification of pseudocode
Bidirectional RNN

Assuming time-synchronous output

# Subscript f represents forward net, b is backward net
# Assuming \( h_f(-1,*) \) and \( h_b(inf,*) \) are known

#forward pass
for t = 0:T-1 # Going forward in time
    \( h_f(t,0) = x(t) \) # Vectors. Initialize \( h(0) \) to input
    for l = 1:L_f # \( L_f \) is depth of forward network hidden layers
        \( z_f(t,l) = W_{fc}(l)h_f(t,l-1) + W_{fr}(l)h_f(t-1,l) + b_f(l) \)
        \( h_f(t,l) = \text{tanh}(z_f(t,l)) \) # Assuming tanh activ.

#backward
h(T,:,:,:) = h(inf,:,:,:) # Just the initial value
for t = T-1:downto:0 # Going backward in time
    \( h_b(t,0) = x(t) \) # Vectors. Initialize \( h(0) \) to input
    for l = 1:L_b # \( L_b \) is depth of backward network hidden layers
        \( z_b(t,l) = W_{bc}(l)h_b(t,l-1) + W_{br}(l)h(t+1,l) + b_b(l) \)
        \( h_b(t,l) = \text{tanh}(z_b(t,l)) \) # Assuming tanh activ.

for t = 0:T-1 # The output combines forward and backward
    \( z_o(t) = W_{fo}h_f(t,L_f) + W_{bo}h_b(t,L_b) + b_o \)
    \( Y(t) = \text{softmax}(z_o(t)) \)
Bidirectional RNN: Simplified code

• Code can be made modular and simplified for better interpretability...
First: Define basic RNN with only hidden units

# Inputs:
#   L : Number of hidden layers
#   \( W_c, W_r, b \): current weights, recurrent weights, biases
#   hinit: initial value of \( h \)(representing \( h(-1,*)) \)
#   x: input vector sequence
#   T: Length of input vector sequence
# Output:
#   h, z: sequence of pre-and post activation hidden representations from all layers of the RNN

function \([h,z] = \text{RNN\_forward}(L, W_c, W_r, b, hinit, x, T)\)
  \( h(-1,:) = hinit \) # hinit is the initial value for all layers
  for \( t = 0:T-1 \) # Going forward in time
    \( h(t,0) = x(t) \) # Vectors. Initialize \( h(0) \) to input
    for \( l = 1:L \)
      \( z(t,l) = W_c(l)h(t,l-1) + W_r(l)h(t-1,l) + b(l) \)
      \( h(t,l) = \text{tanh}(z(t,l)) \) # Assuming tanh activ.
  return \( h,z \)
Bidirectional RNN
Assuming time-synchronous output

# Subscript f represents forward net, b is backward net
# Assuming h_f(-1,*) and h_b(inf,*) are known

#forward pass
[h_f, z_f] = RNN_forward(L_f, W_{fc}, W_{fr}, b_f, h(-1,:), x, T)

#backward pass
x_{rev} = fliplr(x)  # Flip it in time
[h_{brev}, z_{brev}] = RNN_forward(L_b, W_{bc}, W_{br}, b_b, h(inf,:), x_{rev}, T)
h_b = fliplr(h_{brev})  # Flip back to straighten time
z_b = fliplr(z_{brev})

#combine the two for the output
for t = 0:T-1  # The output combines forward and backward
    z_o(t) = W_{fo}h_f(t,L_f) + W_{bo}h_b(t,L_b) + b_o
    Y(t) = softmax( z_o(t) )
Backpropagation in BRNNs

- Forward pass: Compute both forward and backward networks and final output

### Diagram

```
<table>
<thead>
<tr>
<th>Y(0)</th>
<th>Y(1)</th>
<th>Y(2)</th>
<th>Y(T-2)</th>
<th>Y(T-1)</th>
<th>Y(T)</th>
</tr>
</thead>
<tbody>
<tr>
<td>X(0)</td>
<td>X(1)</td>
<td>X(2)</td>
<td>X(T-2)</td>
<td>X(T-1)</td>
<td>X(T)</td>
</tr>
<tr>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
```

- Initial states:
  - $h_t(-1)$
  - $h_b(\text{inf})$

- Time steps:
  - $t$
Backpropagation in BRNNs

- Backward pass: Define a divergence from the desired output

- Separately perform back propagation on both nets
  - From $t=T$ down to $t=0$ for the forward net
  - From $t=0$ up to $t=T$ for the backward net
Backpropagation in BRNNs

- Backward pass: Define a divergence from the desired output
- Separately perform back propagation on both nets
  - From $t=T$ down to $t=0$ for the forward net

$X(0), Y(0)$
$X(1), X(2), X(T-2), X(T-1), X(T)$
$Y(0), Y(1), Y(2), Y(T-2), Y(T-1), Y(T)$
$Div()$
• Backward pass: Define a divergence from the desired output
• Separately perform back propagation on both nets
  – From $t=T$ down to $t=0$ for the forward net
  – From $t=0$ up to $t=T$ for the backward net
Backpropagation: Pseudocode

• As before we will use a 2-step code:
  – A basic backprop routine that we will call
  – Two calls to the routine within a higher-level wrapper
First: backprop through a recurrent net

# Inputs:
#  (In addition to inputs used by L: Number of hidden layers
#  dh_{top}: derivatives ddiv/dh_{*.}(t,L) at each time (* may be f or b)
#  h, z: h and z values returned by the forward pass
#  T: Length of input vector sequence
# Output:
#  dW_c, dW_r, db, dh_{init}: derivatives w.r.t current and recurrent weights,
#  biases, and initial h.
# Assuming all dz, dh, dW_c, dW_r and db are initialized to 0

function [dW_c,dW_r,db,dh_{init}] = RNN_bptt(L, W_c, W_r, b, hinit, x, T, dh_{top}, h, z)

dh = zeros

for t = T-1:downto:0  # Backward through time
    dh(t,L) += dh_{top}(t)
for l = L:1  # Reverse through layers
    dz(t,l) = dh(t,l)Jacobian(h(t,l),z(t,l))
    dh(t,l-1) += dz(t,l) W_c(l)
    dh(t-1,l) += dz(t,l) W_r(l)
    dW_c(l) += h(t,l-1)dz(t,l)
    dW_r(l) += h(t-1,l)dz(t,l)
    db(l) += dz(t,l)

return dW_c, dW_r, db, dh(-1)  # dh(-1) is actually dh(-1,1:L,:)
Bi-RNN gradient computation

Assuming time-synchronous output

# Subscript f represents forward net, b is backward net
# First compute derivatives that directly relate to dY(t) for all t,
# then pass the derivatives into RNN_bptt to compute forward and backward
# parameter derivatives

for t = 0:T-1  # The output combines forward and backward
  dz_o(t) = dY(t)Jacobian(Y(t),z_o(t))
  dh_fo(t) = dz_o(t)W_fo
  dh_bo(t) = dz_o(t)W_bo
  db_o  += dz_o(t)
  dW_fo += h_f(t,L)dz_o(t)
  dW_bo += h_b(t,L)dz_o(t)

#forward net
[dW_fo, dW_fr, db_f, dh_f(-1)] = RNN_bptt(L, W_fo, W_fr, b_f, h_f(-1), x, T, dh_fo, h_f, z_f)

#backward net
x_rev = fliplr(x)  # Flip it in time
[dW_bc, dW_br, db_b, dh_b(inf)] = RNN_bptt(L, W_bc, W_br, b_b, h_b(inf), x_rev, T, dh_bo, h_b, z_b)
• Time series analysis must consider past inputs along with current input

• Recurrent networks look into the infinite past through a state-space framework
  – Hidden states that recurse on themselves

• Training recurrent networks requires
  – Defining a divergence between the actual and desired output *sequences*
  – Backpropagating gradients over the entire chain of recursion
    • Backpropagation through time
  – Pooling gradients with respect to individual parameters over time

• Bidirectional networks analyze data both ways, begin $\rightarrow$ end and end $\rightarrow$ beginning to make predictions
  – In these networks, backprop must follow the chain of recursion (and gradient pooling) separately in the forward and reverse nets
• Excellent models for time-series analysis tasks
  – Time-series prediction
  – Time-series classification
  – Sequence prediction..
So how did this happen

Naturalism and decision for the majority of Arab countries' capitalide was grounded by the Irish language by [[John Clair]], [[An Imperial Japanese Revolt]], associated with Guangzham's sovereignty. His generals were the powerful ruler of the Portugal in the [[Protestant Immineners]], which could be said to be directly in Cantonese Communication, which followed a ceremony and set inspired prison, training. The emperor travelled back to [[Antioch, Perth, October 25|21]] to note, the Kingdom of Costa Rica, unsuccessful fashioned the [[Thrales]], [[Cynth's Dajoard]], known in western [[Scotland]], near Italy to the conquest of India with the conflict. Copyright was the succession of independence in the slop of Syrian influence that was a famous German movement based on a more popular servicious, non-doctrinal and sexual power post. Many governments recognize the military housing of the [[Civil Liberalization and Infantry Resolution 265 National Party in Hungary]], that is sympathetic to be the [[Punjab Resolution]] (PJS)[http://www.humah.yahoo.com/guardian. cfm/7754800786d17551963s89.htm Official economics Adjoint for the Nazism, Montgomery was swear to advance to the resources for those Socialism's rule, was starting to signing a major tripad of aid exile.]]
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More on this later..
RNNs..

• Excellent models for time-series analysis tasks
  – Time-series prediction
  – Time-series classification
  – Sequence prediction..
  – They can even simplify some problems that are difficult for MLPs
    • Next class..