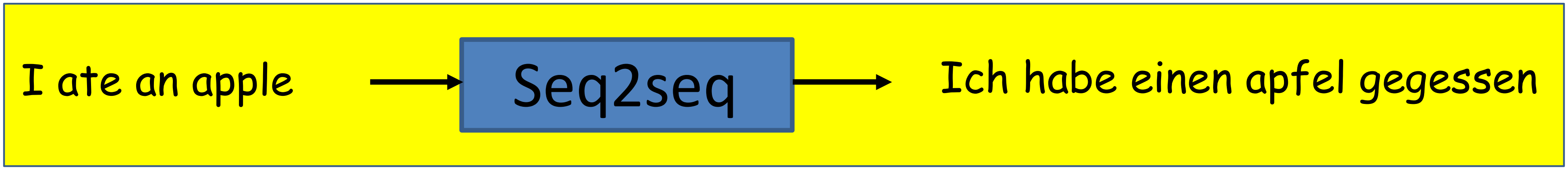
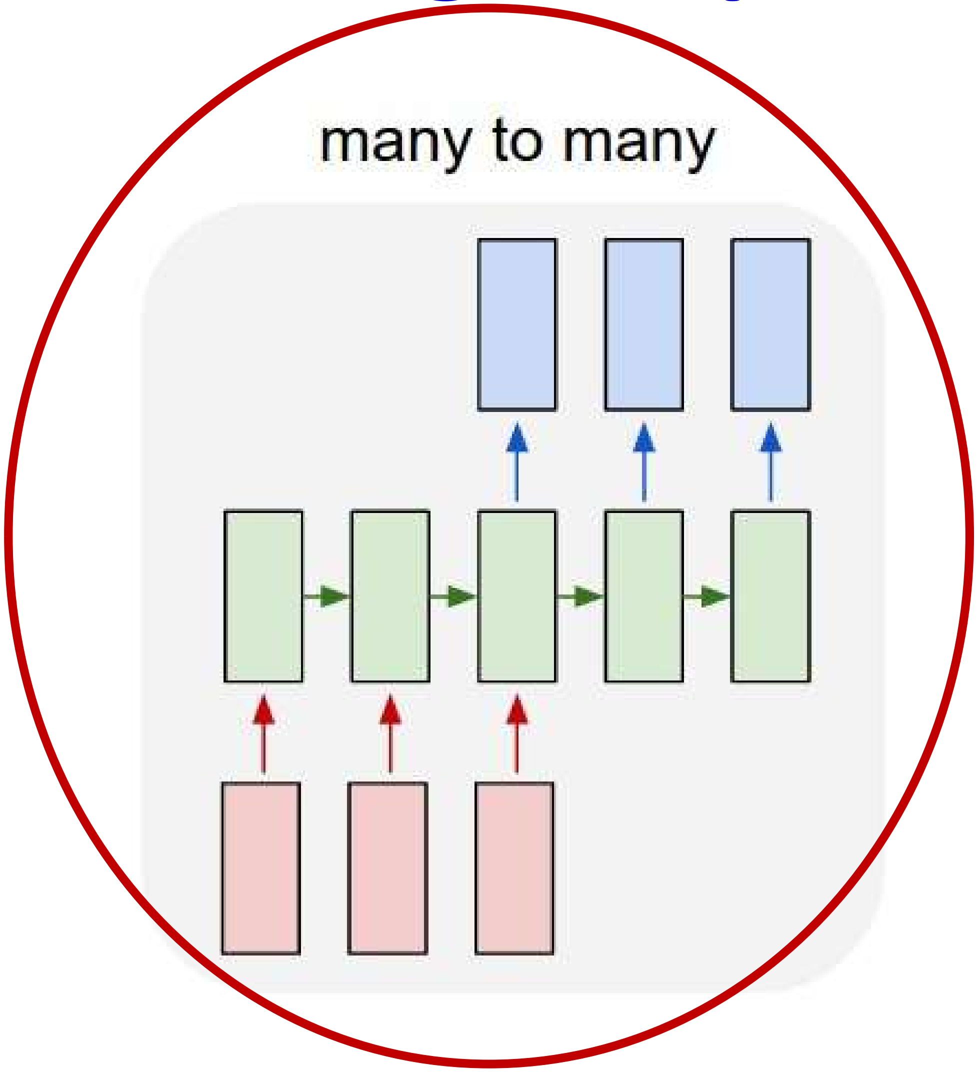
# Deep Learning
# Sequence to Sequence models: Attention Models

# Sequence to sequence models



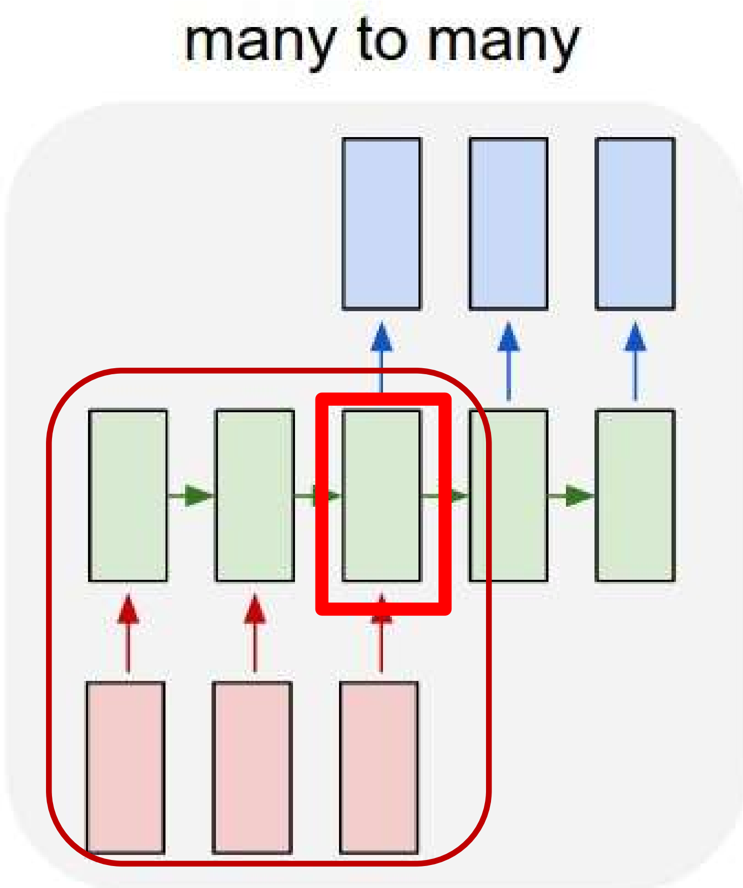

- Sequence goes in, sequence comes out
- No notion of "time synchrony" between input and output
  - May even not even maintain order of symbols
    - E.g. "I ate an apple" → "Ich habe einen apfel gegessen"
  - Or even seem related to the input
    - E.g. "My screen is blank" → "Please check if your computer is plugged in."

# Modelling the problem




- *Delayed* sequence to sequence

# Modelling the problem




First process the input and generate a hidden representation for it

- *Delayed* sequence to sequence

# Modelling the problem

many to many

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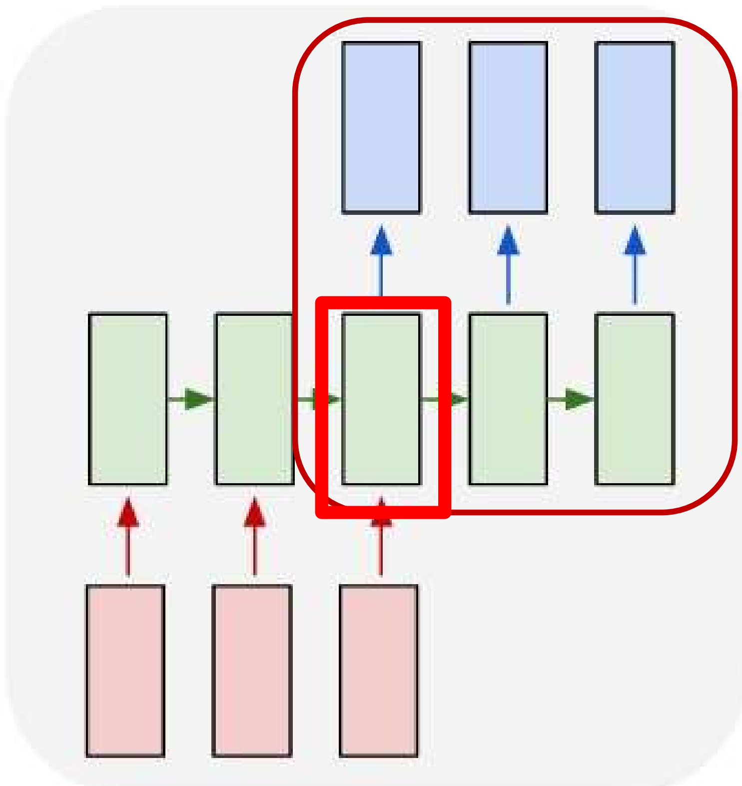

Then use it to generate an output

- *Delayed* sequence to sequence

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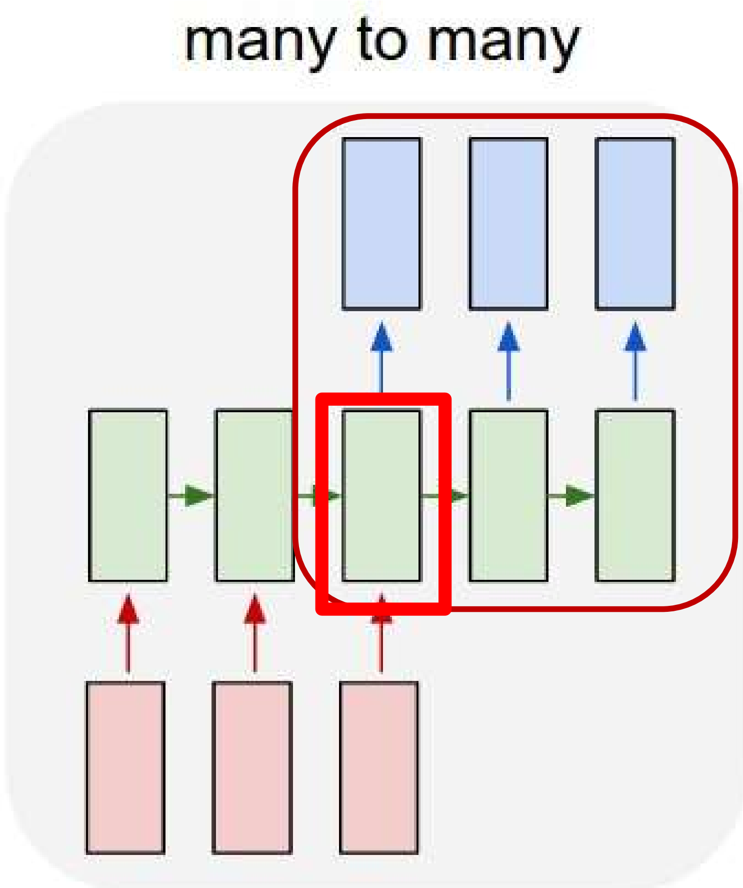
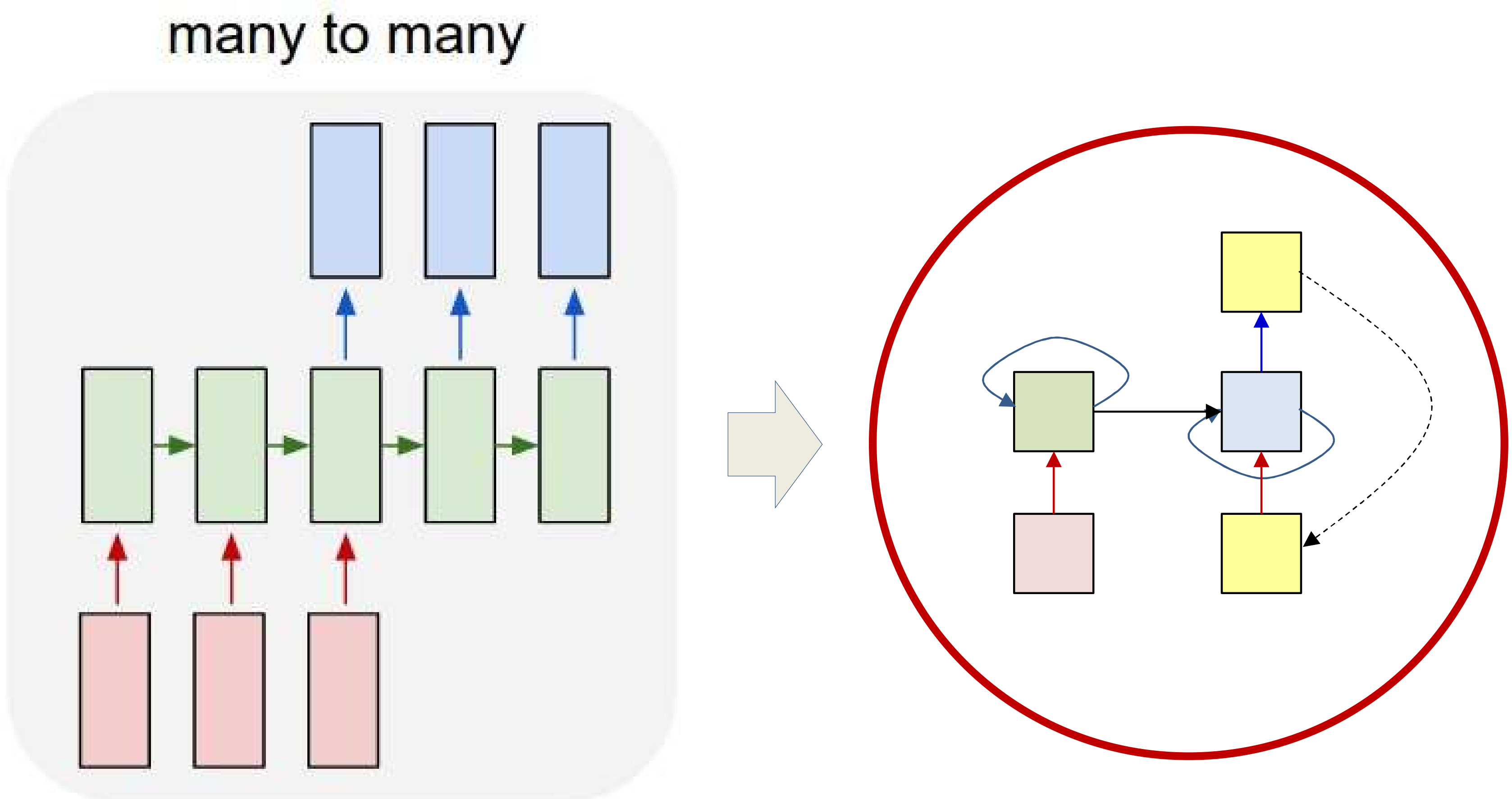



Then use it to generate an output

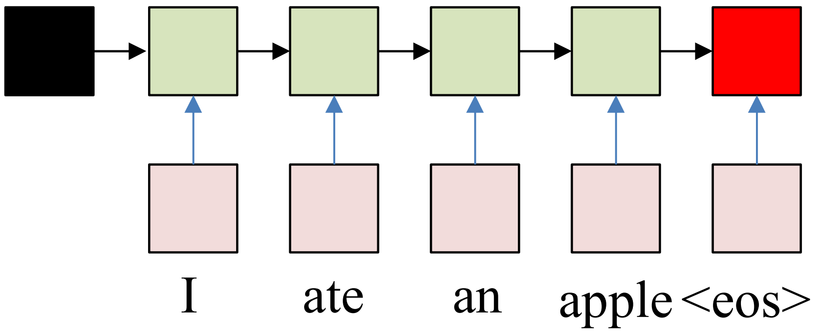
- *Problem:* Each word that is output depends only on current hidden state, and not on previous outputs

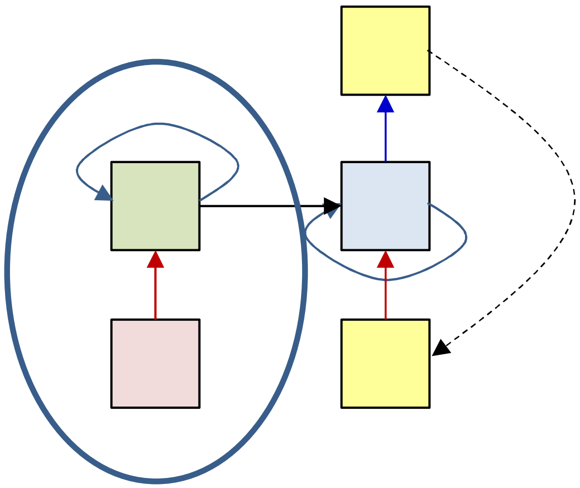
# Modelling the problem




- *Delayed* sequence to sequence
  - Delayed *self-referencing* sequence-to-sequence

# The “simple” translation model



- The input sequence feeds into a recurrent structure
- The input sequence is terminated by an explicit <eos> symbol
  - The hidden activation at the <eos> “stores” all information about the sentence

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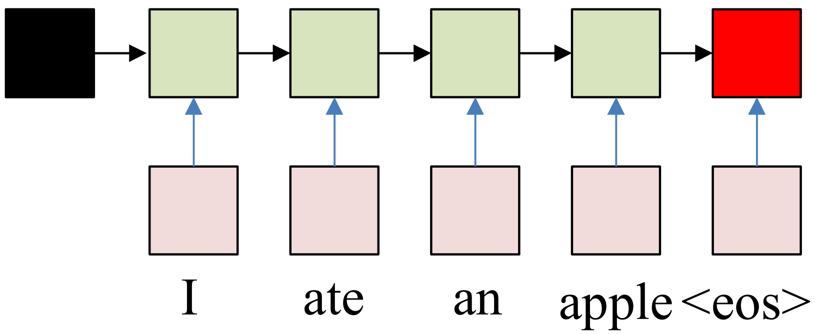


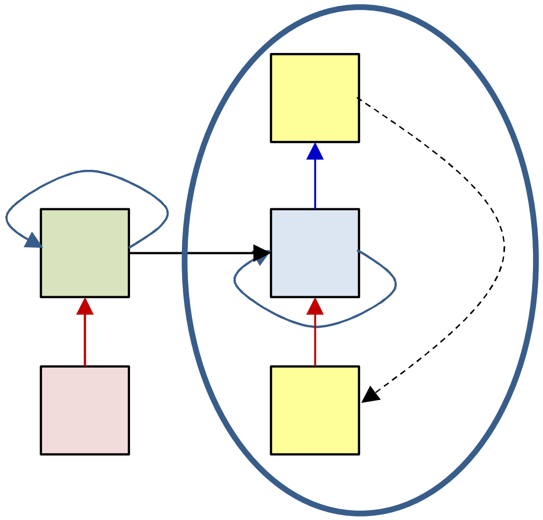

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- Subsequently a *second* RNN uses the hidden activation as initial state, and <sos> as initial symbol, to produce a sequence of outputs
  - The output at each time becomes the input at the next time
  - Output production continues until an <eos> is produced

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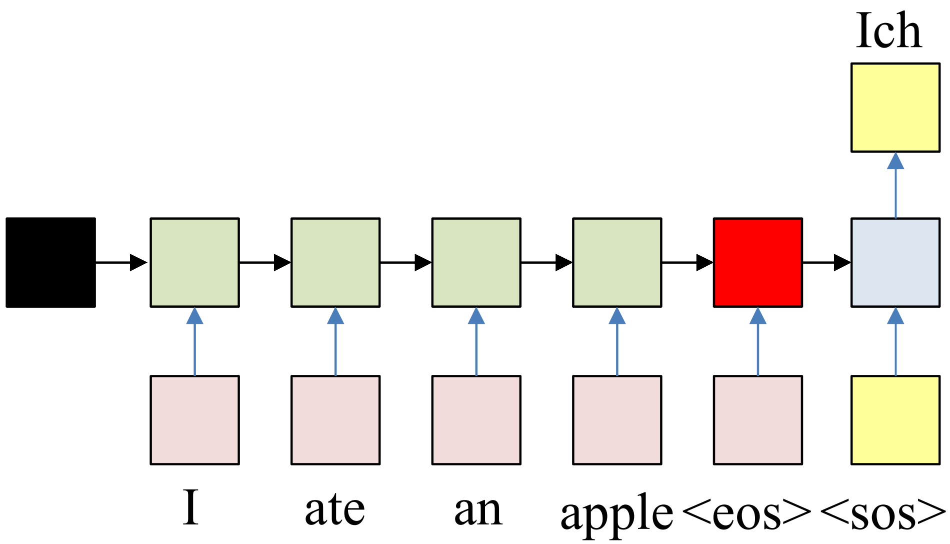


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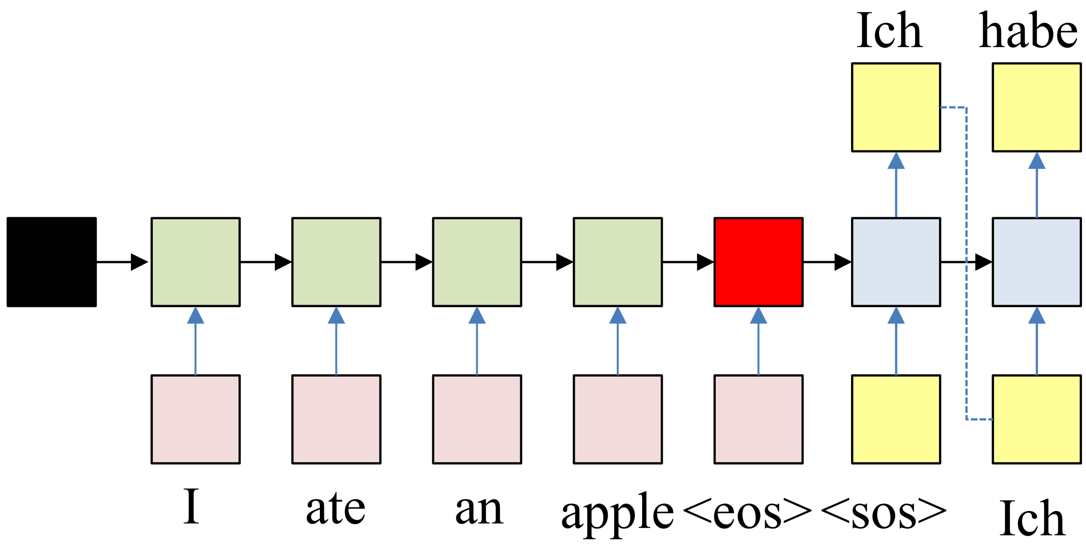


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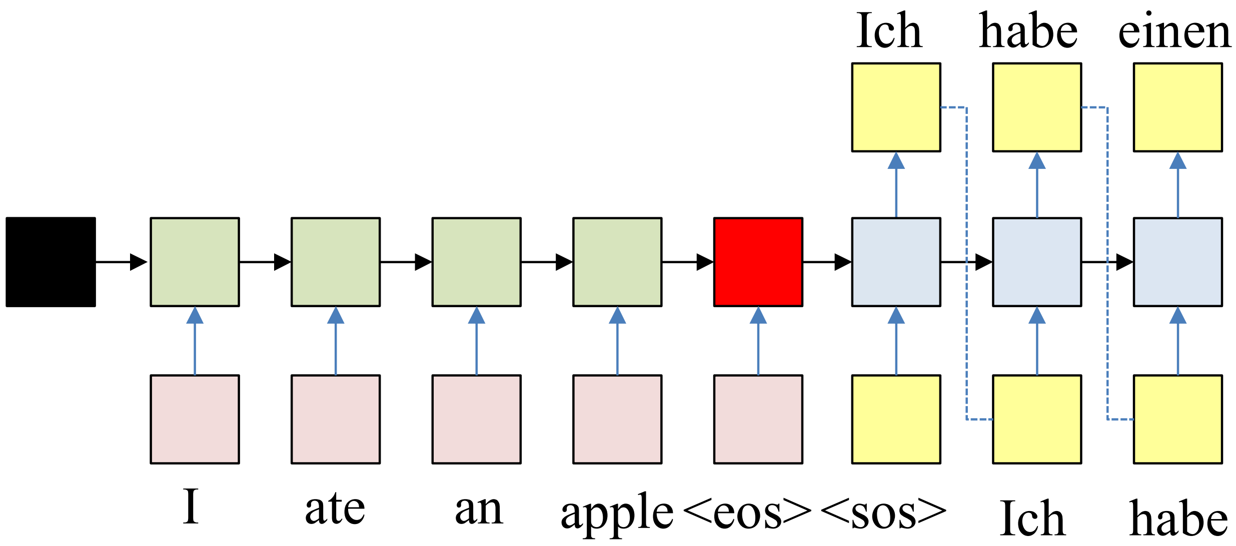


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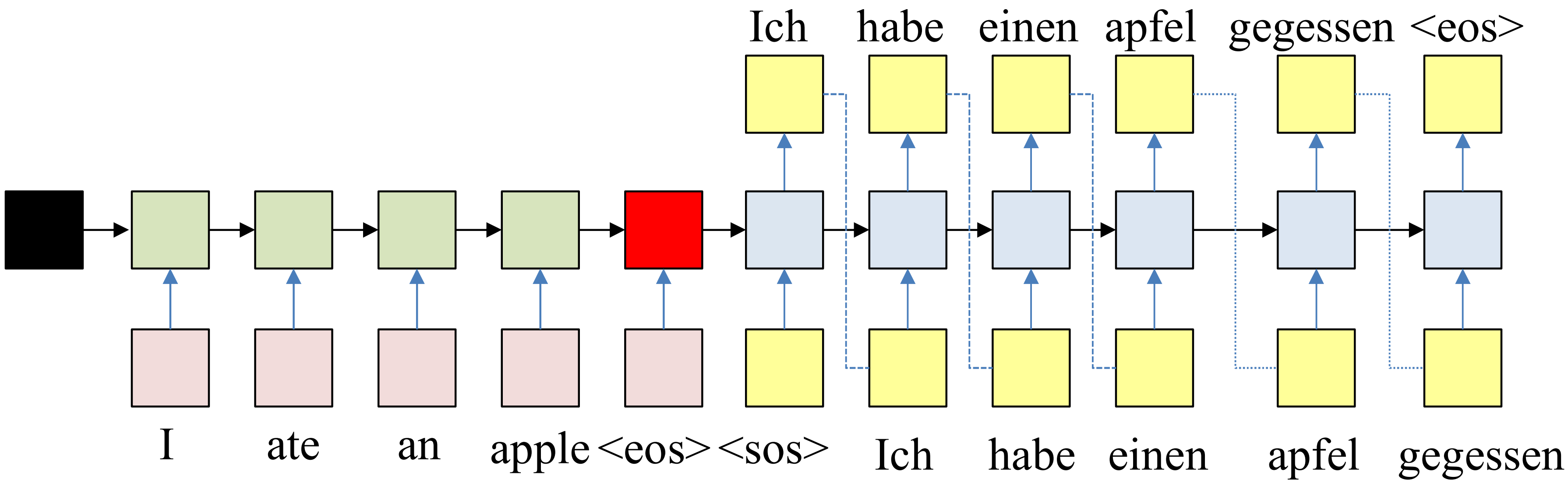


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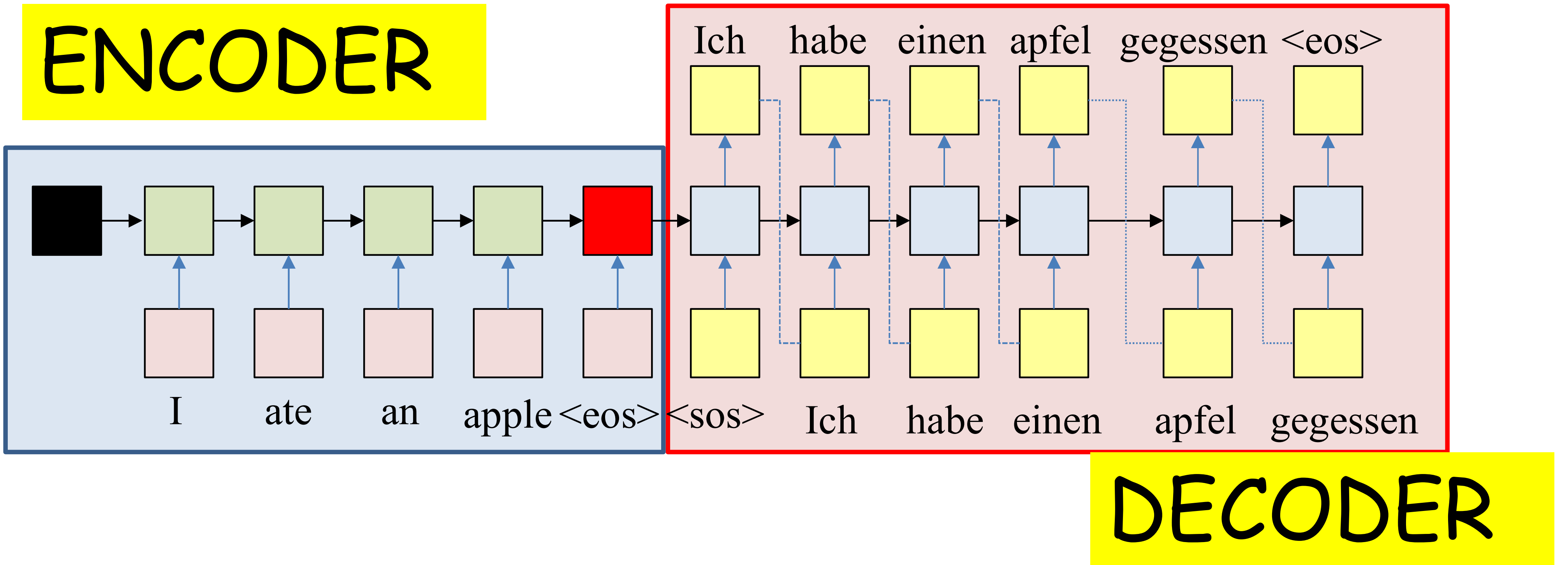
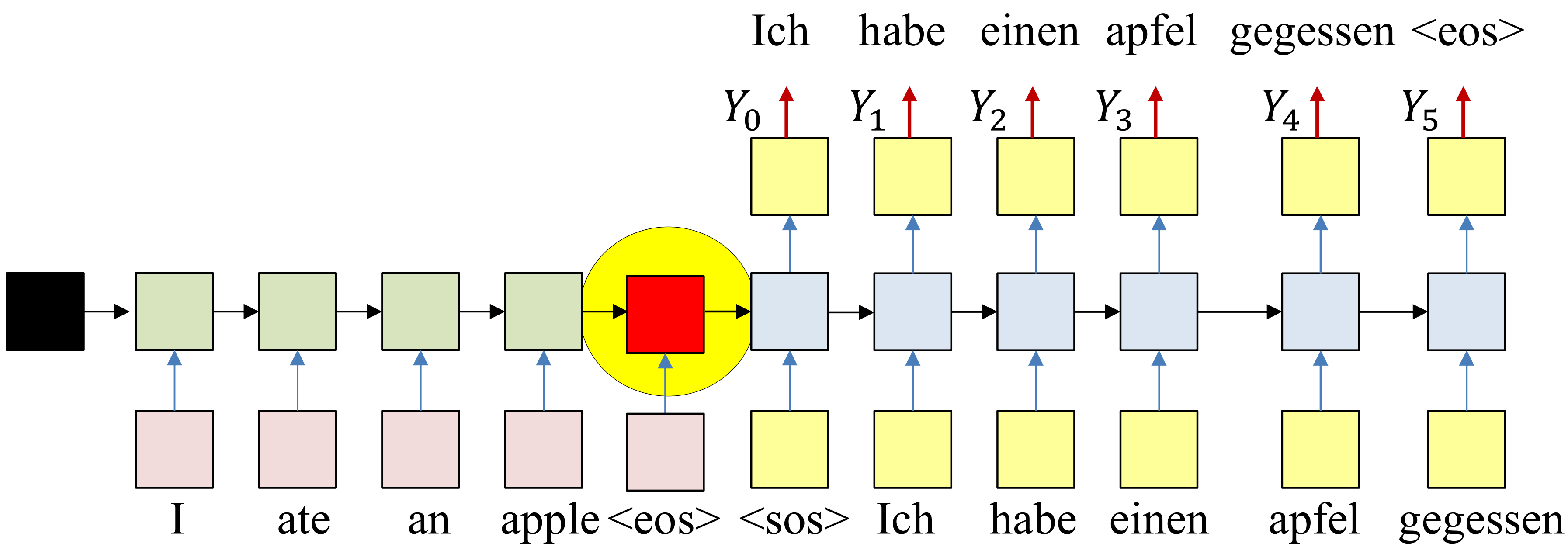


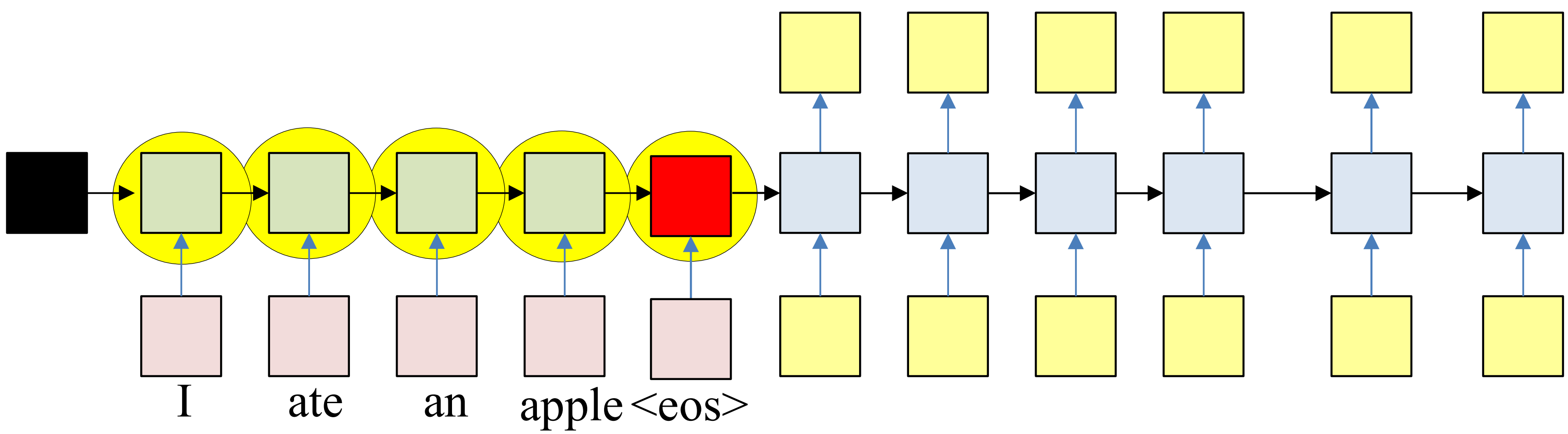

- The recurrent structure that extracts the hidden representation from the input sequence is the *encoder*
- The recurrent structure that utilizes this representation to produce the output sequence is the *decoder*

# A problem with this framework




- *All* the information about the input sequence is embedded into a *single* vector
  - The "hidden" node layer at the end of the input sequence
  - This one node is "overloaded" with information
    - Particularly if the input is long

# A problem with this framework




- In reality: *All* hidden values carry information
  - Some of which may be diluted by the time we get to the final state of the encoder

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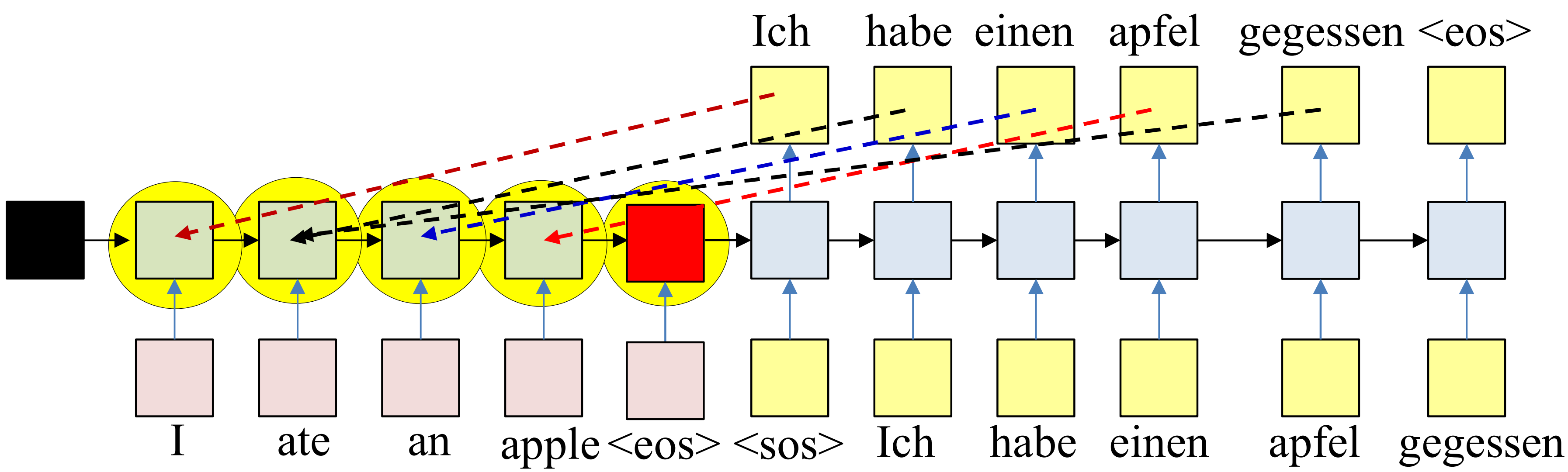


- In reality: *All* hidden values carry information
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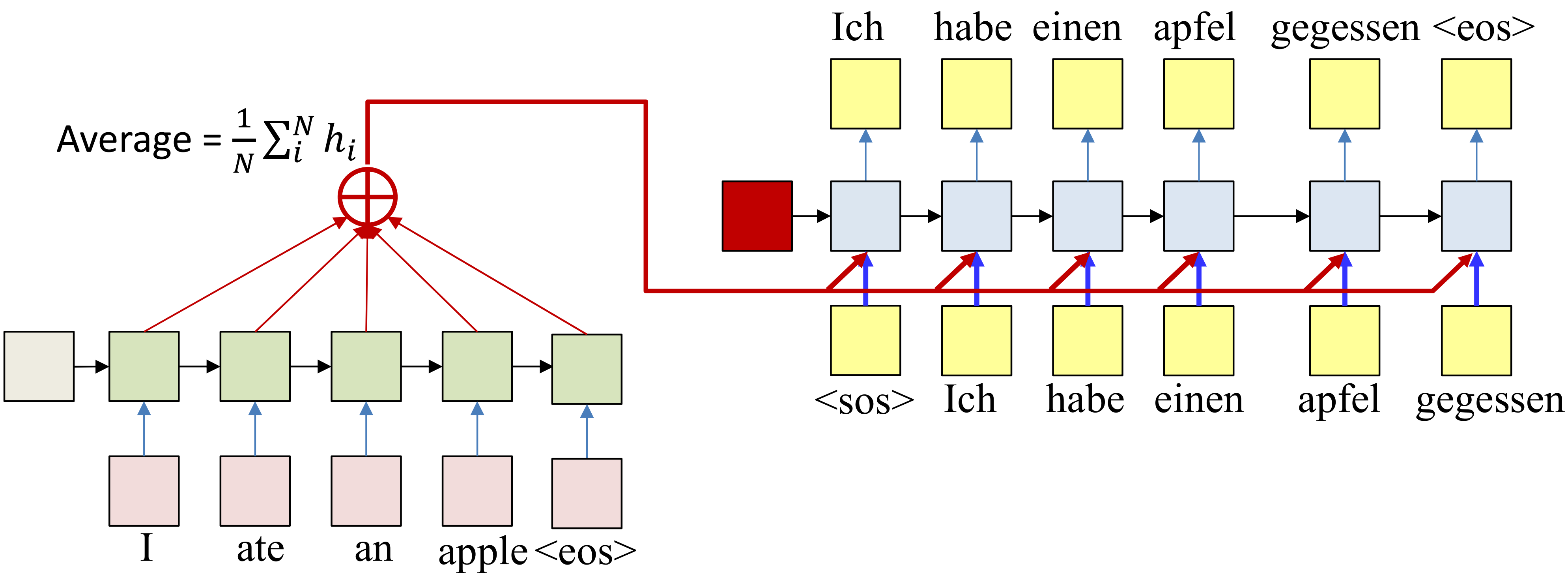
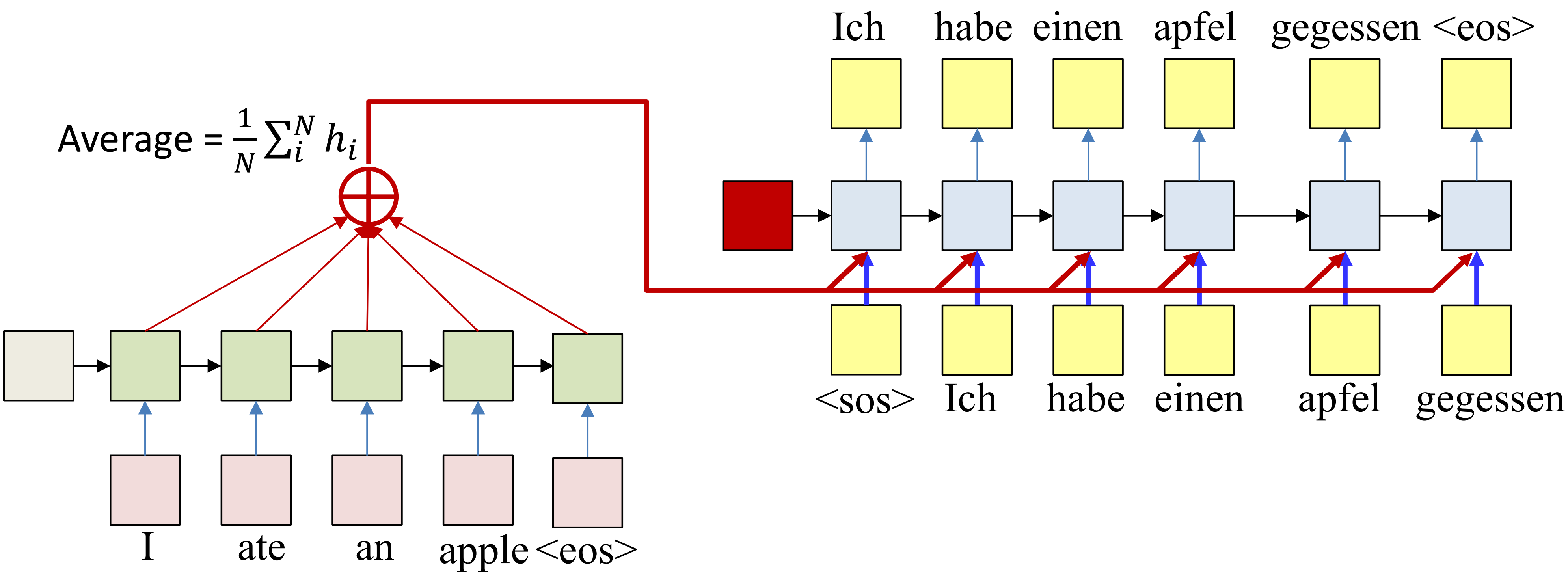
- *Every* output is related to the input directly
  - Not sufficient to have the encoder hidden state to *only* the initial state of the decoder
  - Misses the direct relation of the outputs to the inputs

# Using all input hidden states



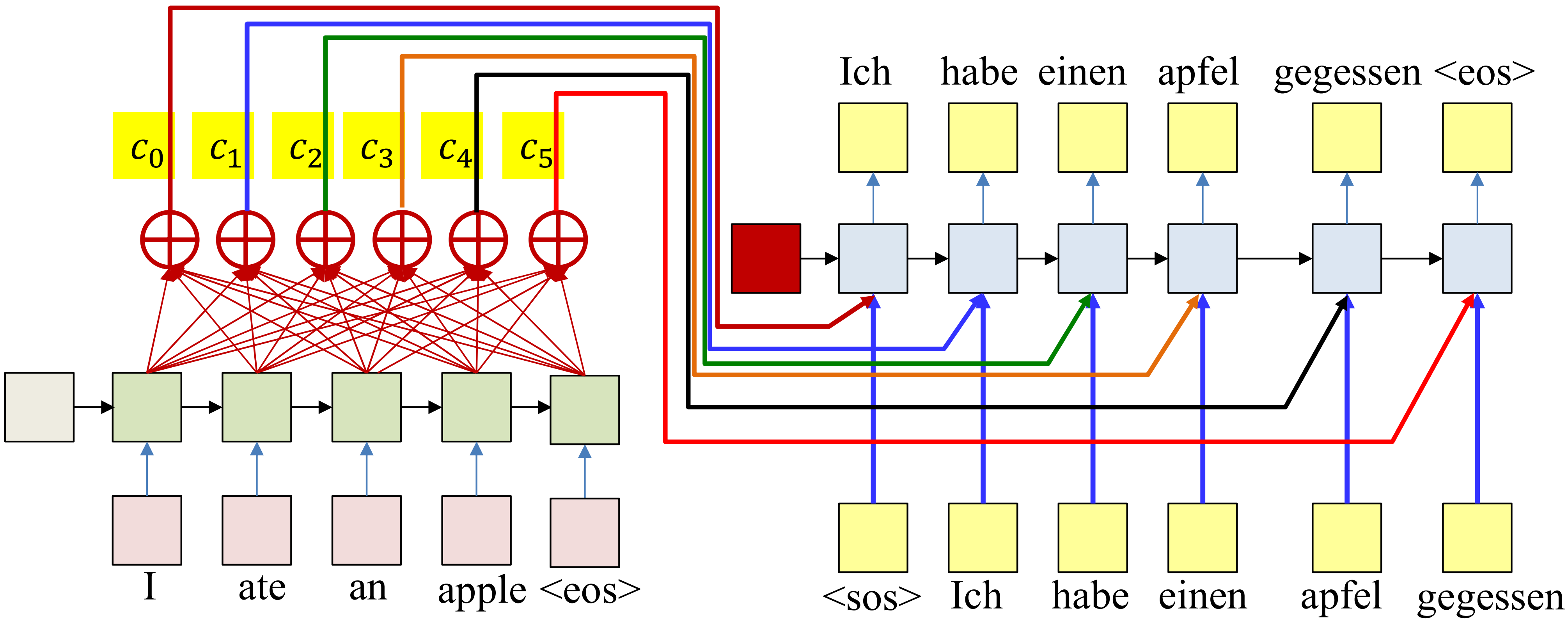

- Simple solution: Compute the average of all encoder hidden states
- Input this average to every stage of the decoder
- The initial decoder hidden state is now separate from the encoder
  - And may be a learnable parameter

# Using all input hidden states




- **Problem:** The average applies the same weight to every input
- It supplies the same average to every output word
- In practice, different outputs may be related to different inputs
  - E.g. “Ich” is most related to “I”, and “habe” and “gegessen” are both most related to “ate”

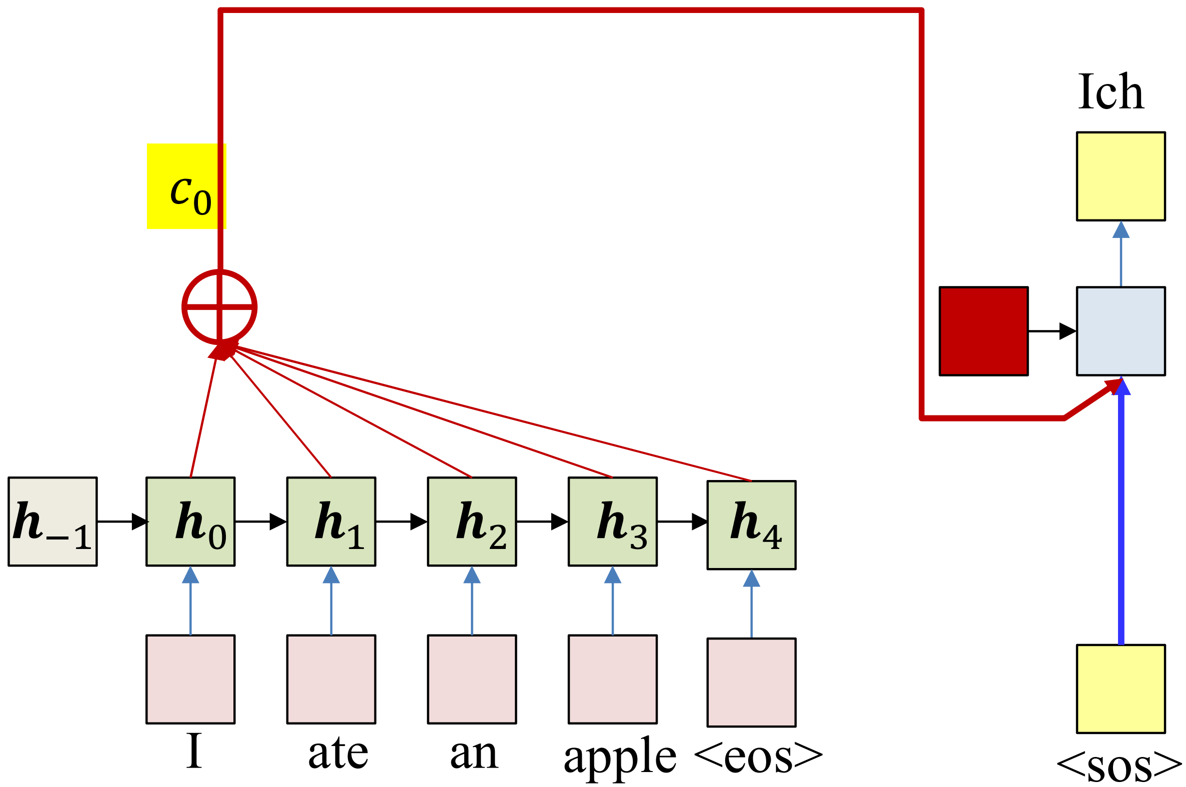
# Using all input hidden states




- **Solution:** Use a *different* weighted average for each output word
  - The weighted average provided for the kth output word is:

$$c_t = \frac{1}{N} \sum_{i}^{N} w_i(t) h_i$$

# Using all input hidden states




- **Solution:** Use a *different* weighted average for each output word
  - The weighted average provided for the kth output word is:

$$c_0 = \frac{1}{N}\sum_{i}^{N} w_i(0)h_i$$

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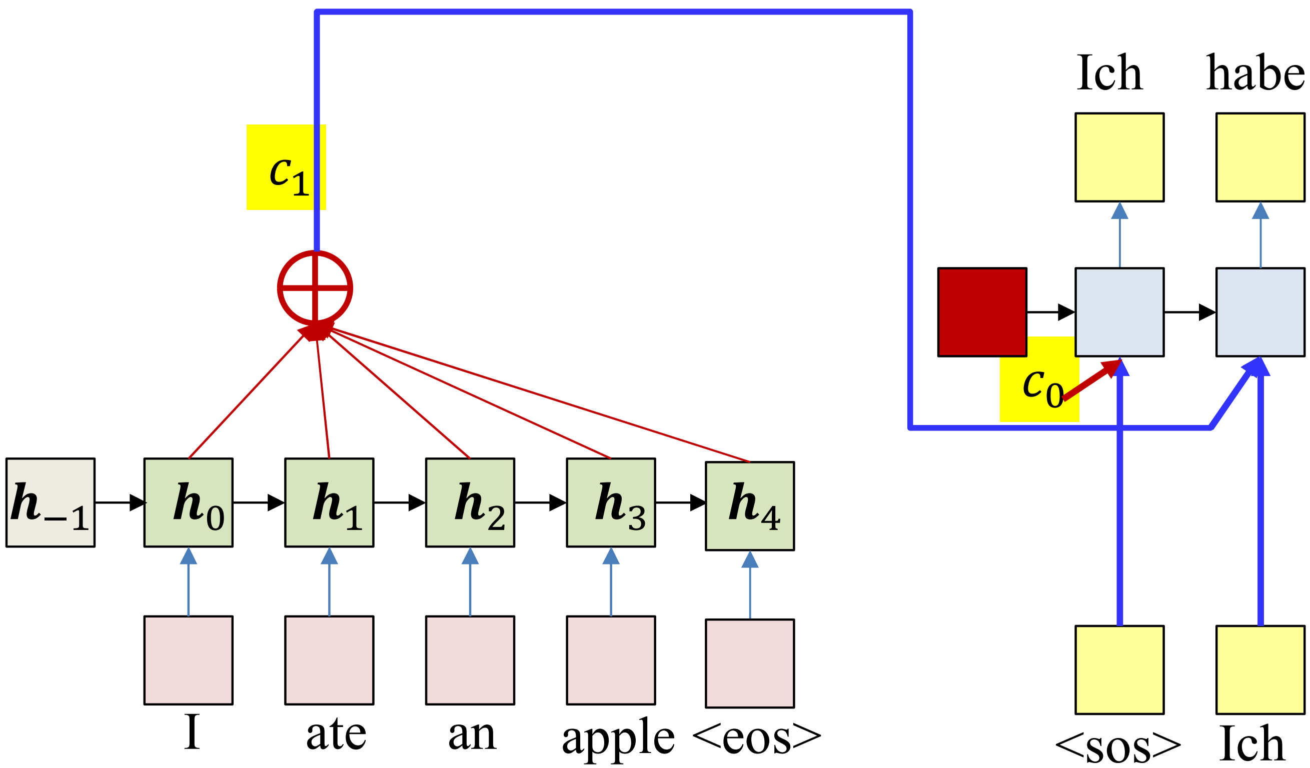
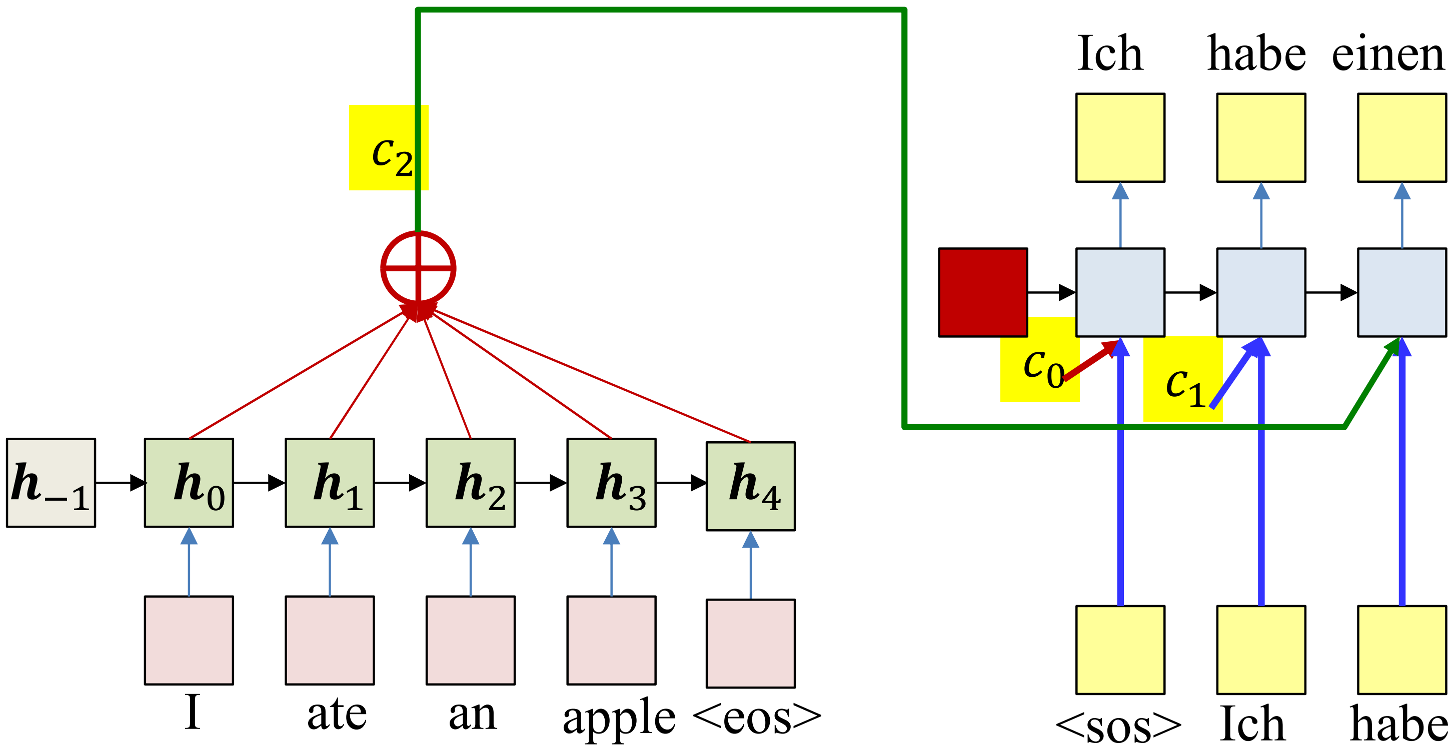



- **Solution:** Use a *different* weighted average for each output word
  - The weighted average provided for the kth output word is:

$$c_1 = \frac{1}{N}\sum_{i}^{N} w_i(1)h_i$$

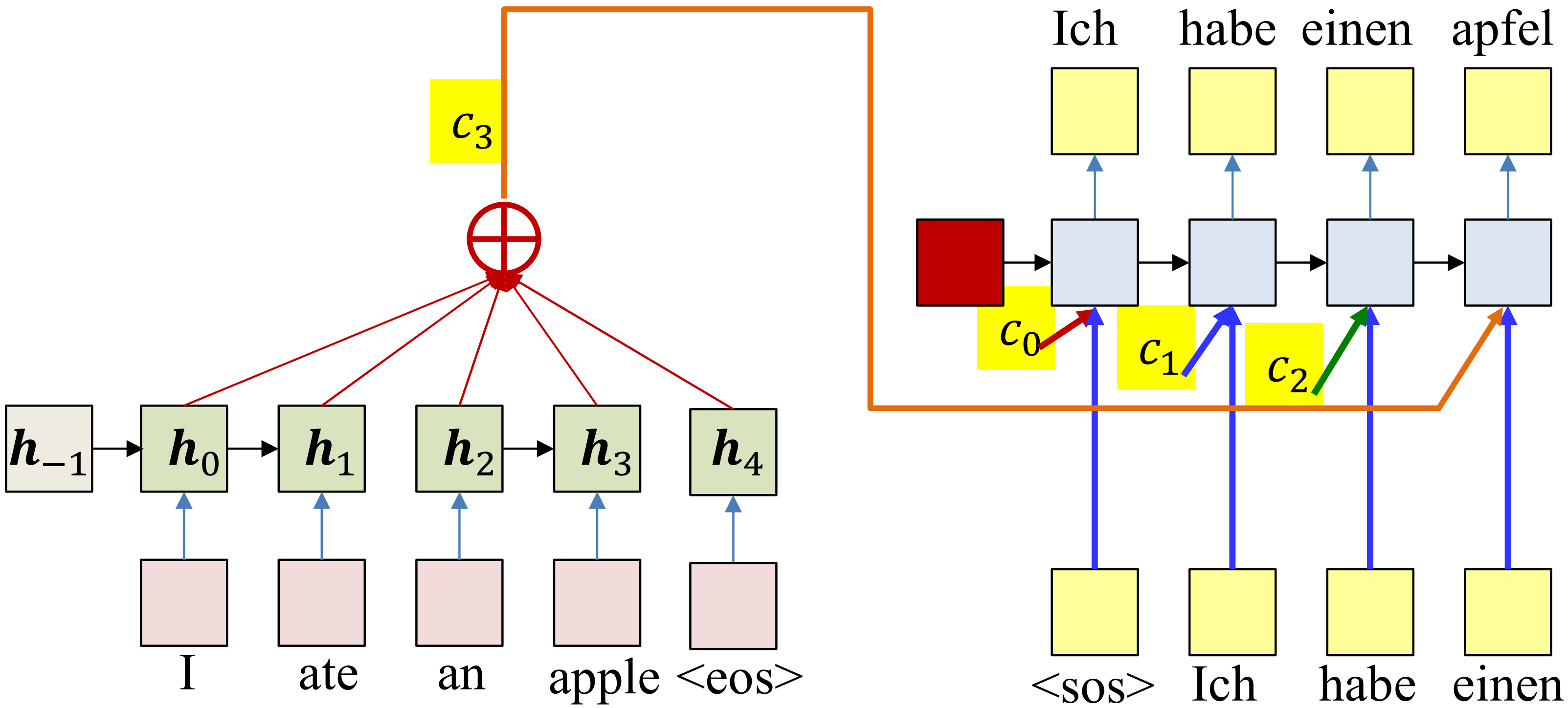
# Using all input hidden states




- **Solution:** Use a *different* weighted average for each output word
  - The weighted average provided for the kth output word is:

$$c_2 = \frac{1}{N}\sum_{i}^{N} w_i(2)h_i$$

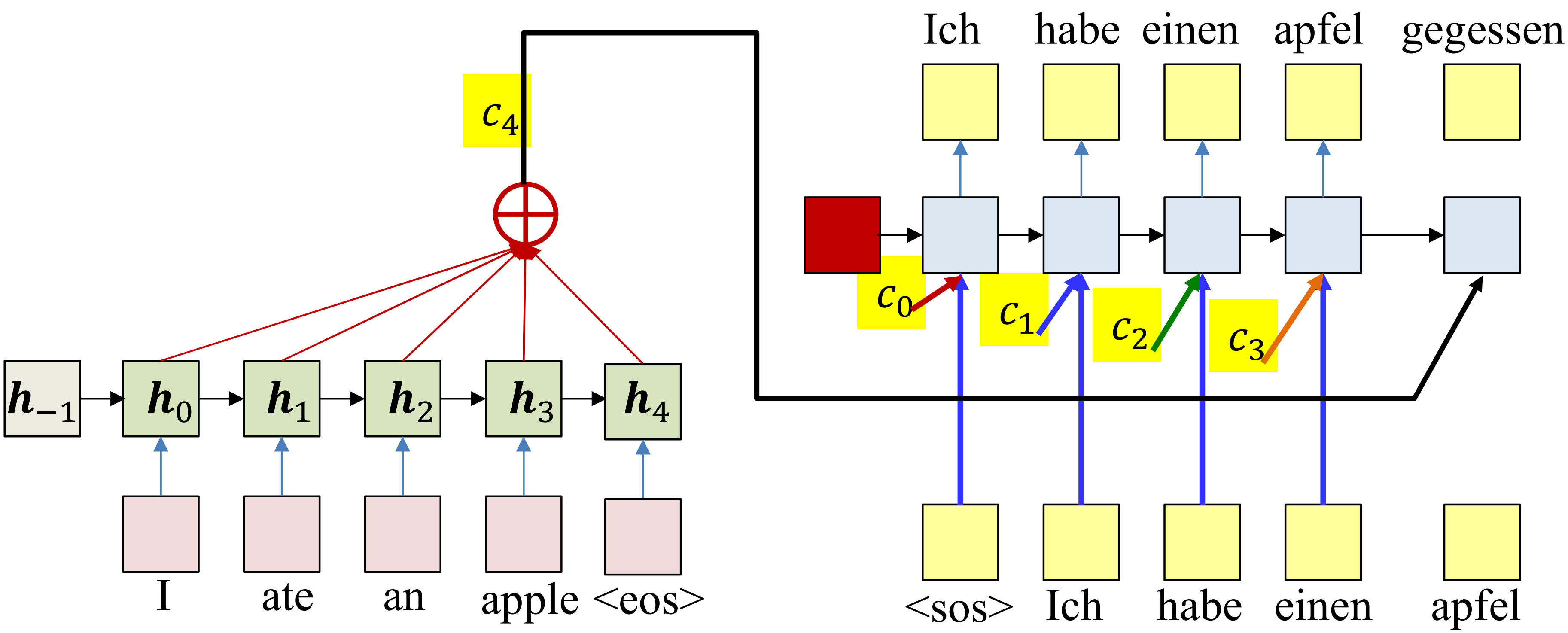
# Using all input hidden states




- **Solution:** Use a *different* weighted average for each output word
  - The weighted average provided for the kth output word is:

$$c_3 = \frac{1}{N}\sum_{i}^{N} w_i(3)h_i$$

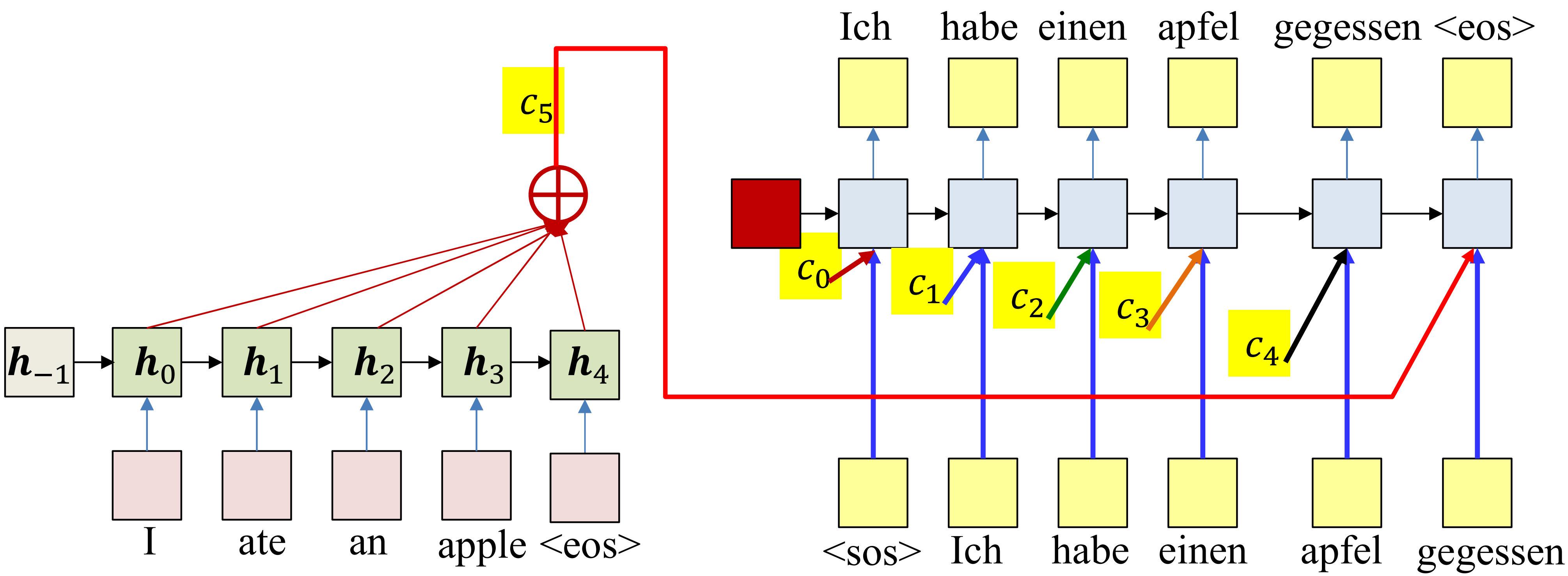
# Using all input hidden states




- **Solution:** Use a *different* weighted average for each output word
  - The weighted average provided for the kth output word is:

$$c_4 = \frac{1}{N}\sum_{i}^{N} w_i(4)h_i$$

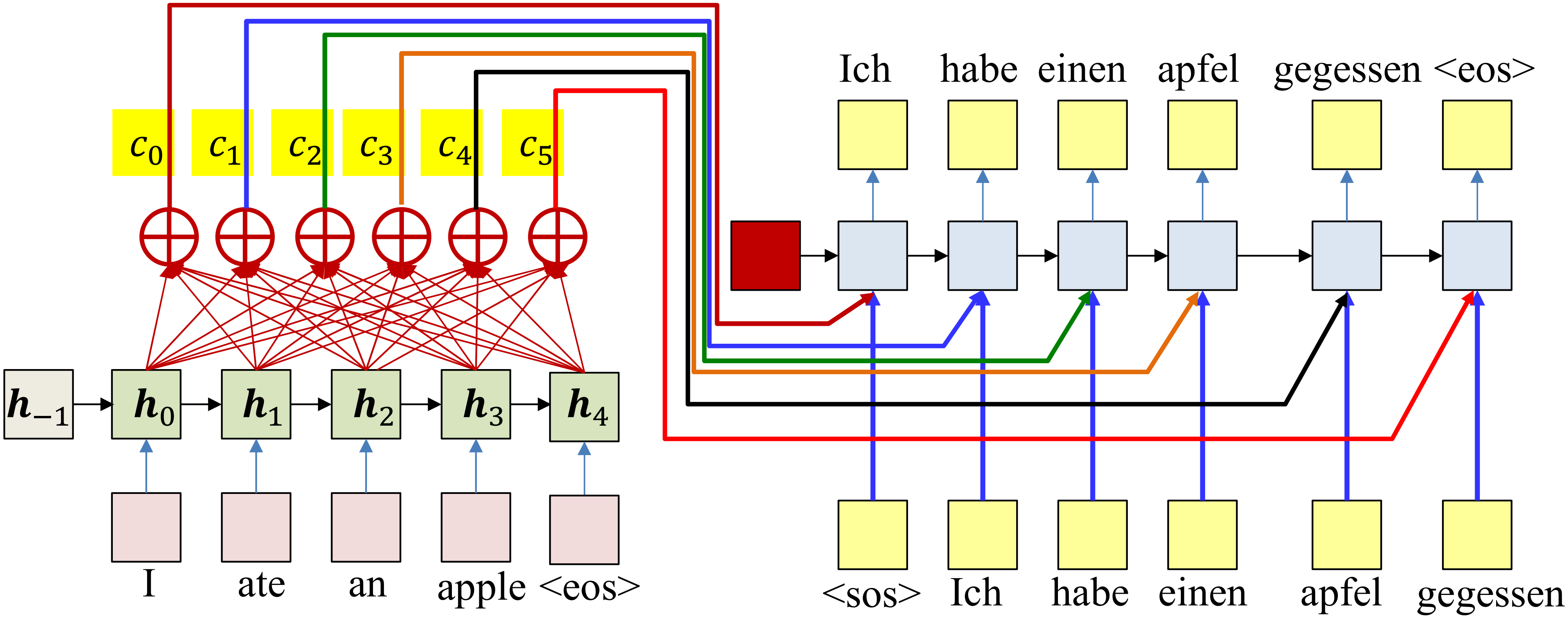
# Using all input hidden states




- **Solution:** Use a *different* weighted average for each output word
  - The weighted average provided for the kth output word is:

$$c_5 = \frac{1}{N}\sum_{i}^{N} w_i(5)h_i$$

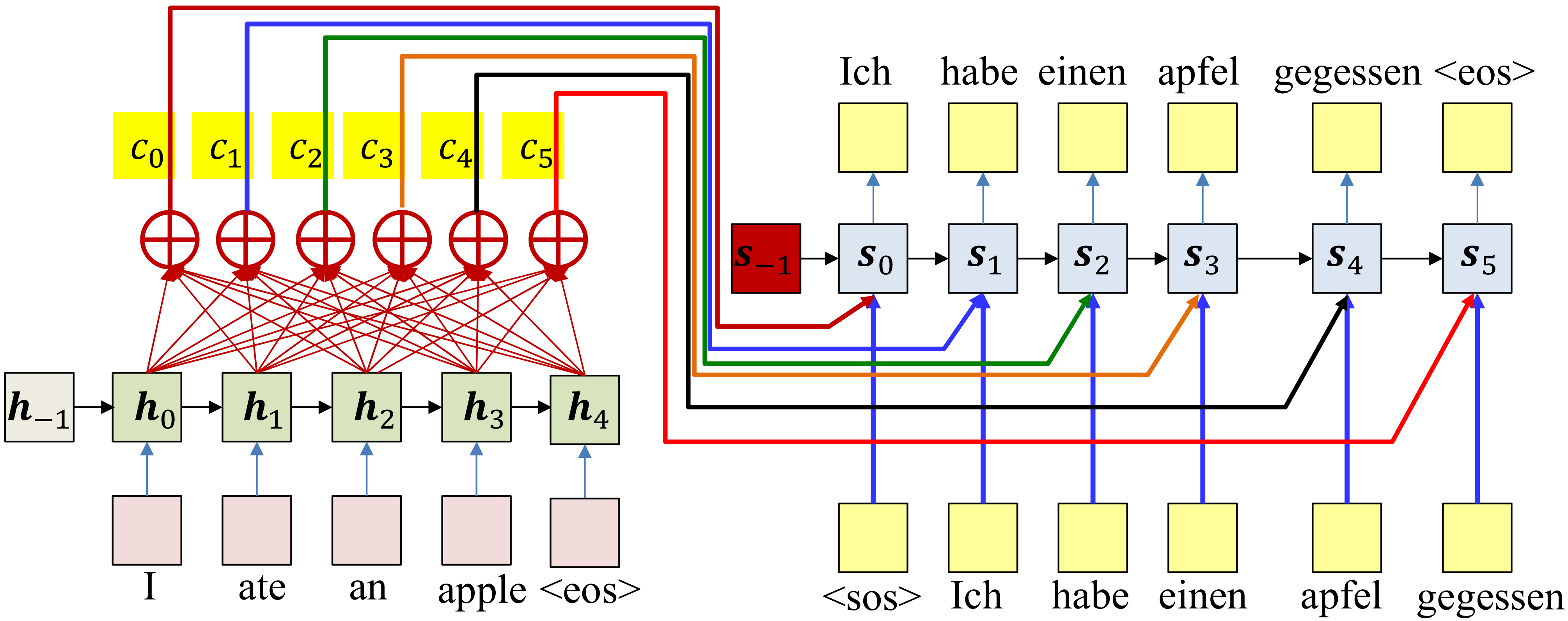
# Using all input hidden states




$$c_t = \frac{1}{N}\sum_{i}^{N} w_i(t)h_i$$

- This solution will work if the weights $w_{ki}$ can somehow be made to "focus" on the right input word
  - E.g., when predicting the word "apfel", $w_3(4)$, the weight for "apple" must be high while the rest must be low
- How do we generate such weights??

# Attention Models

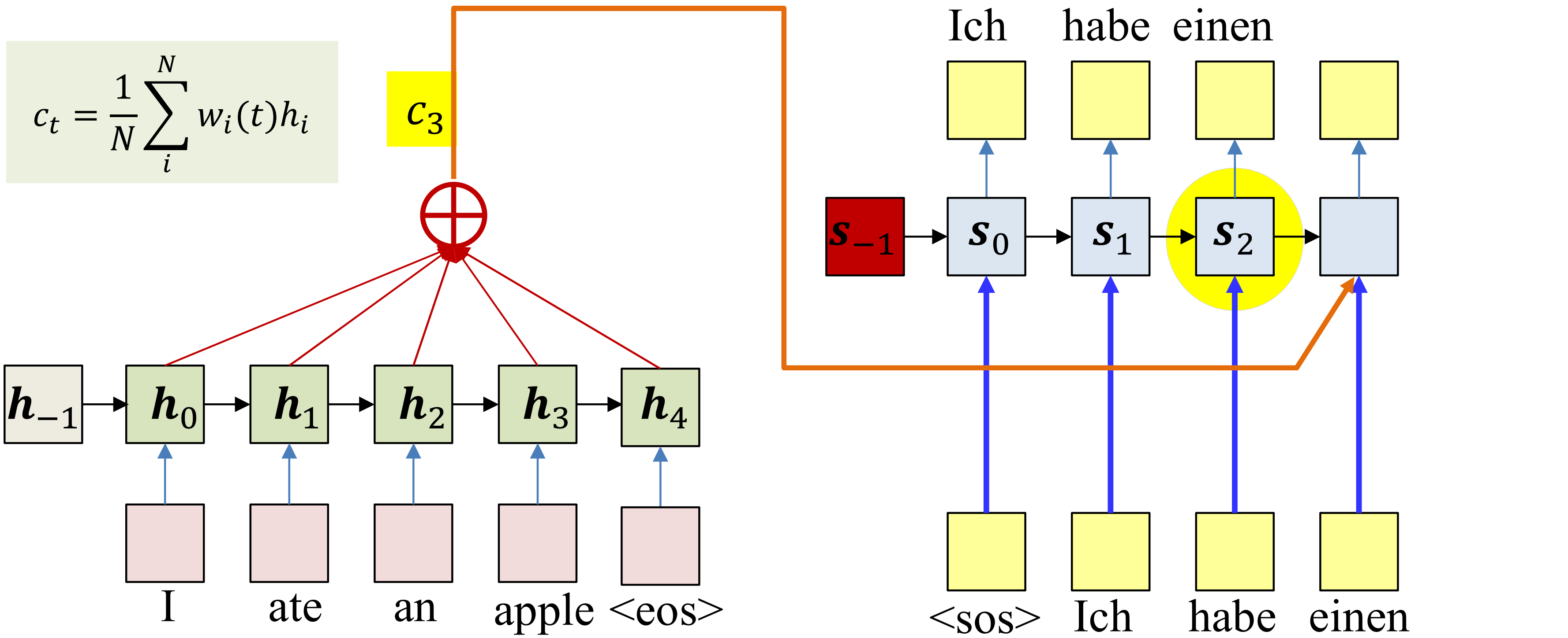



$$c_t = \frac{1}{N}\sum_{i}^{N} w_i(t) h_i$$

- **Attention weights:** The weights $w_i(t)$ are dynamically computed as functions of decoder state
  - Expectation: if the model is well-trained, this will automatically "highlight" the correct input
- But how are these computed?

# Attention weights at time $t$

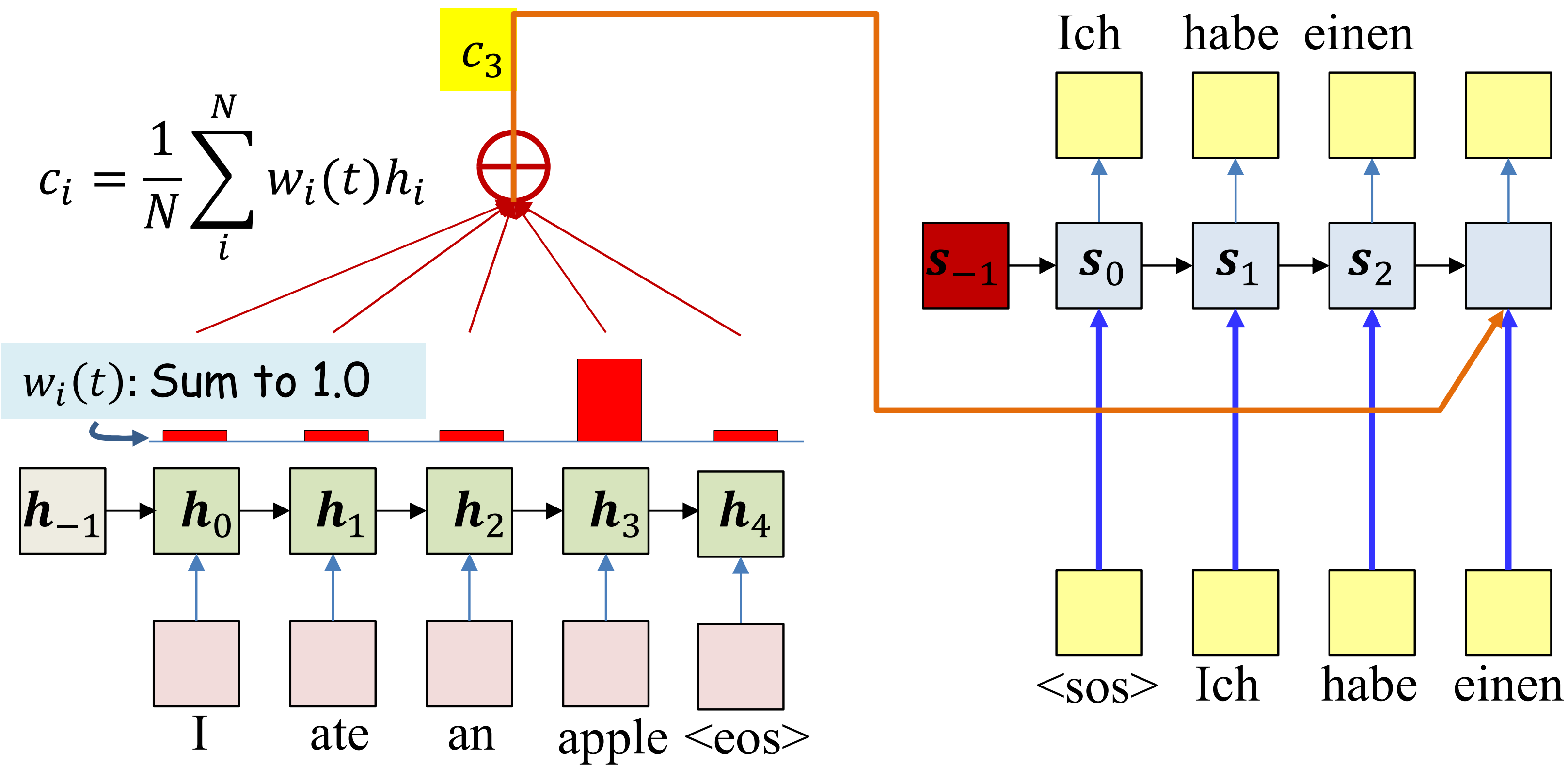
$$c_t = \frac{1}{N}\sum_i^N w_i(t)h_i$$




- The “attention” weights $w_i(t)$ at time $t$ must be computed from available information at time $t$
- The primary information is $s_{t-1}$ (the state at time time $t-1$)
  - Also, the input word at time $t$, but generally not used for simplicity

$$w_i(t) = a(\boldsymbol{h}_i, \boldsymbol{s}_{t-1})$$

# Requirement on attention weights




- The weights $w_i(t)$ must be positive and sum to 1.0
  - I.e. be a distribution
  - Ideally, they must be high for the most relevant inputs for the ith output and low elsewhere

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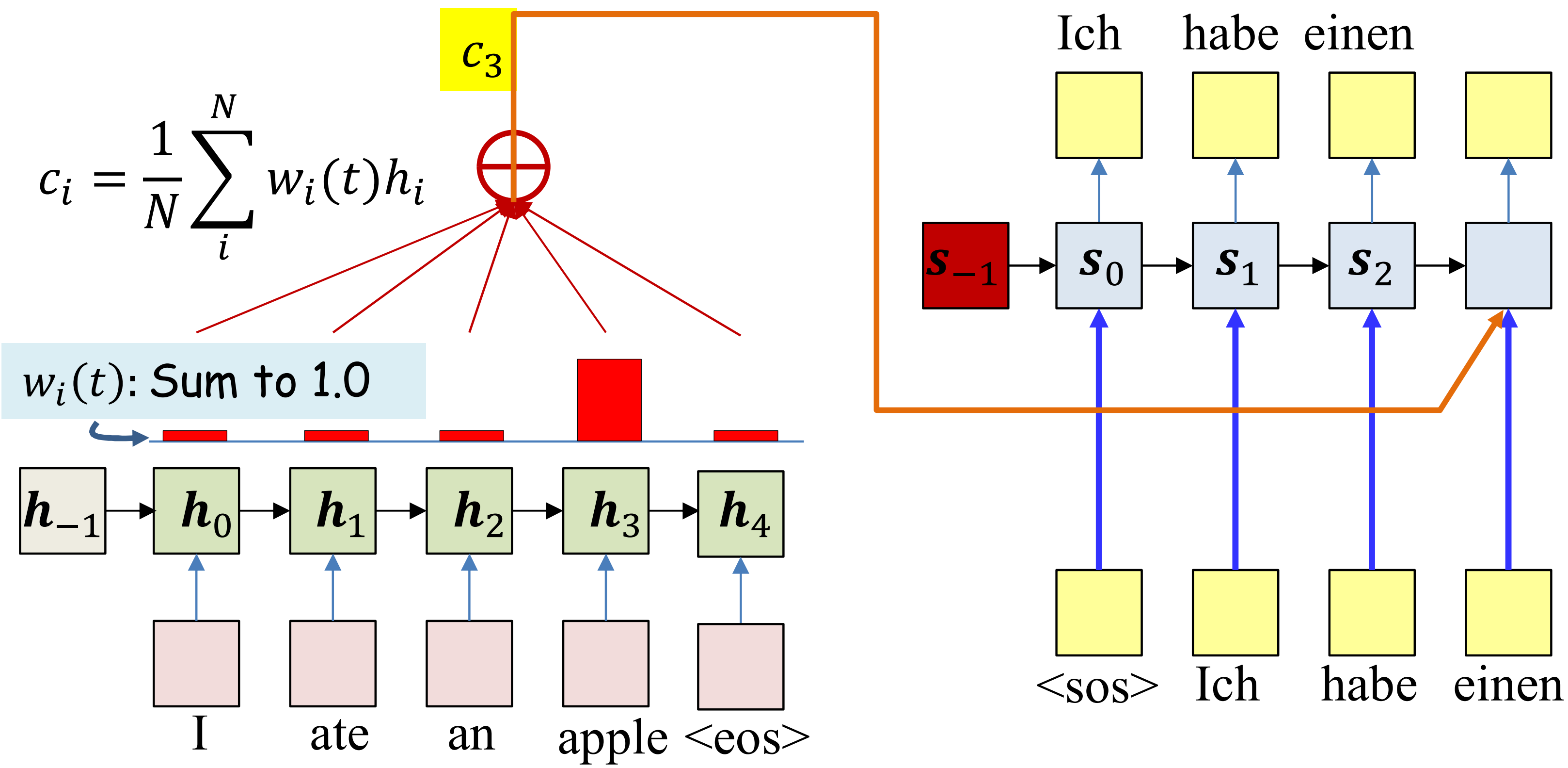


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- Solution: A two step weight computation
  - First compute *raw* weights (which could be +ve or –ve)
  - Then softmax them to convert them to a distribution

$$e_i(t) = g(\boldsymbol{h}_i, \boldsymbol{s}_{t-1})$$

$$w_i(t) = \frac{\exp(e_i(t))}{\sum_j \exp(e_j(t))}$$

# Poll 1 (@1209,@1210,@1212)

The attention framework computes a different “context” vector at each output step (T/F)

- True
- False

The context vector is chosen as the hidden (encoder) representation of the input word that is assigned the highest attention weight (T/F)

- True
- False

The attention weight to any input word is a function of the hidden encoder representation of the word and the most recent decoder state (T/F)

- True
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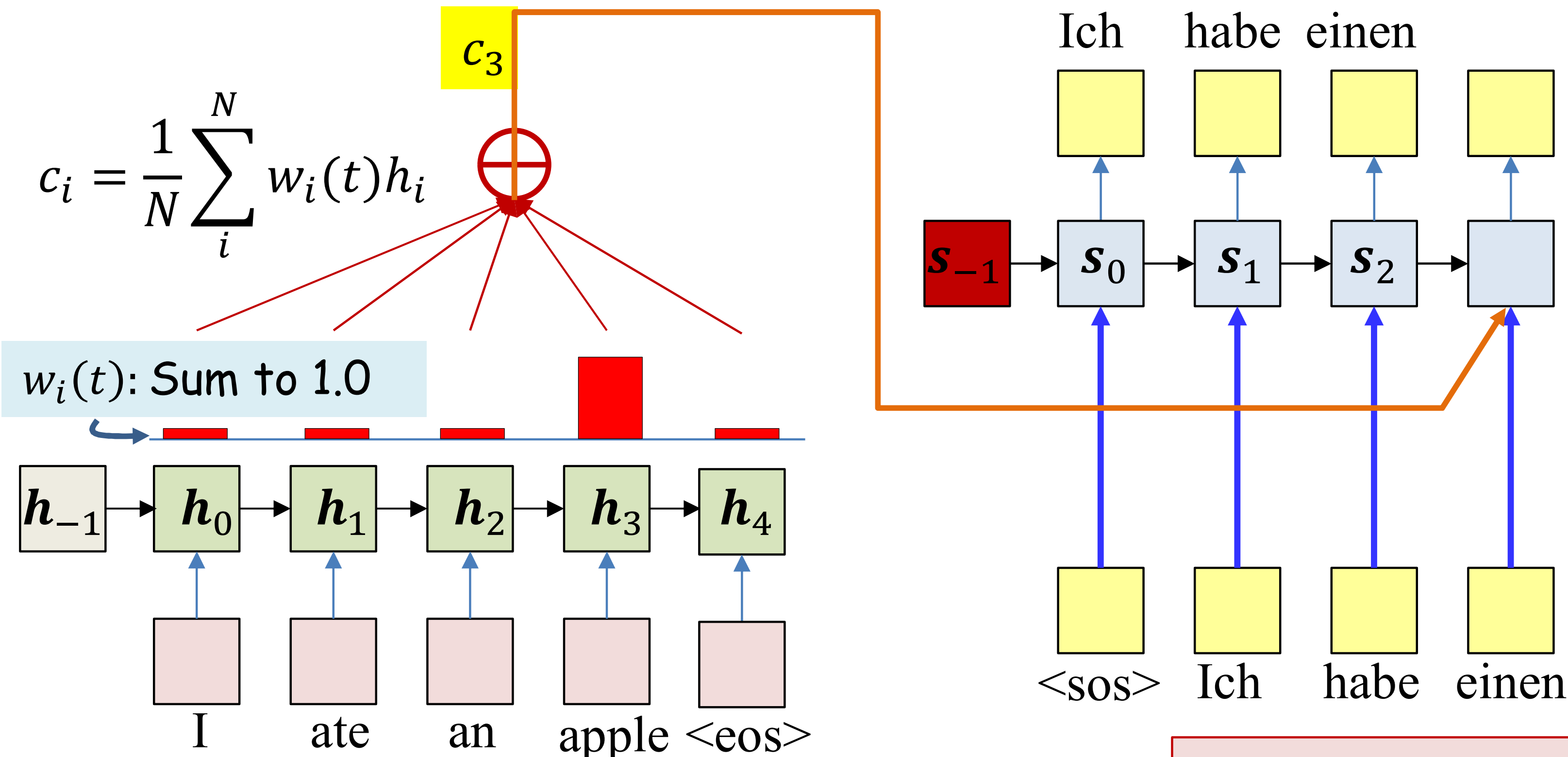


What is this function?

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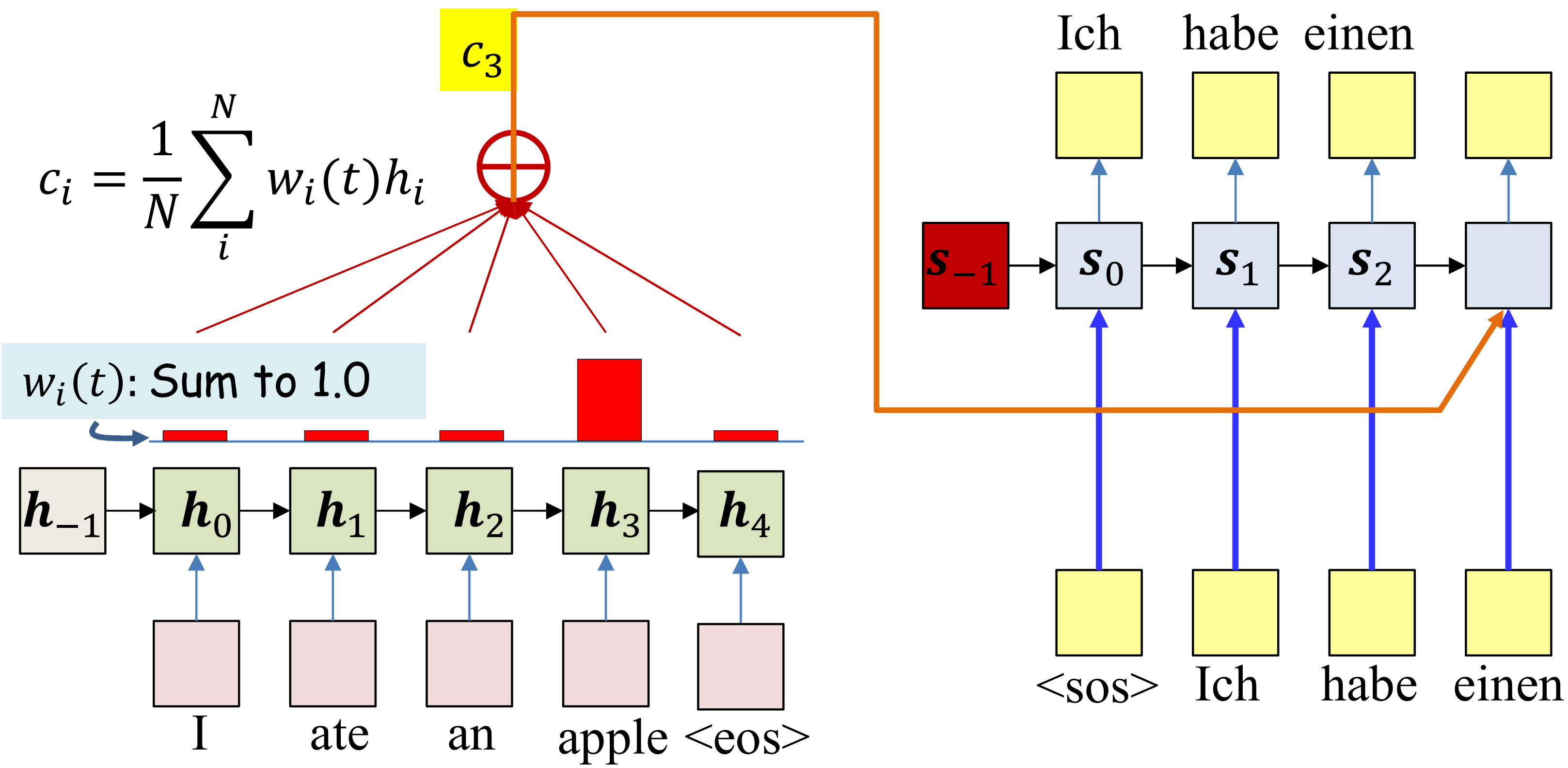


- Typical options for $g()$ (**variables in red must learned**)

$$g(\boldsymbol{h}_i, \boldsymbol{s}_{t-1}) = \boldsymbol{h}_i^T \boldsymbol{s}_{t-1}$$

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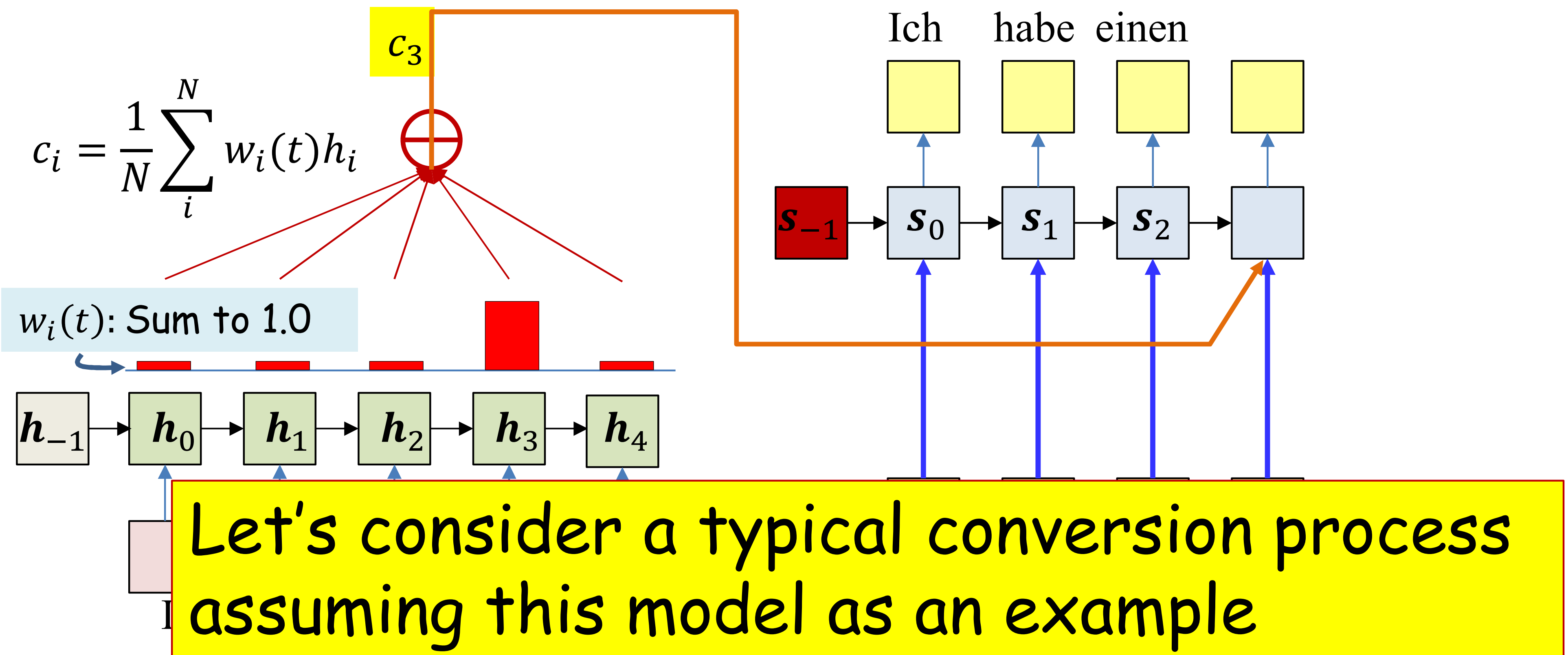


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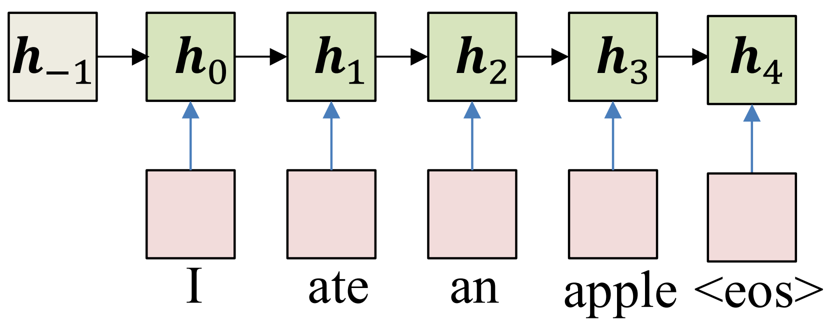
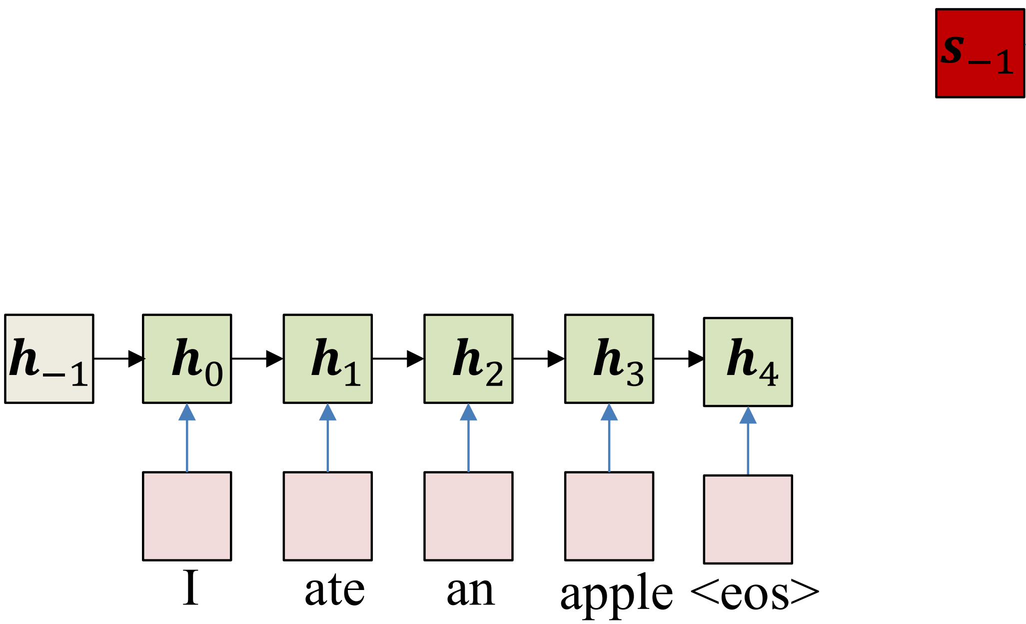
# Converting an input: Inference




- Pass the input through the encoder to produce hidden representations $\boldsymbol{h}_i$

# Converting an input: Inference

This may be
- a learned parameter, or
- Or just set to some fixed value, e.g. a vector of 1s or 0s, or
- Or the average of all the encoder embeddings: $mean(h_0, \dots, h_4)$
- Or $W_{init}\ mean(h_0, \dots, h_4)$ where $W_{init}$ is a learned parameter

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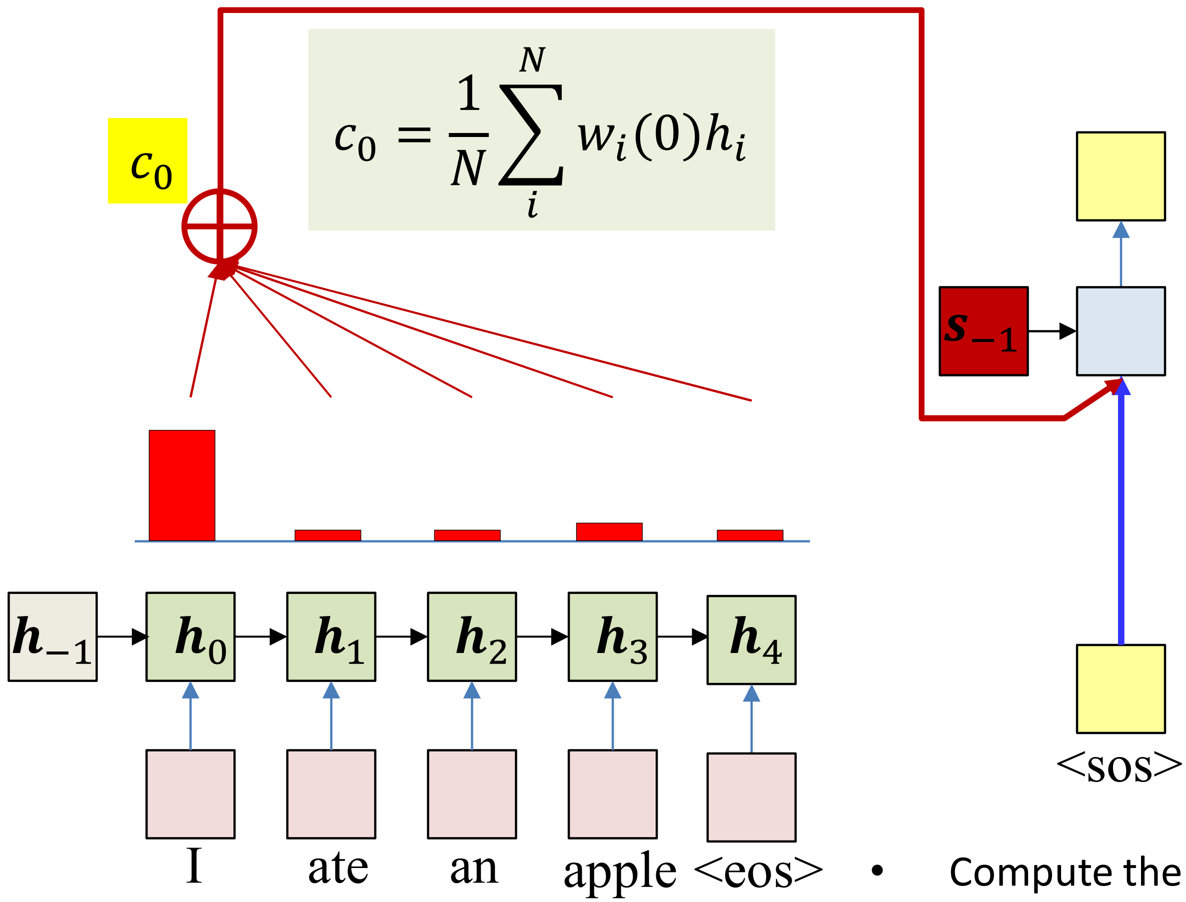


$$g(\boldsymbol{h}_i, \boldsymbol{s}_{-1}) = \boldsymbol{h}_i^T \boldsymbol{W}_g \boldsymbol{s}_{-1}$$

$$e_i(0) = g(\boldsymbol{h}_i, \boldsymbol{s}_{-1})$$

$$w_i(0) = \frac{\exp(e_i(0))}{\sum_j \exp(e_j(0))}$$

- Compute the attention weights $w_i(0)$ for the first output using $s_{-1}$
  - Will be a distribution over the input words
- Compute “context” $c_0$
  - Weighted sum of input word hidden states
- Input $c_0$ and <sos> to the decoder at time 0
  - <sos> because we are starting a new sequence
  - In practice we will enter the *embedding* of <sos>

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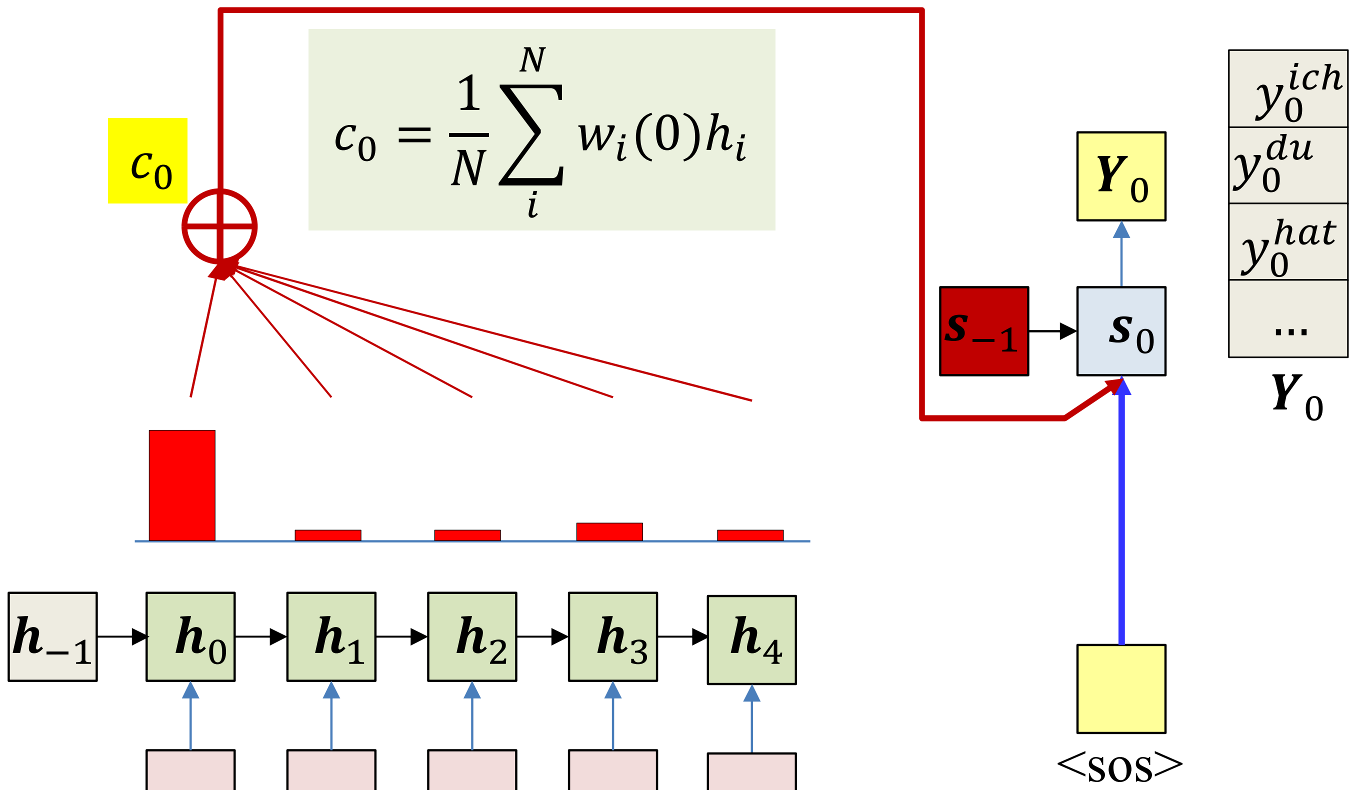


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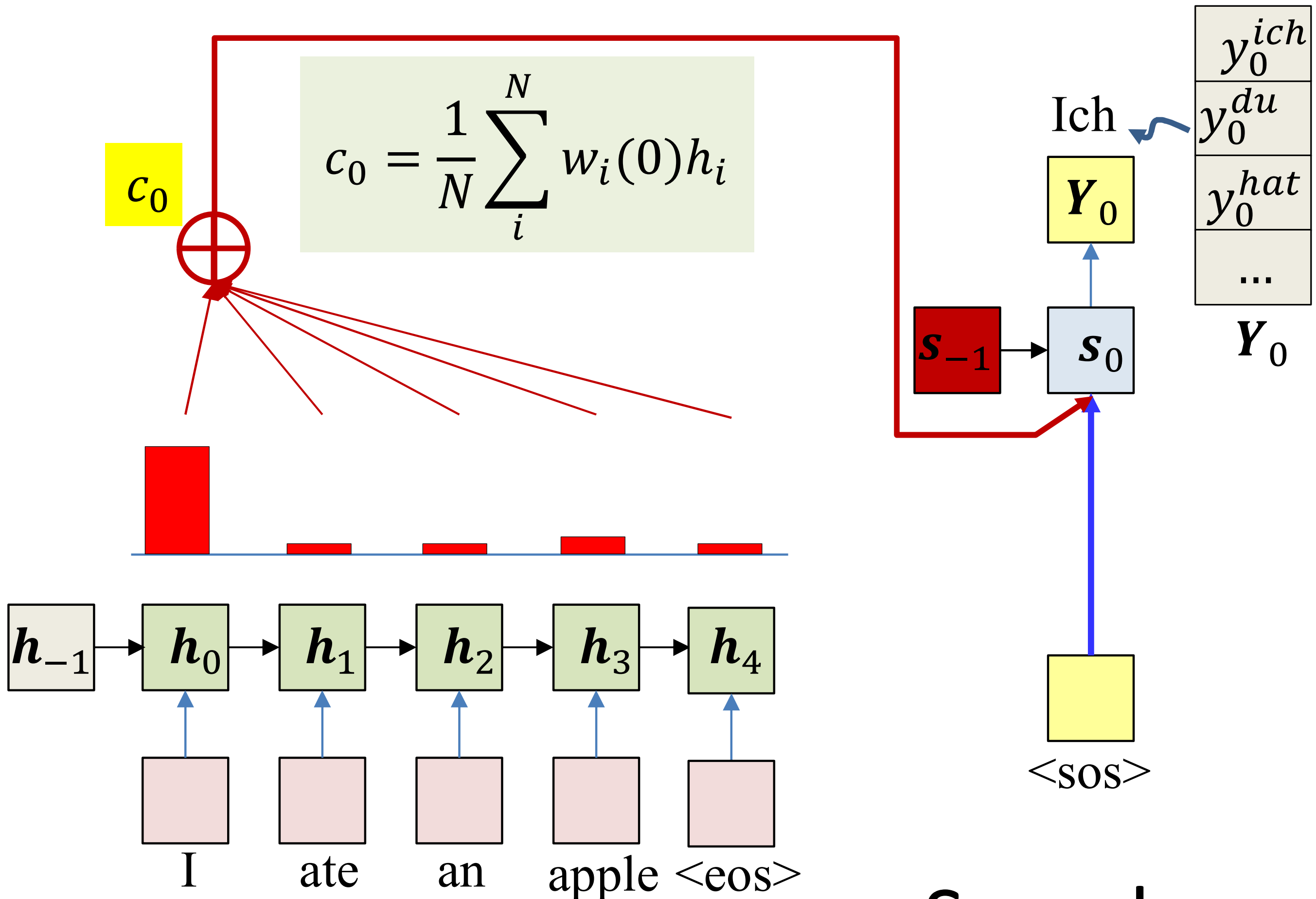
- The decoder computes
  - $s_0$
  - A probability distribution over the output vocabulary
    - Output of softmax output layer

# Converting an input: Inference

$$c_0 = \frac{1}{N}\sum_{i}^{N} w_i(0) h_i$$



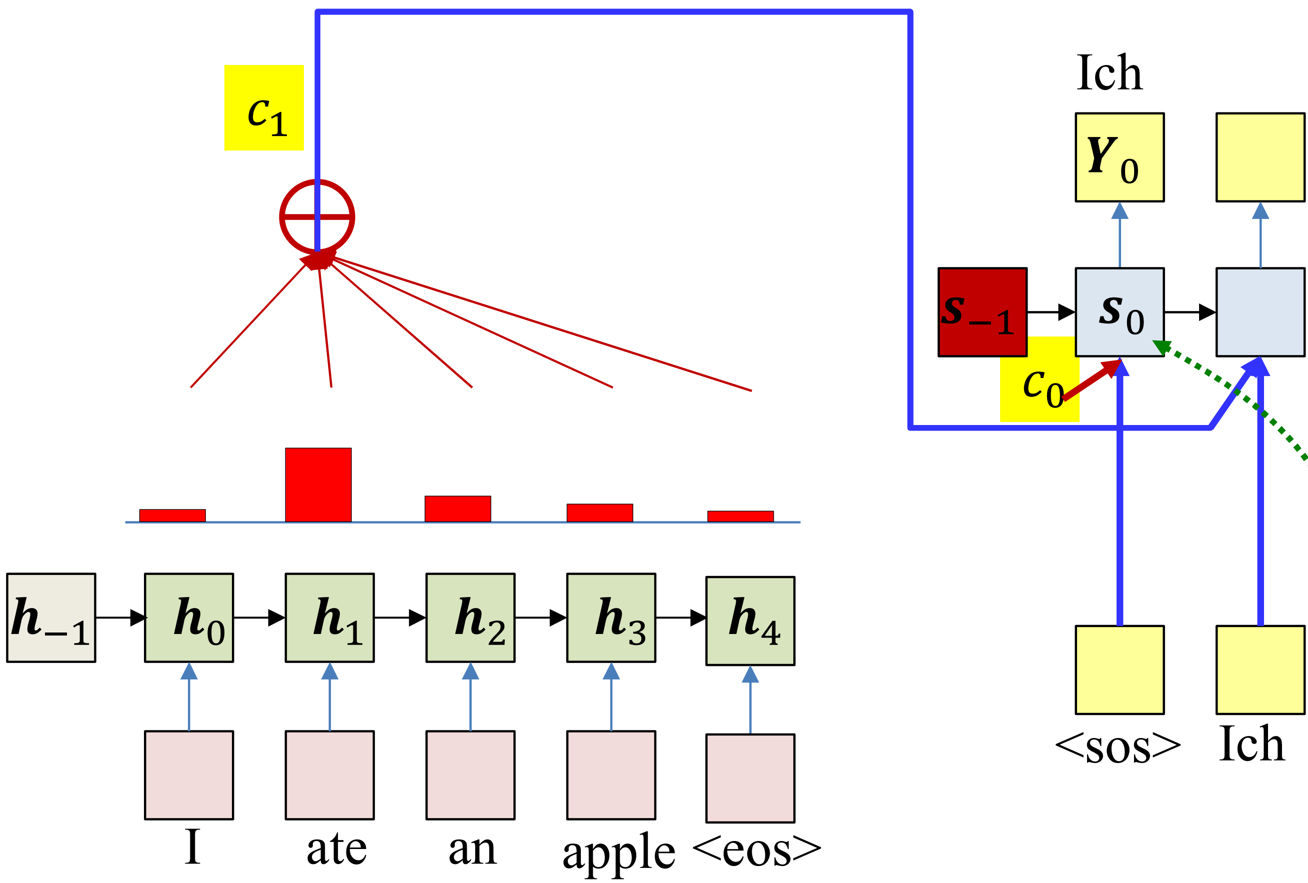

$$g(\boldsymbol{h}_i, \boldsymbol{s}_{-1}) = \boldsymbol{h}_i^T \boldsymbol{W}_g \boldsymbol{s}_{-1}$$

$$e_i(0) = g(\boldsymbol{h}_i, \boldsymbol{s}_{-1})$$

$$w_i(0) = \frac{\exp(e_i(0))}{\sum_j \exp(e_j(0))}$$

- Sample a word from the output distribution

# Converting an input: Inference




- Compute the attention weights $w_i(1)$ over all inputs for the *second* output using $s_0$
  - Compute raw weights, followed by softmax
- Compute "context" $c_1$
  - Weighted sum of input hidden representations
- Input $c_1$ and first output word to the decoder
  - In practice we enter the *embedding* of the word

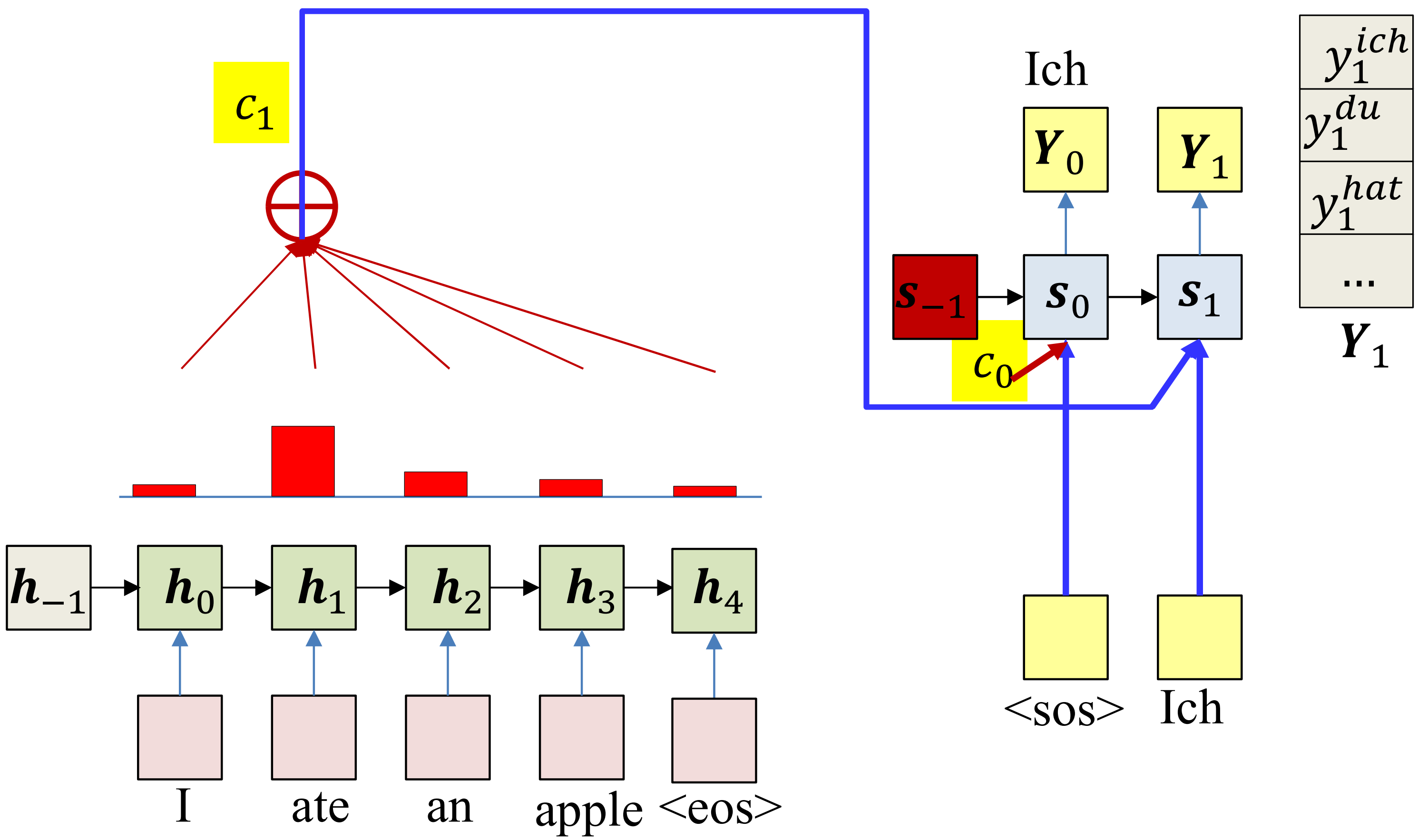
$$g(\boldsymbol{h}_i, \boldsymbol{s}_0) = \boldsymbol{h}_i^T \boldsymbol{W}_g \boldsymbol{s}_0$$

$$e_i(1) = g(\boldsymbol{h}_i, \boldsymbol{s}_0)$$

$$w_i(1) = \frac{\exp(e_i(1))}{\sum_j \exp(e_j(1))}$$

$$c_1 = \frac{1}{N} \sum_i^N w_i(1) h_i$$

# Converting an input: Inference




- The decoder computes
  - $s_1$
  - A probability distribution over the output vocabulary

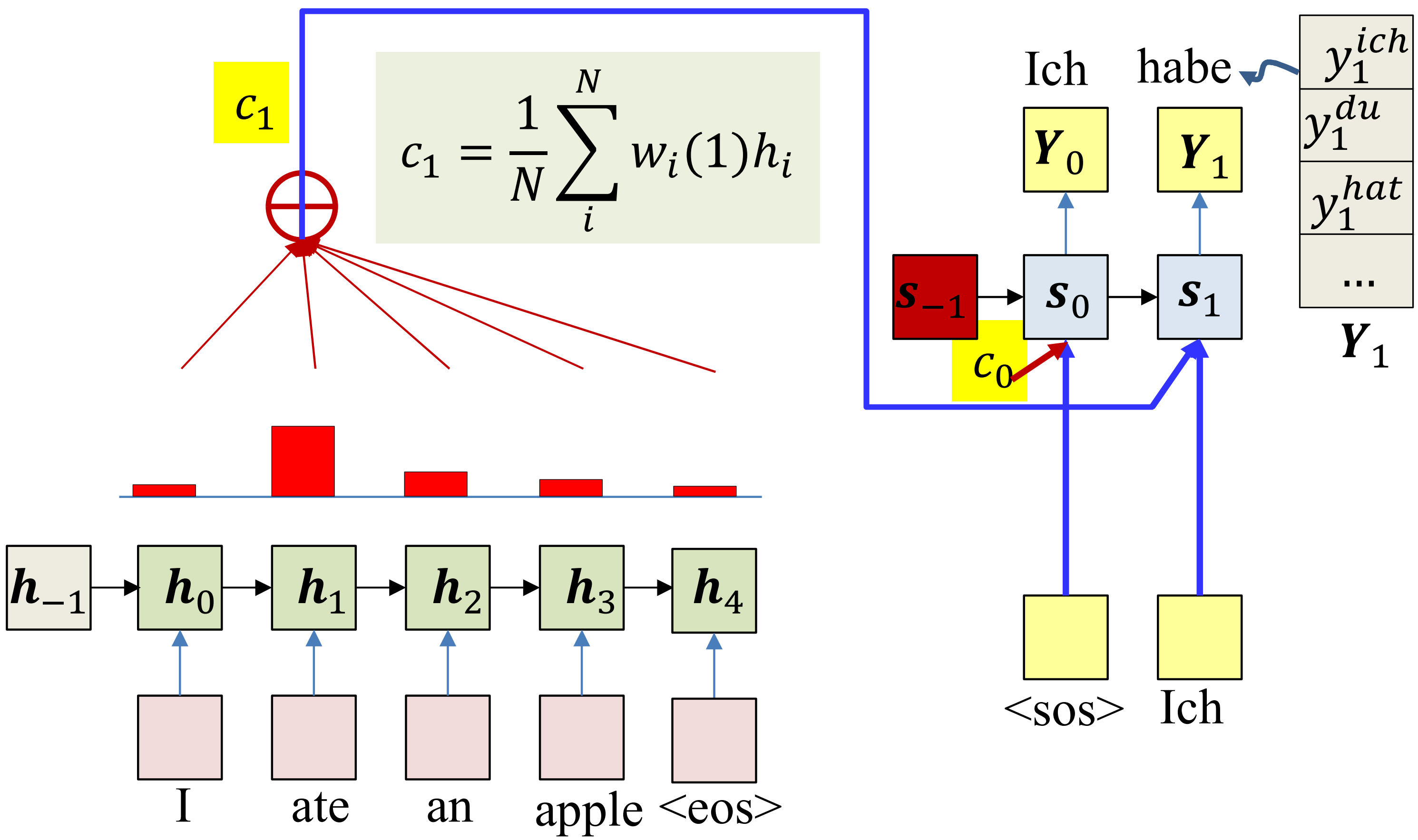
$$g(\boldsymbol{h}_i, \boldsymbol{s}_0) = \boldsymbol{h}_i^T \boldsymbol{W}_g \boldsymbol{s}_0$$

$$e_i(1) = g(\boldsymbol{h}_i, \boldsymbol{s}_0)$$

$$w_i(1) = \frac{\exp(e_i(1))}{\sum_j \exp(e_j(1))}$$

$$c_1 = \frac{1}{N}\sum_i^N w_i(1) h_i$$

# Converting an input: Inference




- Sample the second word from the output distribution

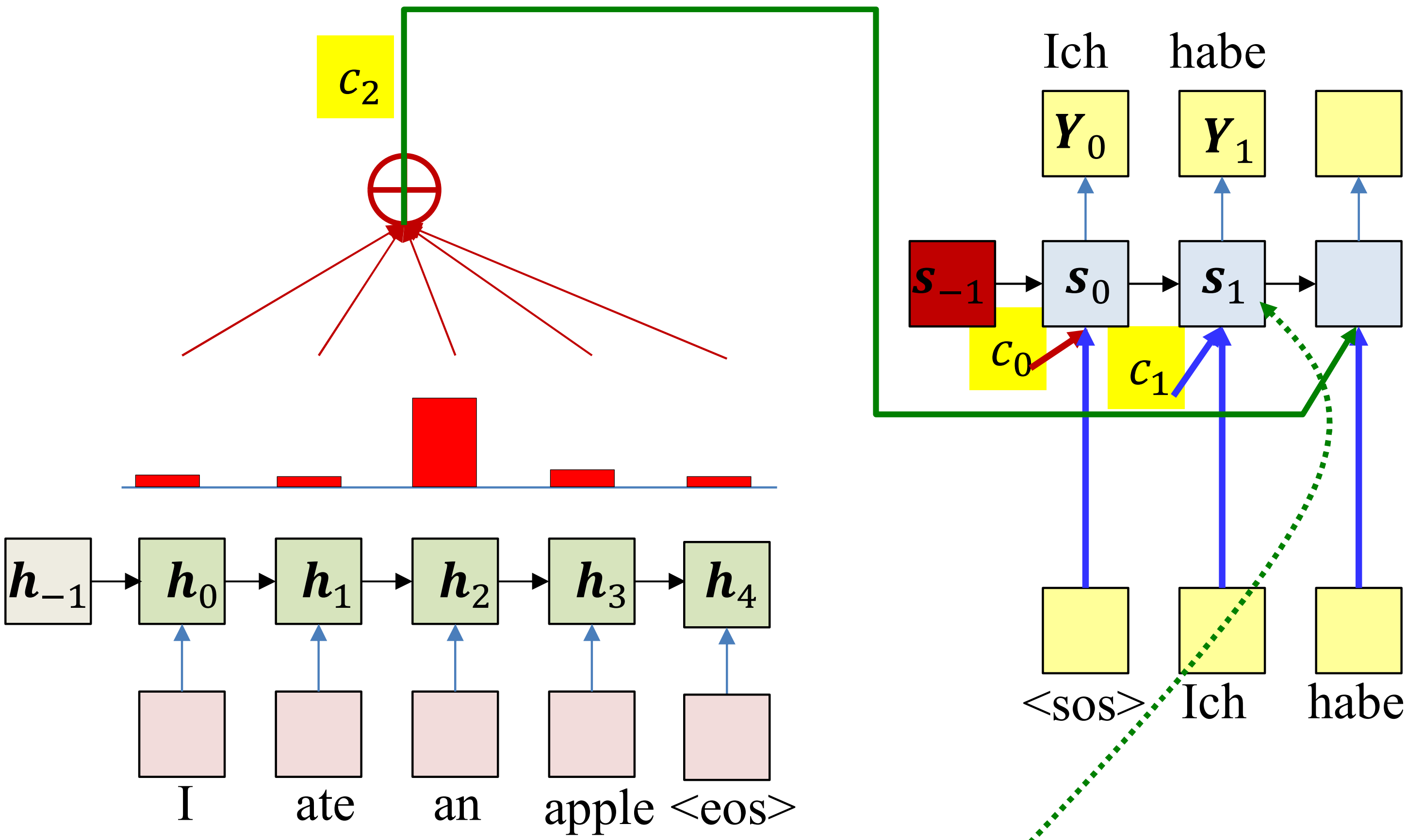
$$g(\boldsymbol{h}_i, \boldsymbol{s}_0) = \boldsymbol{h}_i^T \boldsymbol{W}_g \boldsymbol{s}_0$$

$$e_i(1) = g(\boldsymbol{h}_i, \boldsymbol{s}_0)$$

$$w_i(1) = \frac{\exp(e_i(1))}{\sum_j \exp(e_j(1))}$$

$$c_1 = \frac{1}{N}\sum_i^N w_i(1)h_i$$

# Converting an input: Inference



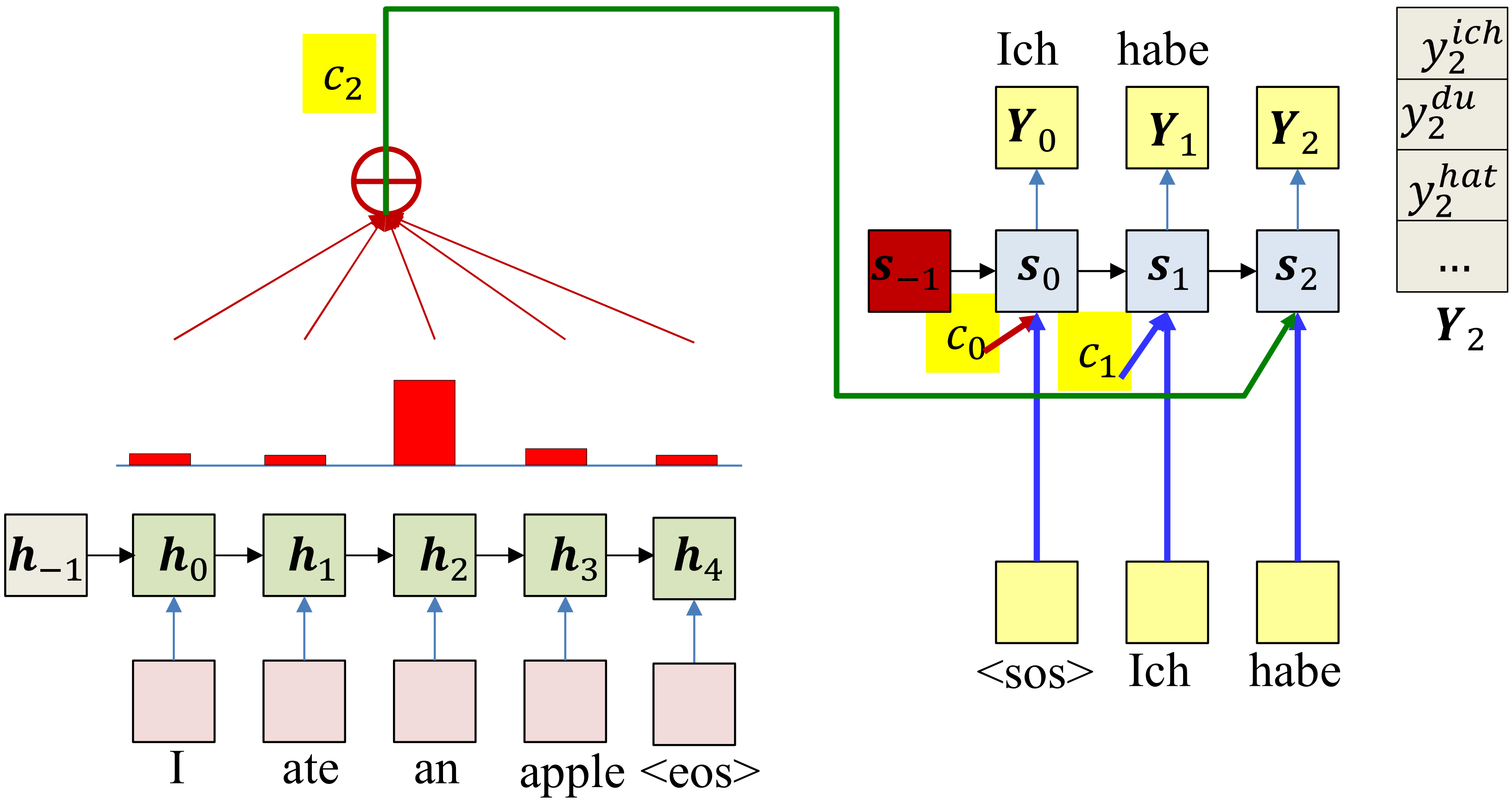

$$g(\boldsymbol{h}_i, \boldsymbol{s}_1) = \boldsymbol{h}_i^T \boldsymbol{W}_g \boldsymbol{s}_1$$

$$e_i(2) = g(\boldsymbol{h}_i, \boldsymbol{s}_1)$$

$$w_i(2) = \frac{\exp(e_i(2))}{\sum_j \exp(e_j(2))}$$

$$c_2 = \frac{1}{N}\sum_i^N w_i(2) h_i$$

# Converting an input: Inference



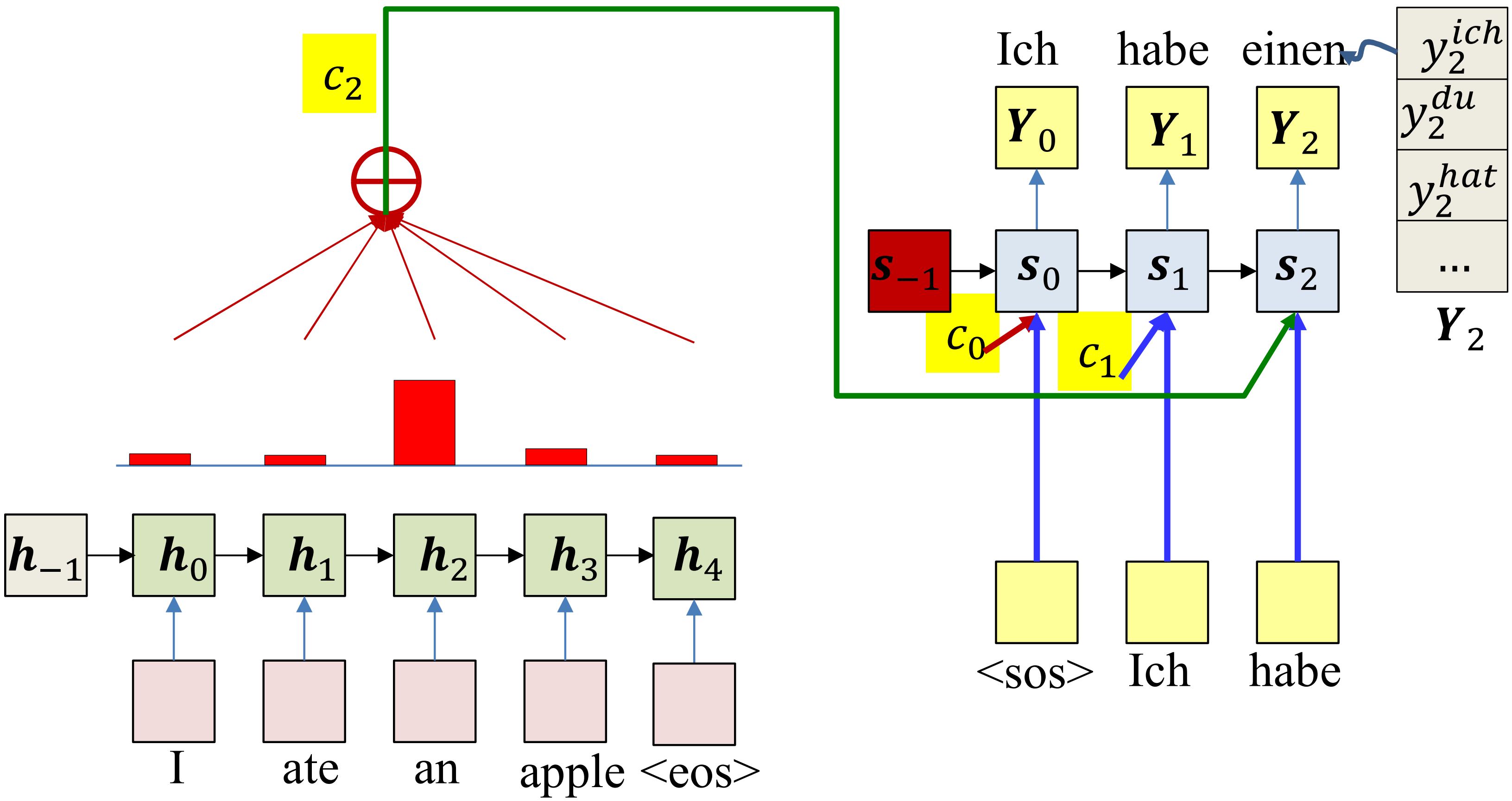

$$g(\boldsymbol{h}_i, \boldsymbol{s}_1) = \boldsymbol{h}_i^T \boldsymbol{W}_g \boldsymbol{s}_1$$

$$e_i(2) = g(\boldsymbol{h}_i, \boldsymbol{s}_1)$$

$$w_i(2) = \frac{\exp(e_i(2))}{\sum_j \exp(e_j(2))}$$

$$c_2 = \frac{1}{N} \sum_i^N w_i(2) h_i$$

# Converting an input: Inference



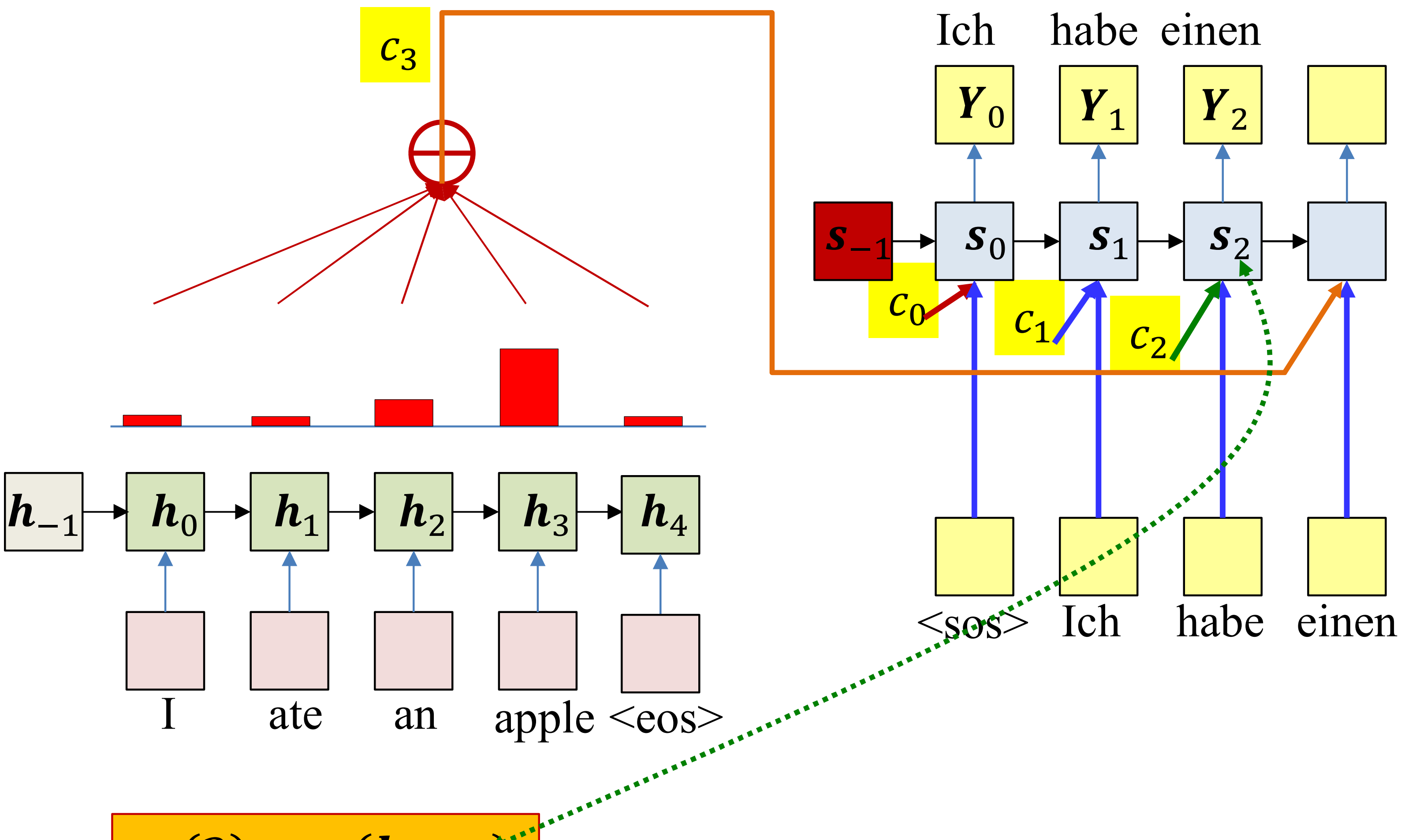

$$g(\boldsymbol{h}_i, \boldsymbol{s}_1) = \boldsymbol{h}_i^T \boldsymbol{W}_g \boldsymbol{s}_1$$

$$e_i(2) = g(\boldsymbol{h}_i, \boldsymbol{s}_1)$$

$$w_i(2) = \frac{\exp(e_i(2))}{\sum_j \exp(e_j(2))}$$

$$c_2 = \frac{1}{N} \sum_i^N w_i(2) h_i$$

# Converting an input: Inference

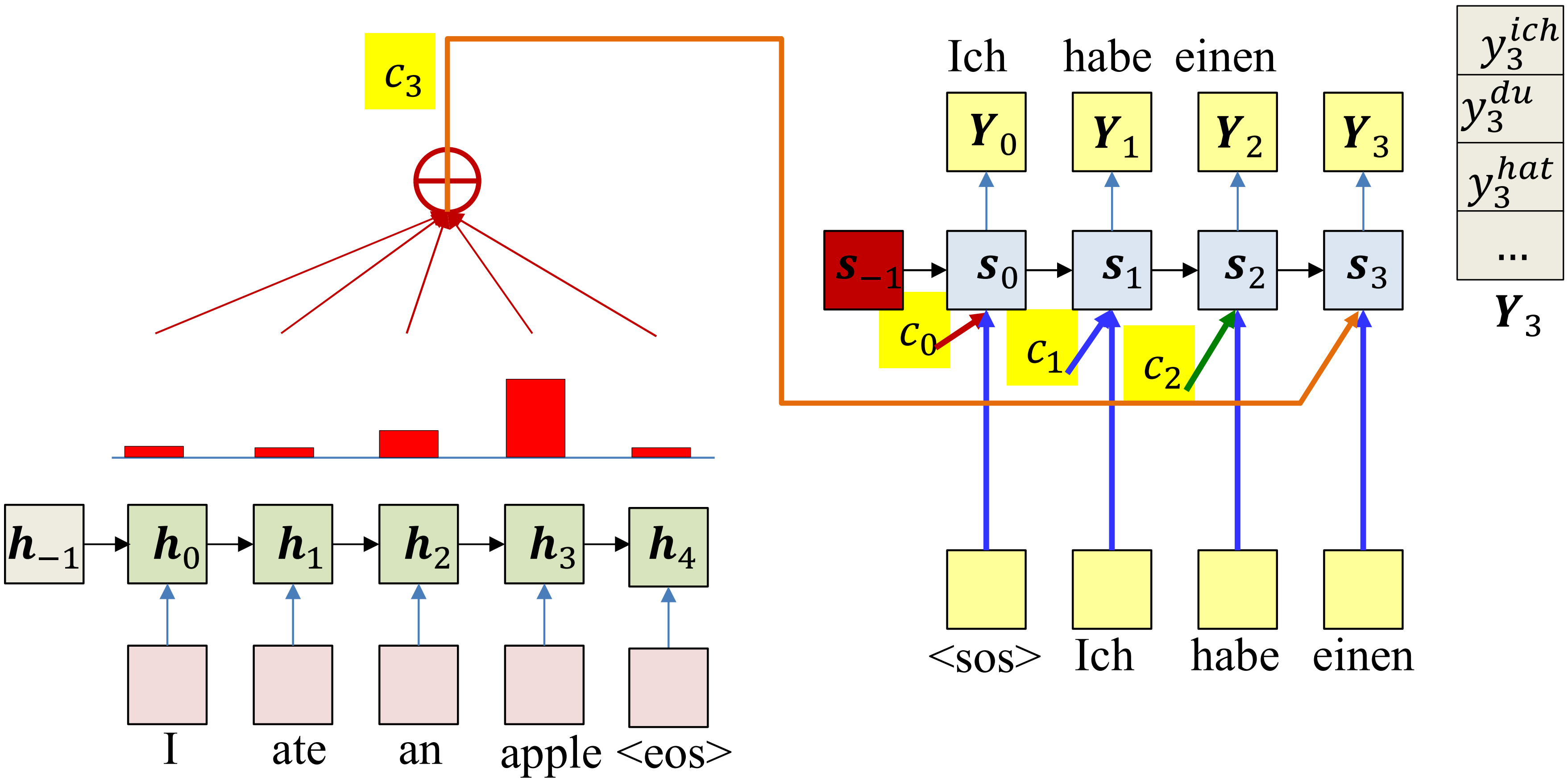



$$e_i(3) = g(h_i, s_2)$$

$$w_i(3) = \frac{\exp(e_i(3))}{\sum_j \exp(e_j(3))}$$

$$c_3 = \frac{1}{N}\sum_i^N w_i(3) h_i$$

# Converting an input: Inference

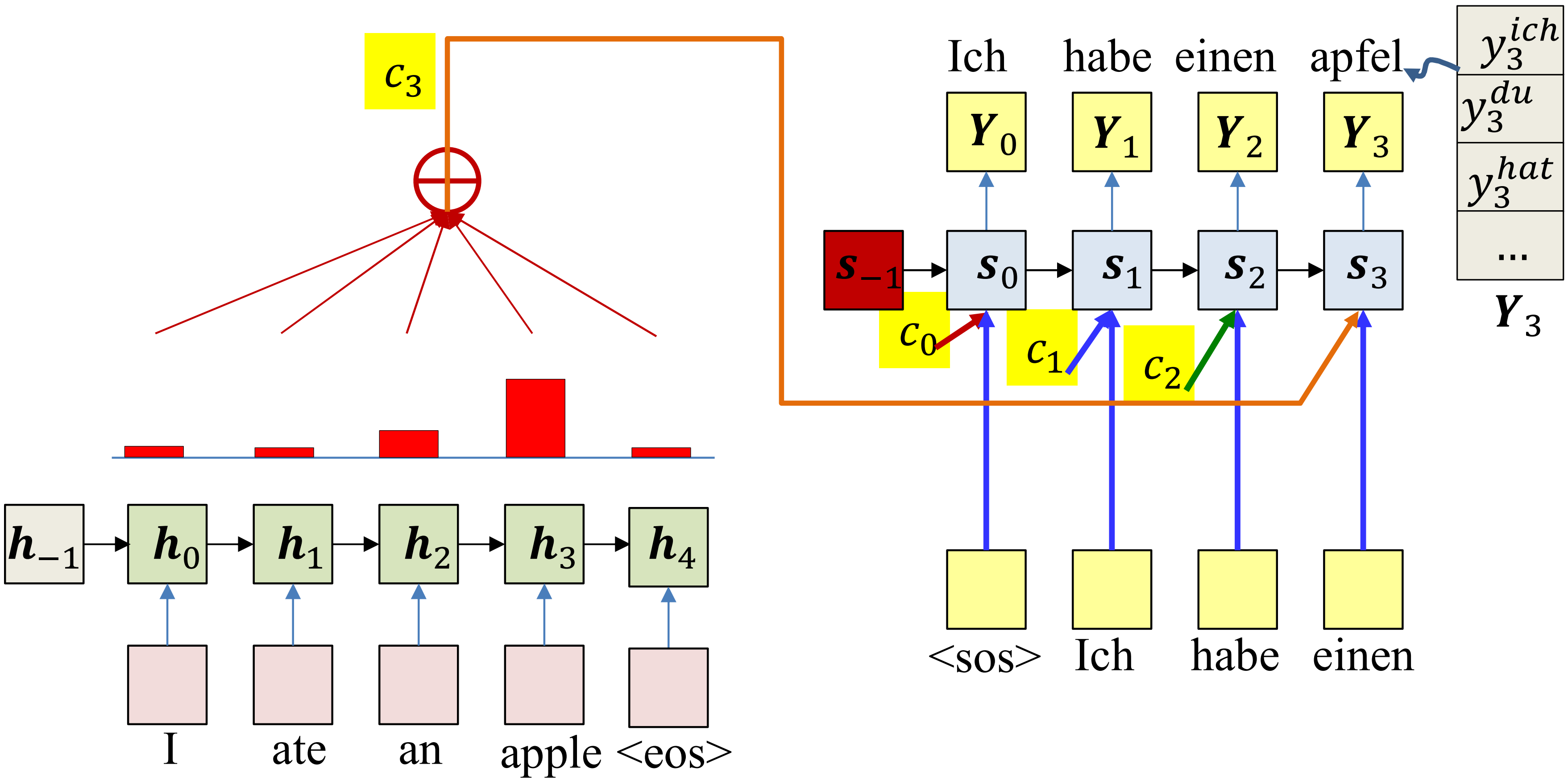



$$e_i(3) = g(\boldsymbol{h}_i, \boldsymbol{s}_2)$$

$$w_i(3) = \frac{\exp(e_i(3))}{\sum_j \exp(e_j(3))}$$

$$c_3 = \frac{1}{N}\sum_i^N w_i(3)h_i$$

# Converting an input: Inference

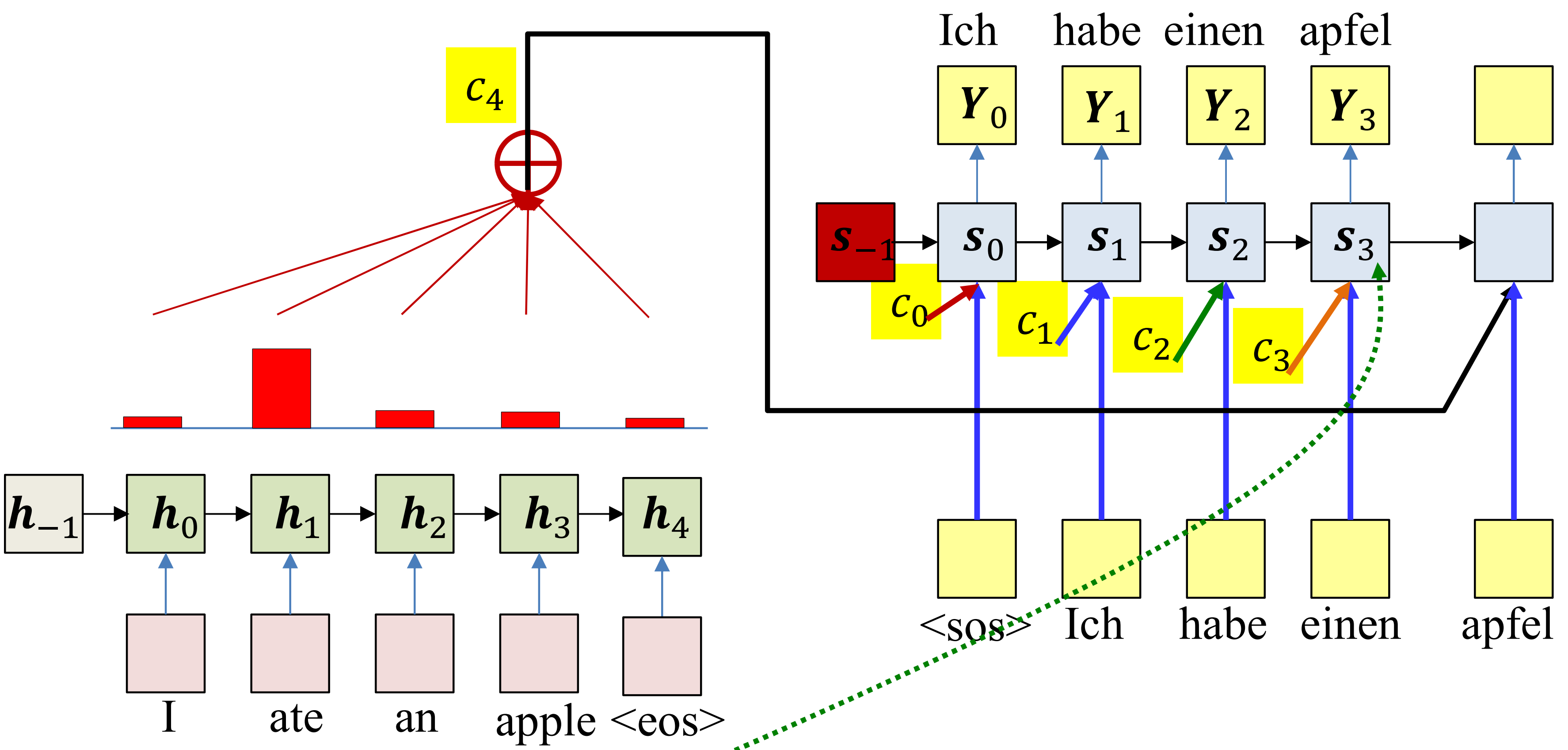



$$e_i(3) = g(\boldsymbol{h}_i, \boldsymbol{s}_2)$$

$$w_i(3) = \frac{\exp(e_i(3))}{\sum_j \exp(e_j(3))}$$

$$c_3 = \frac{1}{N}\sum_i^N w_i(3) h_i$$

# Converting an input: Inference

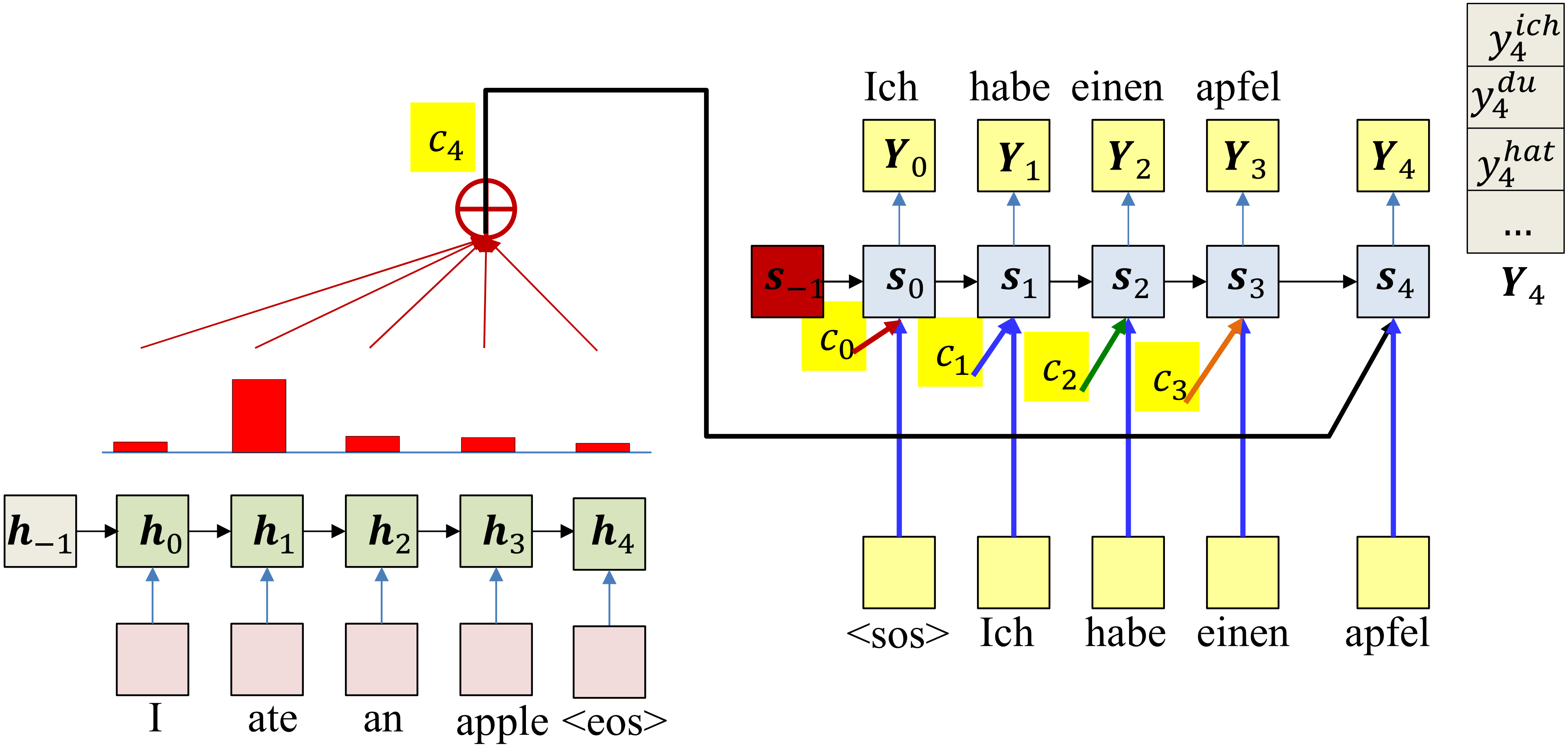



$$e_i(4) = g(\boldsymbol{h}_i, \boldsymbol{s}_3)$$

$$w_i(4) = \frac{\exp(e_i(4))}{\sum_j \exp(e_j(4))}$$

$$c_4 = \frac{1}{N}\sum_i^N w_i(4) h_i$$

# Converting an input: Inference

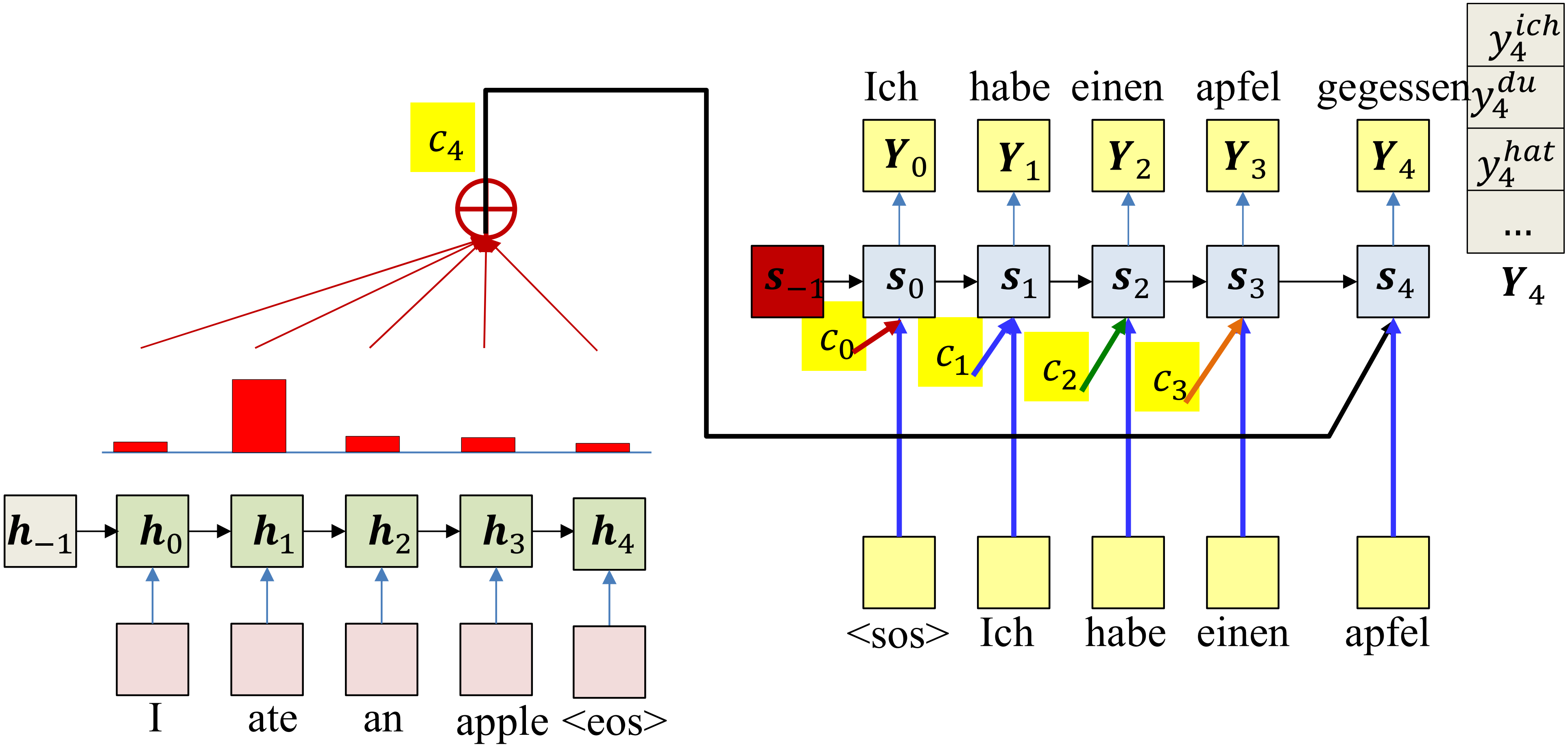



$$e_i(4) = g(\boldsymbol{h}_i, \boldsymbol{s}_3)$$

$$w_i(4) = \frac{\exp(e_i(4))}{\sum_j \exp(e_j(4))}$$

$$c_4 = \frac{1}{N}\sum_{i}^{N} w_i(4) h_i$$

# Converting an input: Inference

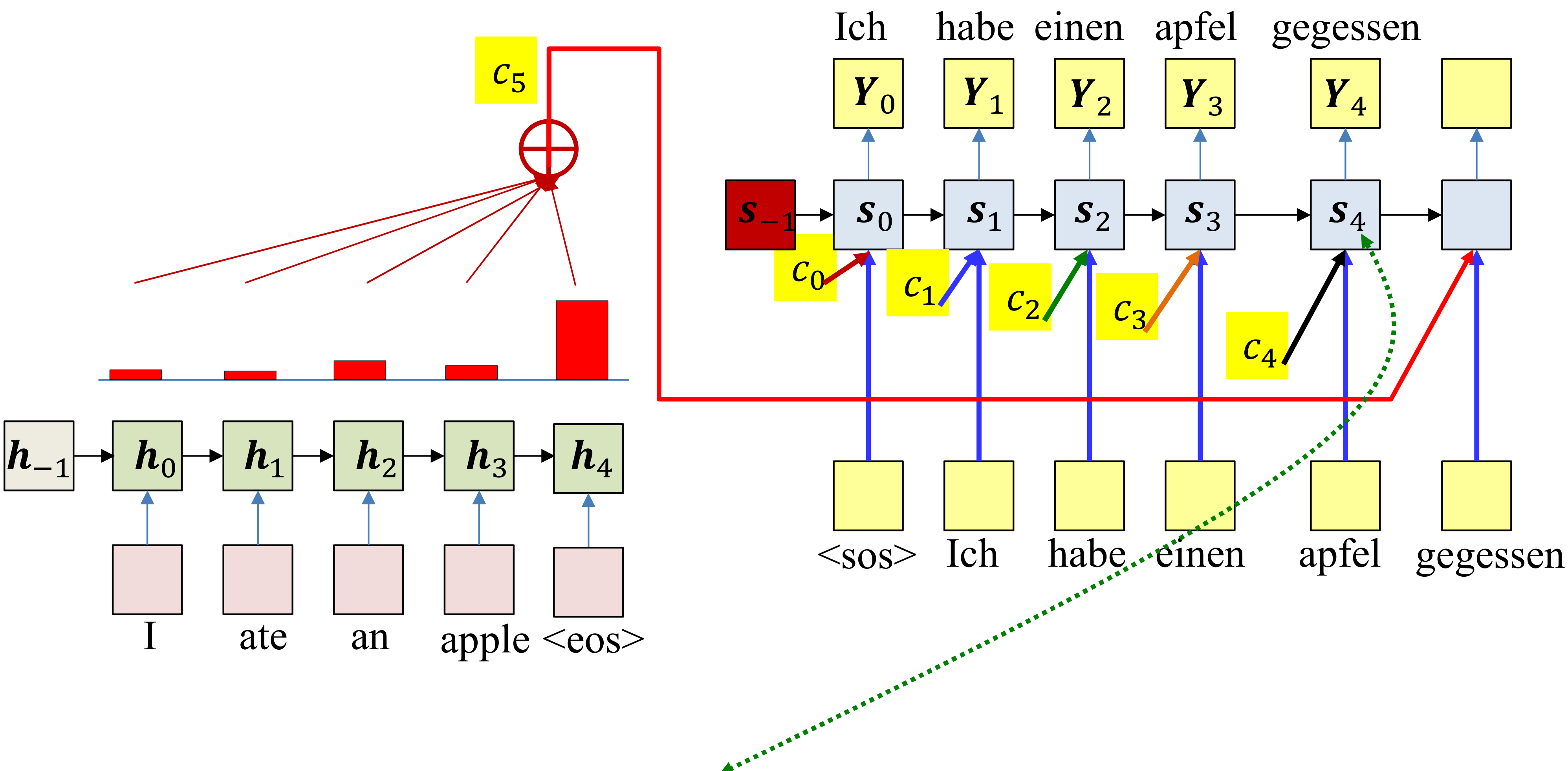



$$e_i(4) = g(\boldsymbol{h}_i, \boldsymbol{s}_3)$$

$$w_i(4) = \frac{\exp(e_i(4))}{\sum_j \exp(e_j(4))}$$

$$c_4 = \frac{1}{N}\sum_i^N w_i(4) h_i$$

# Converting an input: Inference

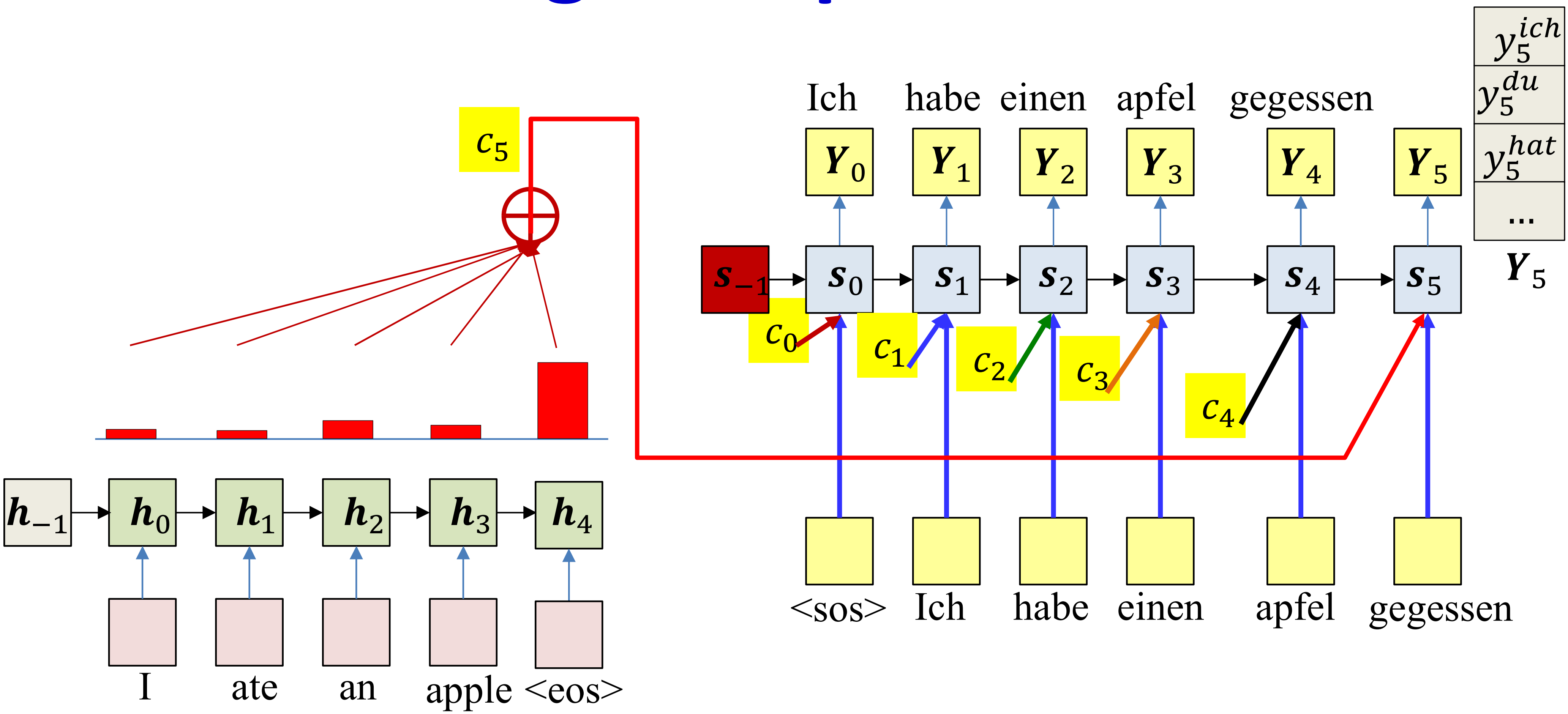



$$e_i(5) = g(\boldsymbol{h}_i, \boldsymbol{s}_4)$$

$$w_i(5) = \frac{\exp(e_i(5))}{\sum_j \exp(e_j(5))}$$

$$c_5 = \frac{1}{N}\sum_i^N w_i(5) h_i$$

# Converting an input: Inference

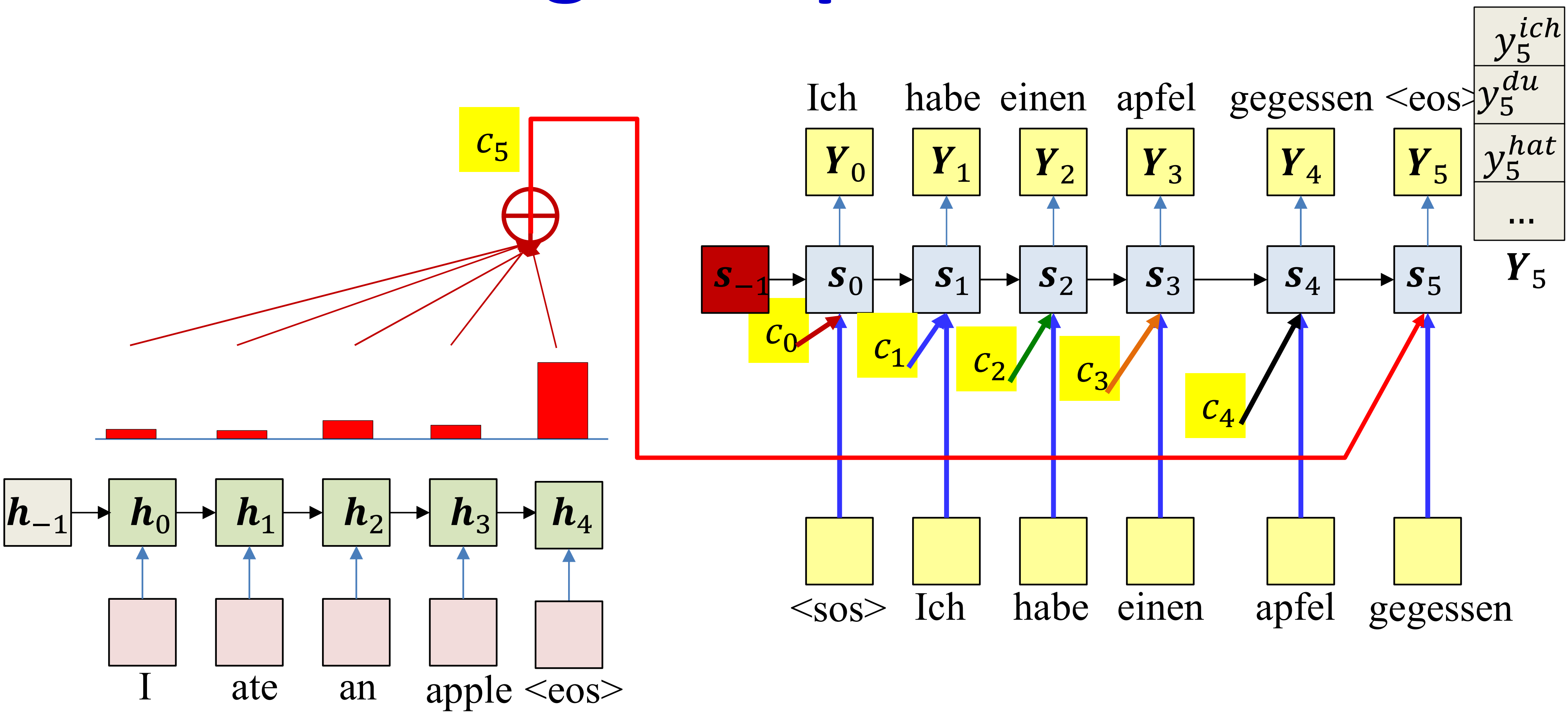



$$e_i(5) = g(\boldsymbol{h}_i, \boldsymbol{s}_4)$$

$$w_i(5) = \frac{\exp(e_i(5))}{\sum_j \exp(e_j(5))}$$

$$c_5 = \frac{1}{N}\sum_i^N w_i(5) h_i$$

# Converting an input: Inference



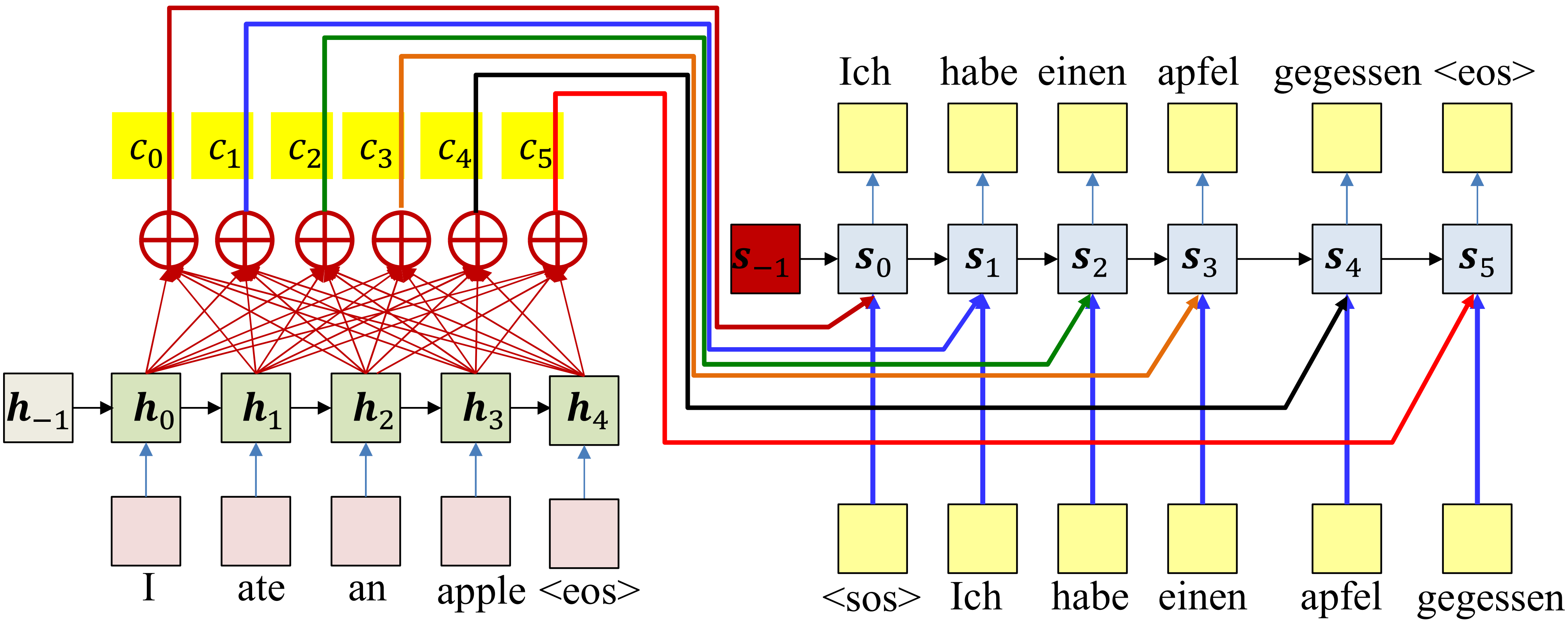

Continue this process until <eos> is drawn

$$e_i(5) = g(\boldsymbol{h}_i, \boldsymbol{s}_4)$$

$$w_i(5) = \frac{\exp(e_i(5))}{\sum_j \exp(e_j(5))}$$

$$c_5 = \frac{1}{N}\sum_i^N w_i(5) h_i$$

# Attention-based decoding

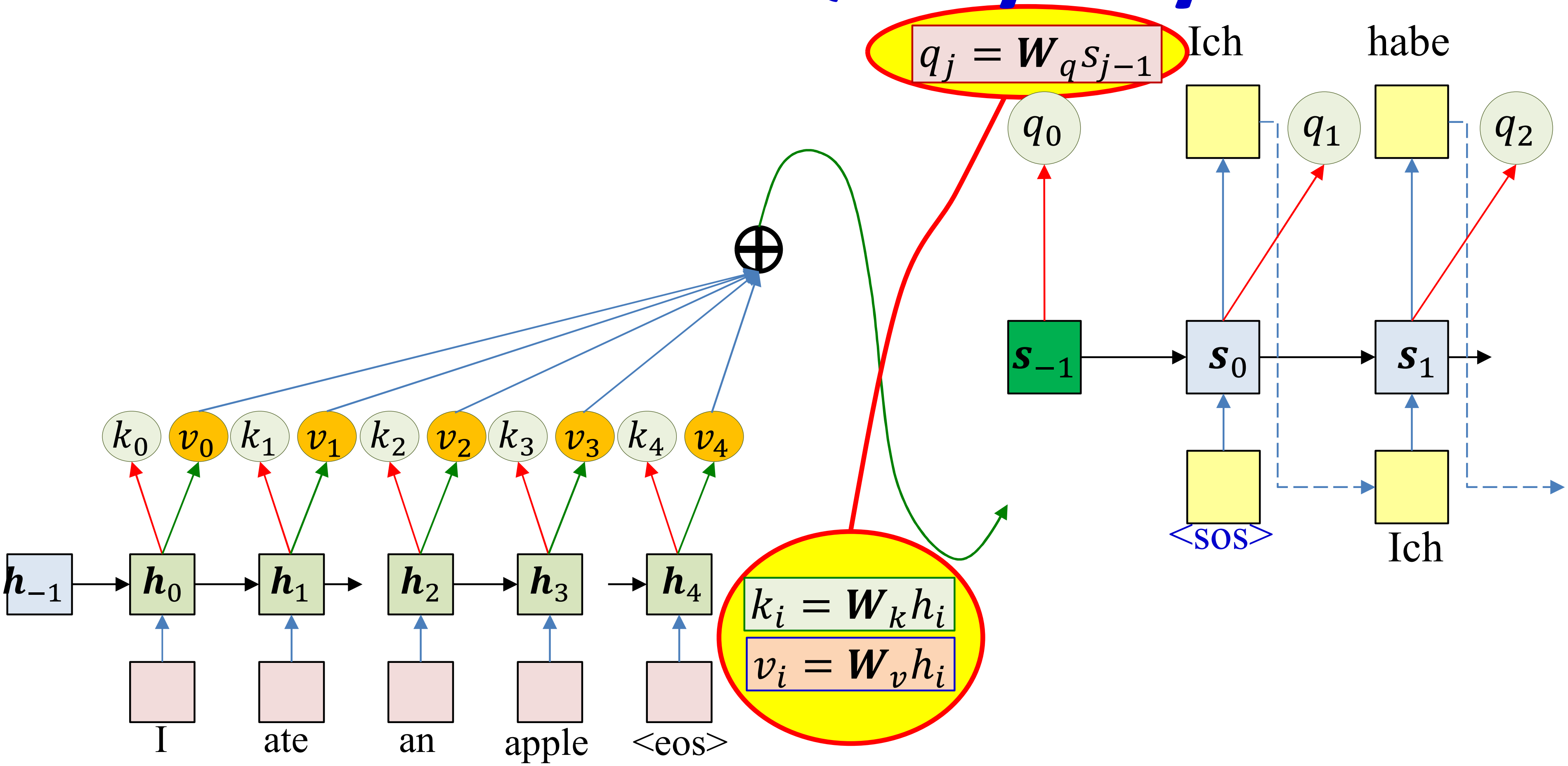



$$e_i(t) = g(\boldsymbol{h}_i, \boldsymbol{s}_{t-1})$$

$$w_i(t) = \frac{\exp(e_i(t))}{\sum_j \exp(e_j(t))}$$

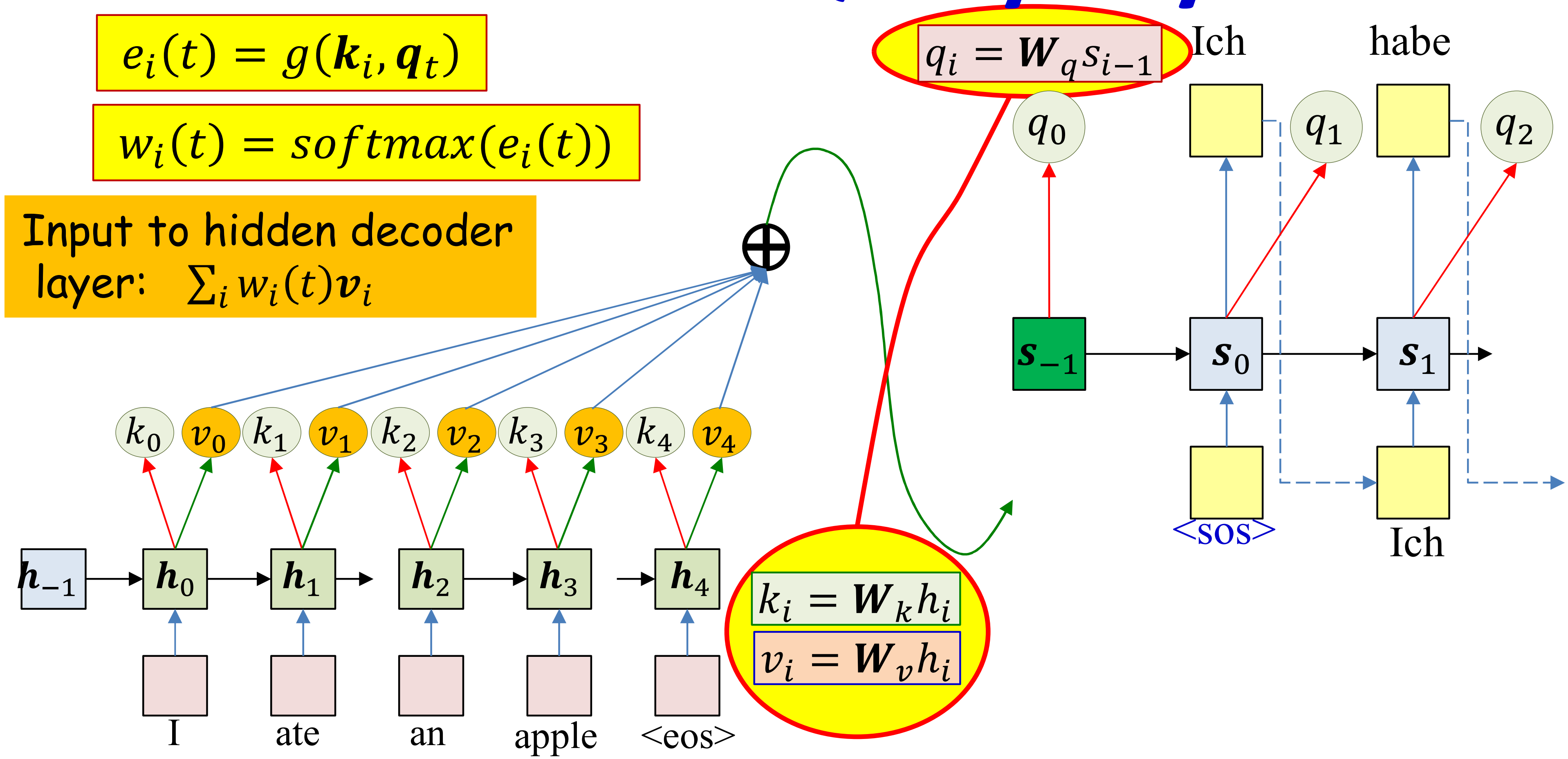
$$c_t = \frac{1}{N}\sum_i^N w_i(t) h_i$$

# Modification: Query key value




- Encoder outputs an explicit "key" and "value" at each input time
  - Key is used to evaluate the importance of the input at that time, for a given output
- Decoder outputs an explicit "query" at each output time
  - Query is used to evaluate which inputs to pay attention to
- The weight is a function of key and query
- The actual context is a weighted sum of value

# Modification: Query key value



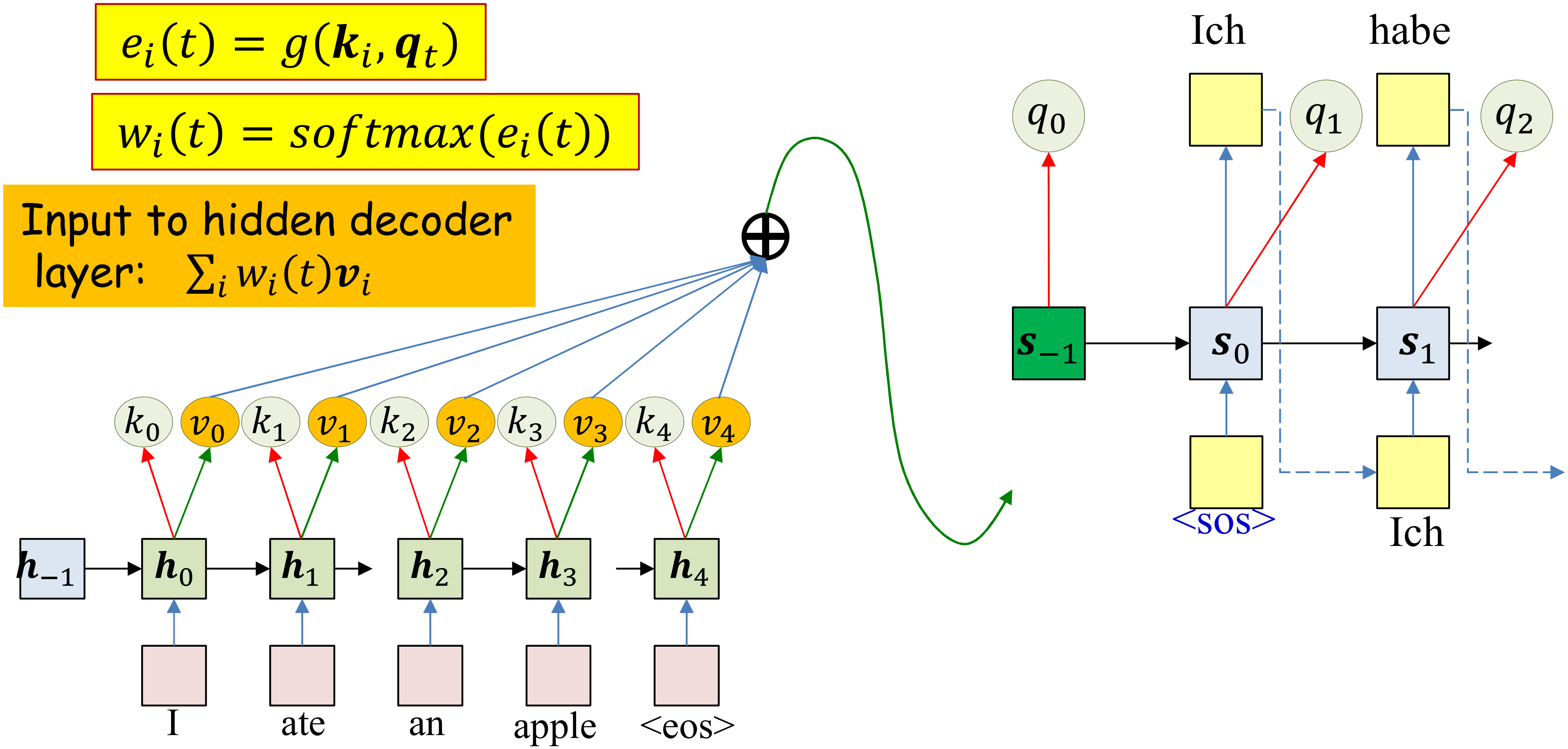

- Encoder outputs an explicit "key" and "value" at each input time
  - Key is used to evaluate the importance of the input at that time, for a given output
- Decoder outputs an explicit "query" at each output time
  - Query is used to evaluate which inputs to pay attention to
- The weight is a function of key and query
- The actual context is a weighted sum of value

# Modification: Query key value

$e_i(t) = g(\boldsymbol{k}_i, \boldsymbol{q}_t)$

$w_i(t) = softmax(e_i(t))$

Input to hidden decoder layer: $\sum_i w_i(t)\boldsymbol{v}_i$




Special case: $k_i = v_i = h_i$

$q_t = s_{t-1}$

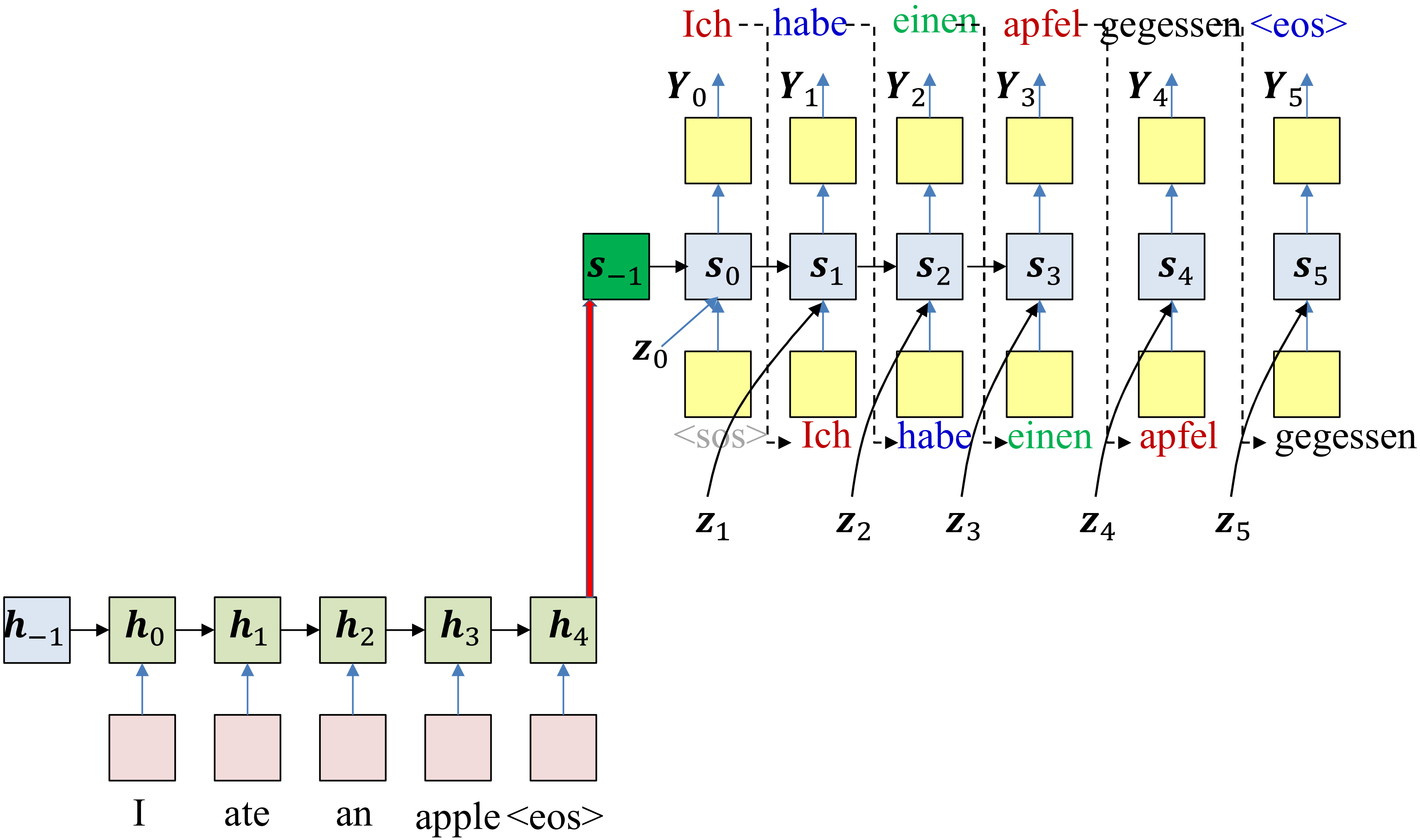
We will continue using this assumption in the following slides but in practice the query-key-value format is used

# Pseudocode

```
# Assuming encoded input
#       (K,V) = [kenc[0]… kenc[T]], [venc[0]… venc[T]]
# is available

t = -1
hout[-1] = 0    # Initial Decoder hidden state
q[0] = 0        # Initial query

# Note: begins with a "start of sentence" symbol
#       <sos> and <eos> may be identical
Yout[0] = <sos>
do
    t = t+1
    C  = compute_context_with_attention(q[t], K, V)
    y[t],hout[t],q[t+1] = RNN_decode_step(hout[t-1], yout[t-1], C)
    yout[t] = generate(y[t]) # Random, or greedy
until yout[t] == <eos>
```

# Pseudocode : Computing context with attention

```
# Takes in previous state, encoder states, outputs attention-weighted context
function compute_context_with_attention(q, K, V)
    # First compute attention
    e = []
    for t = 1:T  # Length of input
        e[t] = raw_attention(q, K[t])
    end
    maxe = max(e) # subtract max(e) from everything to prevent underflow
    a[1..T] = exp(e[1..T] - maxe)    # Component-wise exponentiation
    suma = sum(a)  # Add all elements of a
    a[1..T] = a[1..T]/suma

    C = 0
    for t = 1..T
        C += a[t] * V[t]
    end

    return C
```

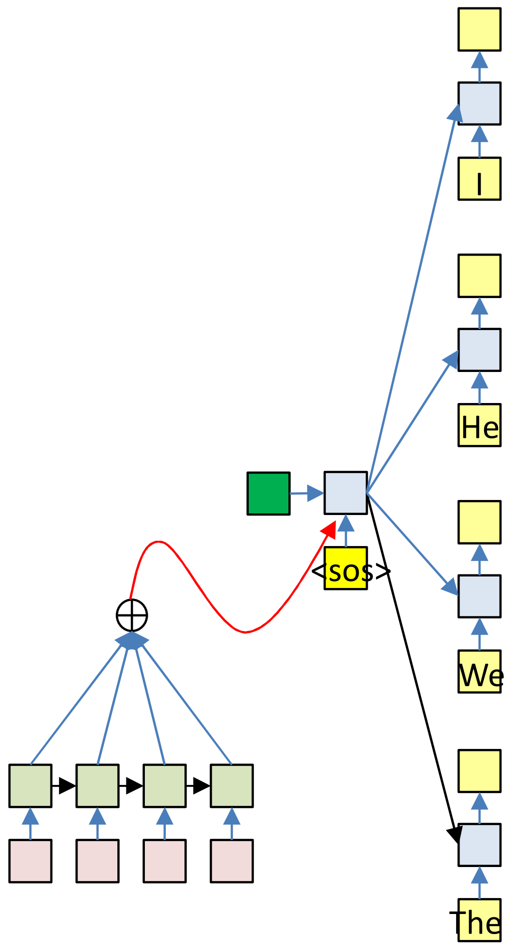



- As before, the objective of drawing: Produce the most likely output (that ends in an <eos>)

$$\underset{O_1,\ldots,O_L}{\operatorname{argmax}}\, y_1^{O_1} y_1^{O_2} \ldots y_1^{O_L}$$

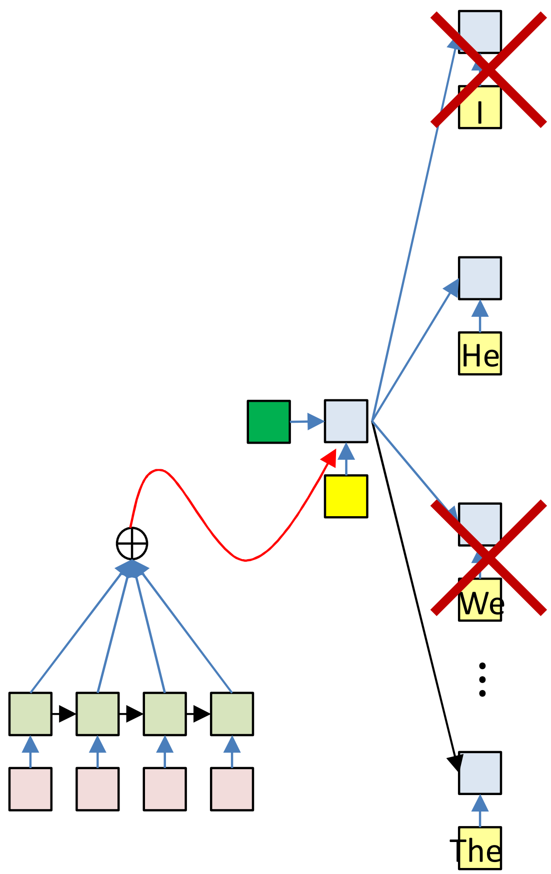
- Simply selecting the most likely symbol at each time may result in suboptimal output

# Solution: Multiple choices




- Retain all choices and *fork* the network
  - With every possible word as input

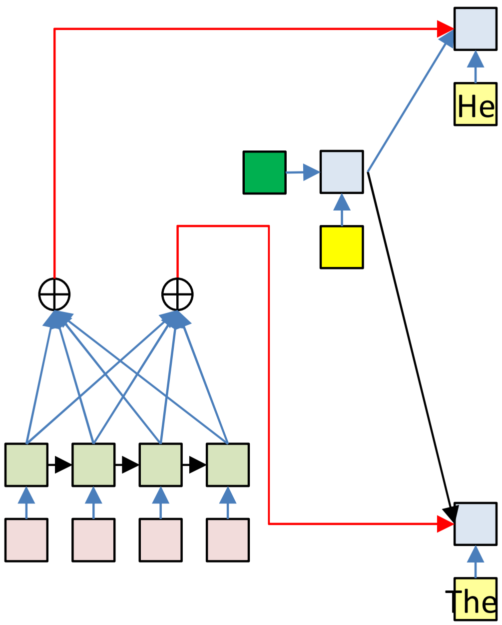
# To prevent blowup: Prune




$$Top_K\ P(O_1|I_1, \ldots, I_N)$$

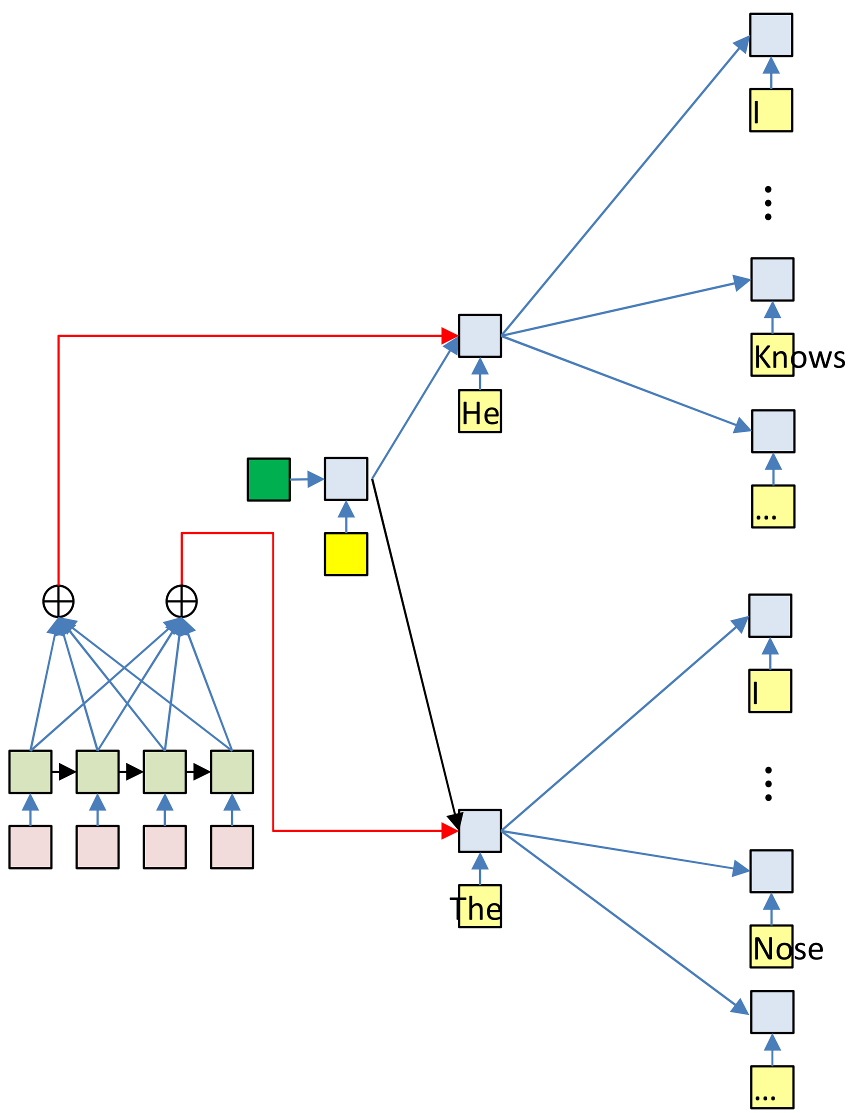
- **Prune**
  - At each time, retain only the top K scoring forks

# Decoding




- At each time, retain only the top K scoring forks

# Decoding



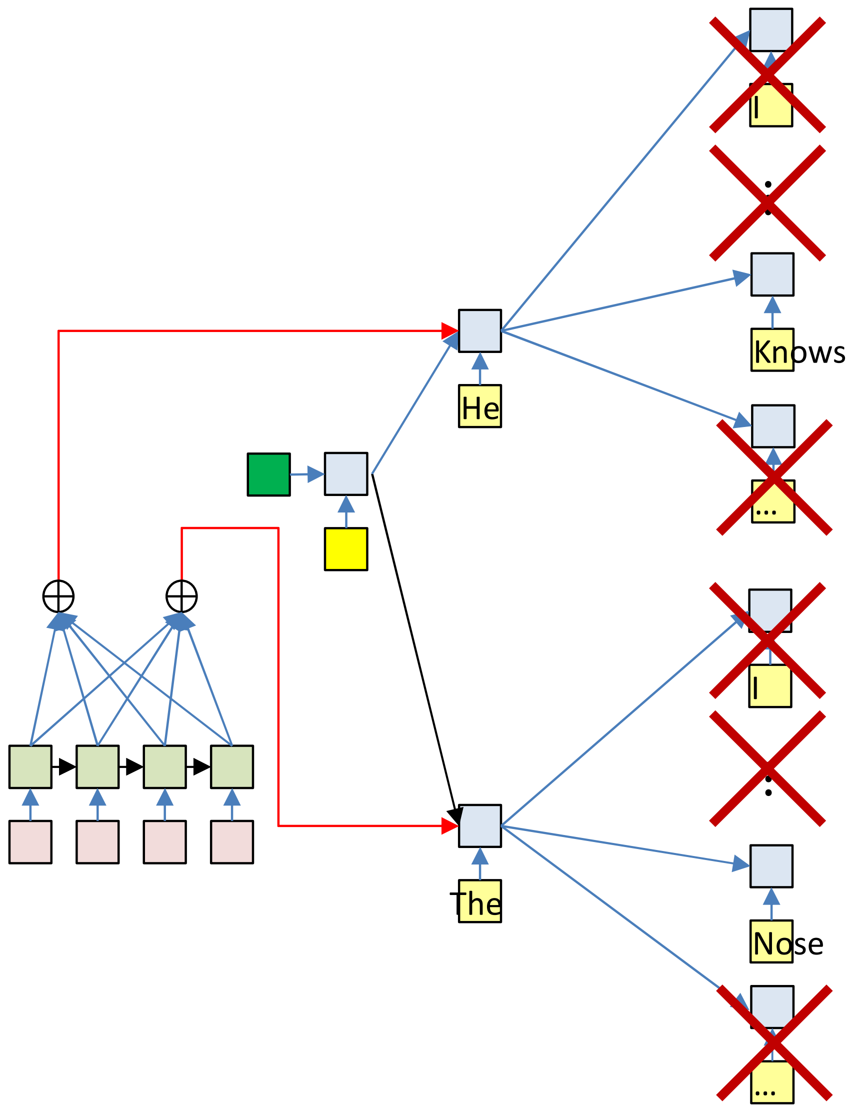

Note: based on product

$$Top_K \; P(O_2 O_1 | I_1, \dots, I_N)$$

$$= Top_K \; P(O_2 | O_1, I_1, \dots, I_N) P(O_1 | I_1, \dots, I_N)$$

- At each time, retain only the top K scoring forks

# Decoding



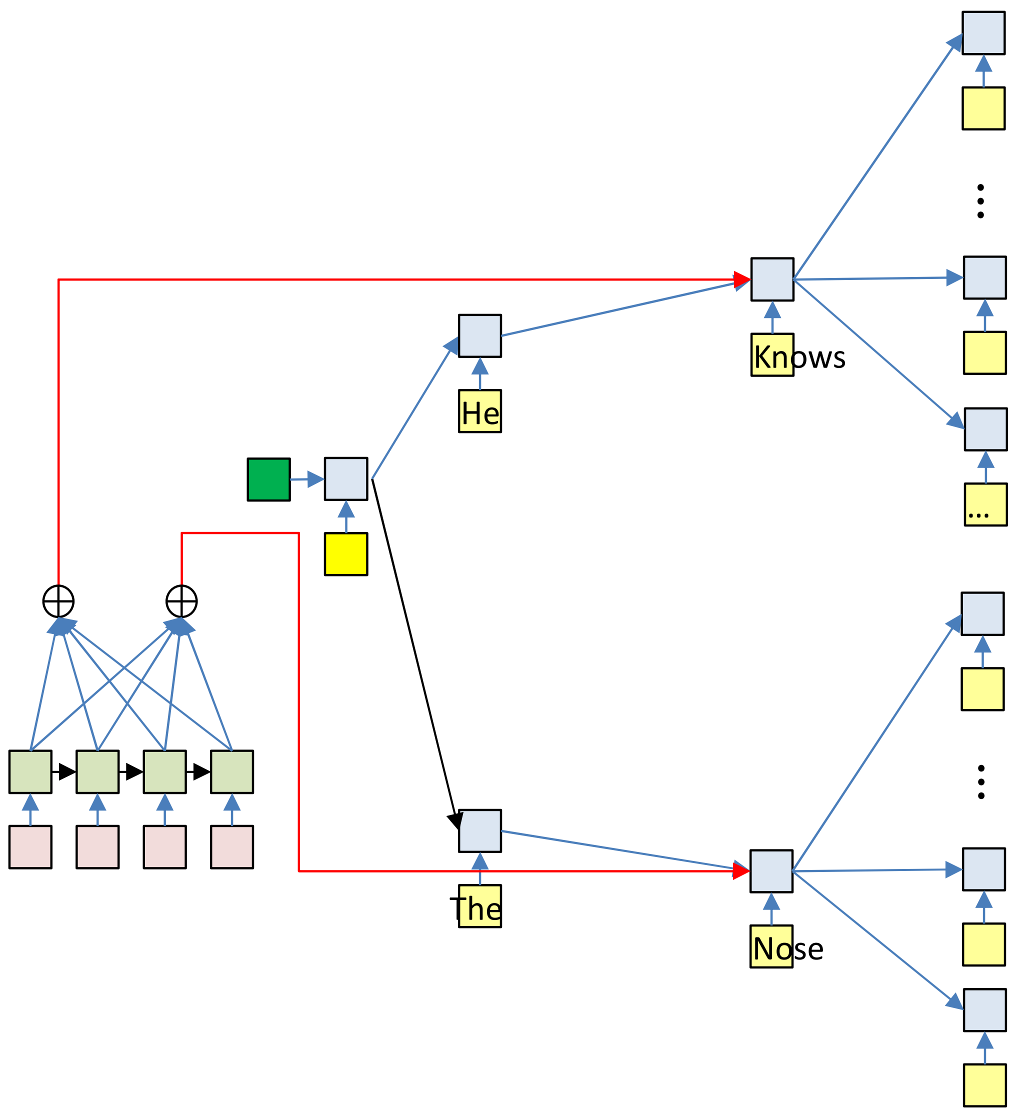

Note: based on product

$$Top_K\ P(O_2 O_1 | I_1, \dots, I_N)$$

$$= Top_K\ P(O_2 | O_1, I_1, \dots, I_N) P(O_1 | I_1, \dots, I_N)$$

- At each time, retain only the top K scoring forks

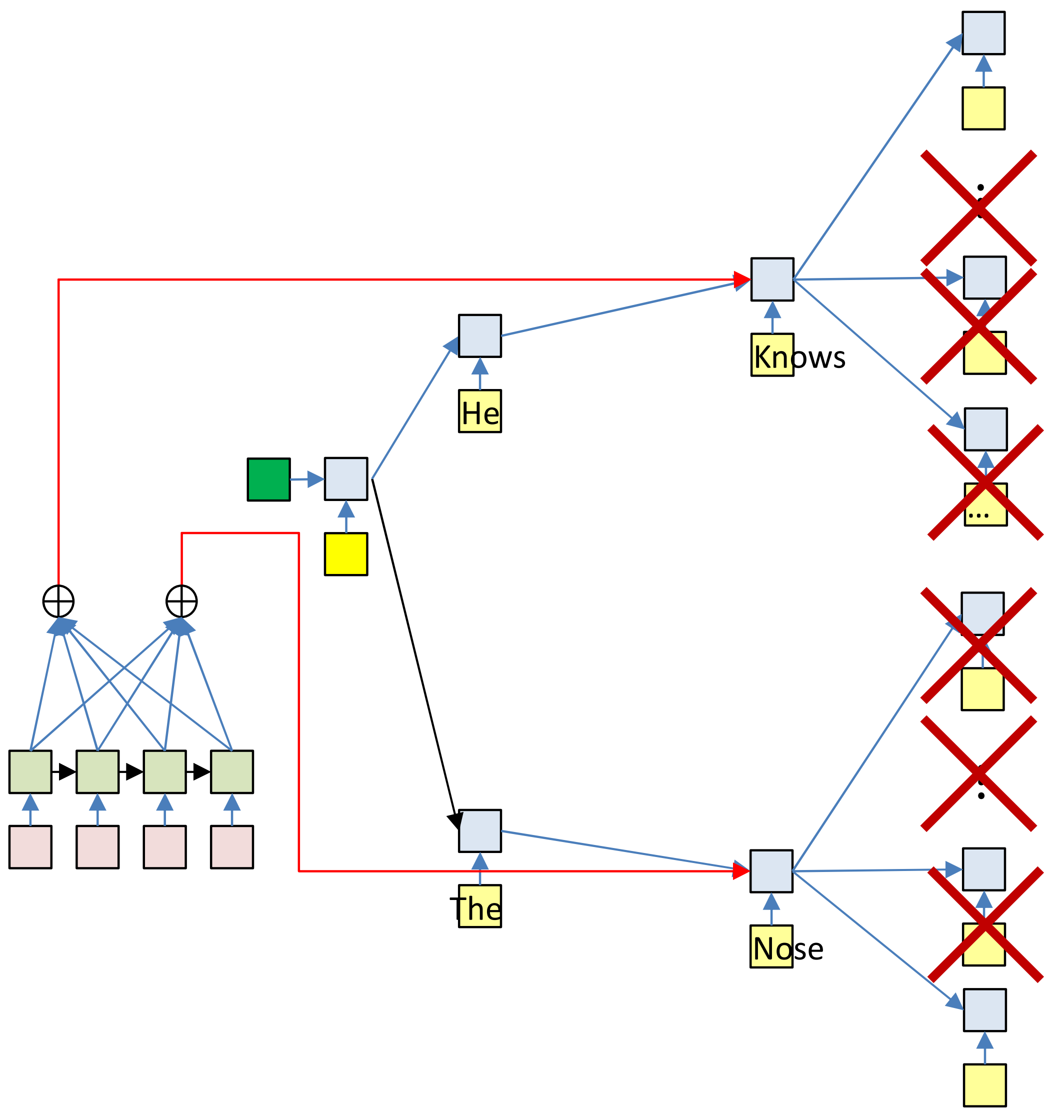
# Decoding




$$= Top_K P(O_2|O_1, I_1, \dots, I_N) \times P(O_2|O_1, I_1, \dots, I_N) \times P(O_1|I_1, \dots, I_N)$$

- At each time, retain only the top K scoring forks

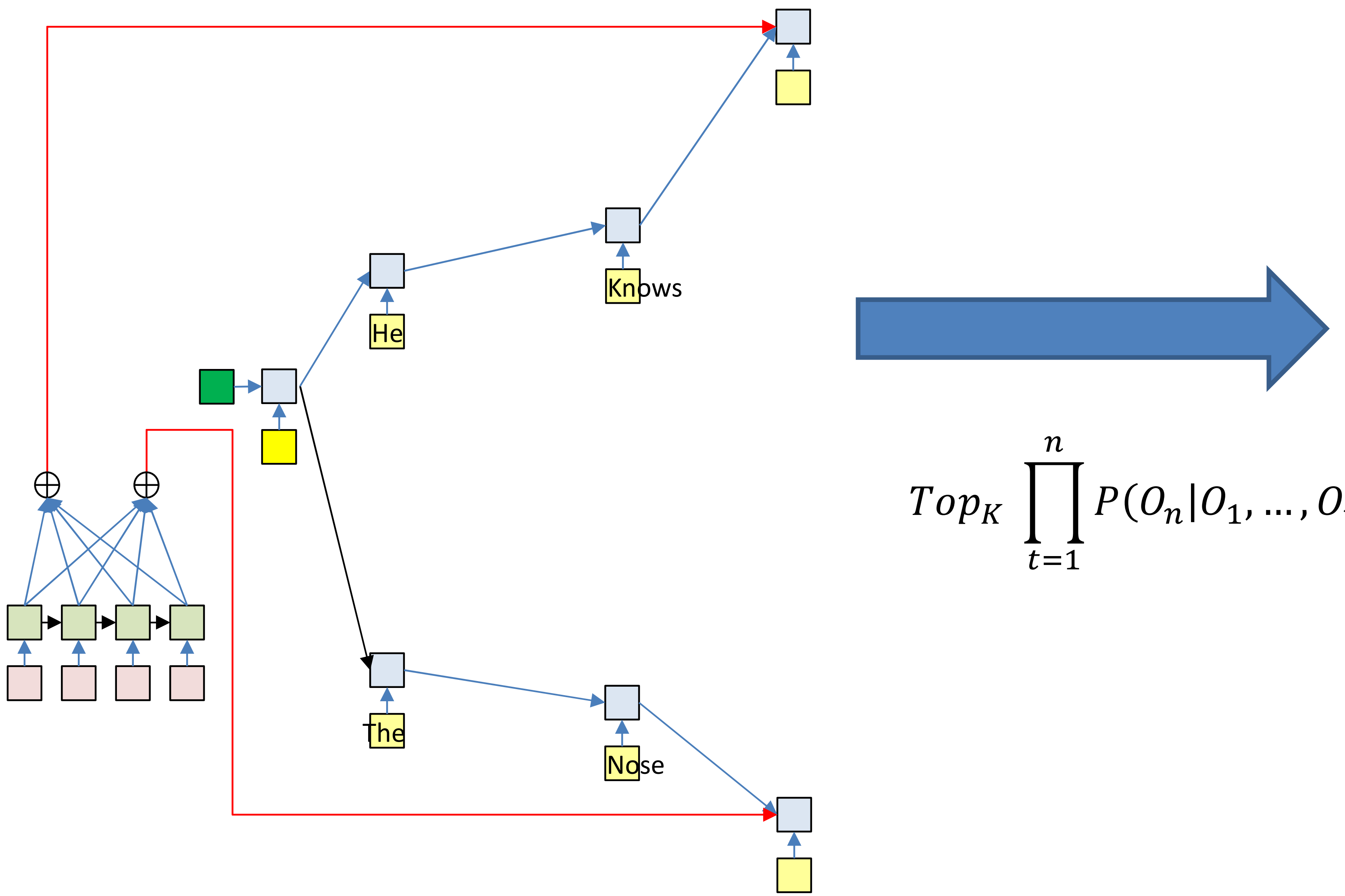
# Decoding




$$= Top_K P(O_2|O_1, I_1, \ldots, I_N) \times P(O_2|O_1, I_1, \ldots, I_N) \times P(O_1|I_1, \ldots, I_N)$$

- At each time, retain only the top K scoring forks

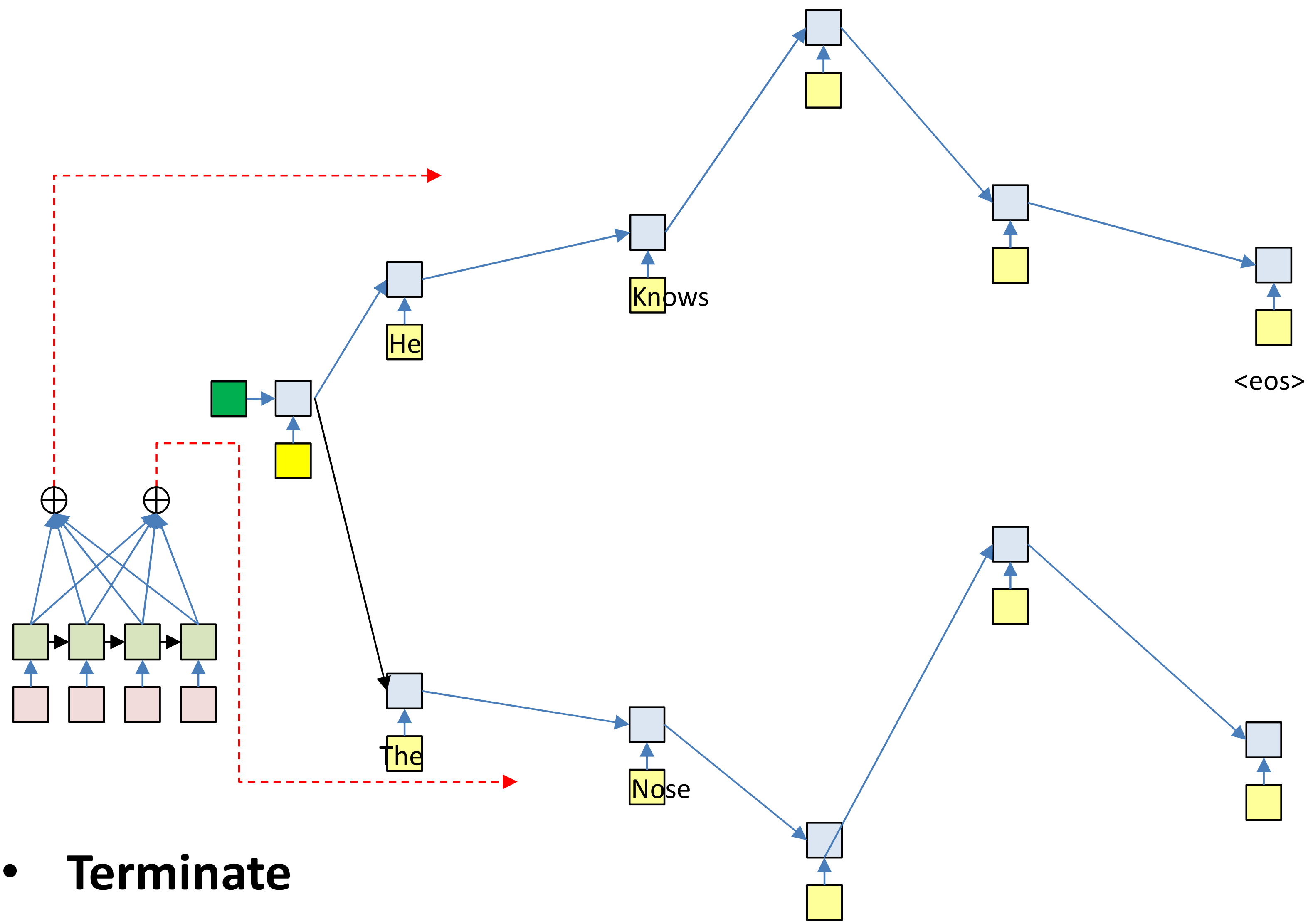
# Decoding




$$Top_K \prod_{t=1}^{n} P(O_n|O_1, \ldots, O_{n-1}, I_1, \ldots, I_N)$$

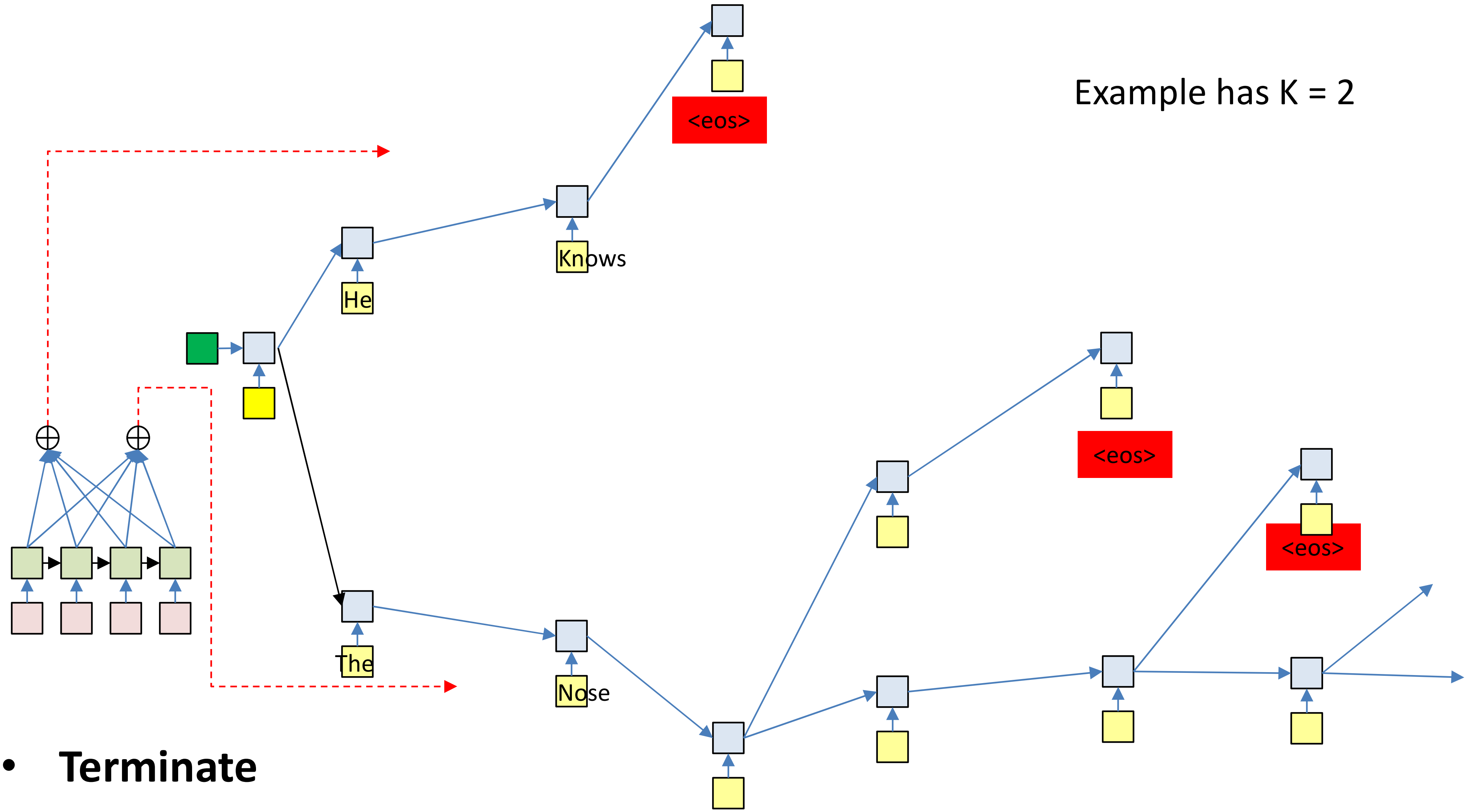
- At each time, retain only the top K scoring forks

# Terminate




- **Terminate**
  – When the current most likely path overall ends in <eos>
    - Or continue producing more outputs (each of which terminates in <eos>) to get N-best outputs

# Termination: <eos>




- **Terminate**
  – Paths cannot continue once the output an <eos>
    - So paths may be different lengths
      – Select the most likely sequence ending in <eos> across *all* terminating sequences

# Pseudocode: Beam search

```
# Assuming encoder output H = hin[1]… hin[T] is available
path = <sos>
beam = {path}
pathscore = [path] = 1
state[path] = h[0]  # initial state (computed using your favorite method)
do  # Step forward
    nextbeam = {}
    nextpathscore = []
    nextstate = {}
    for path in beam:
        cfin = path[end]
        hpath = state[path]
        C  = compute_context_with_attention(hpath, H)
        y,h = RNN_decode_step(hpath, cfin, C)
        for c in Symbolset
            newpath = path + c
            nextstate[newpath] = h
            nextpathscore[newpath] = pathscore[path]*y[c]
            nextbeam += newpath # Set addition
        end
    end
    beam, pathscore, state, bestpath = prune(nextstate,nextpathscore,nextbeam)
until bestpath[end] = <eos>
```

# Pseudocode: Beam search

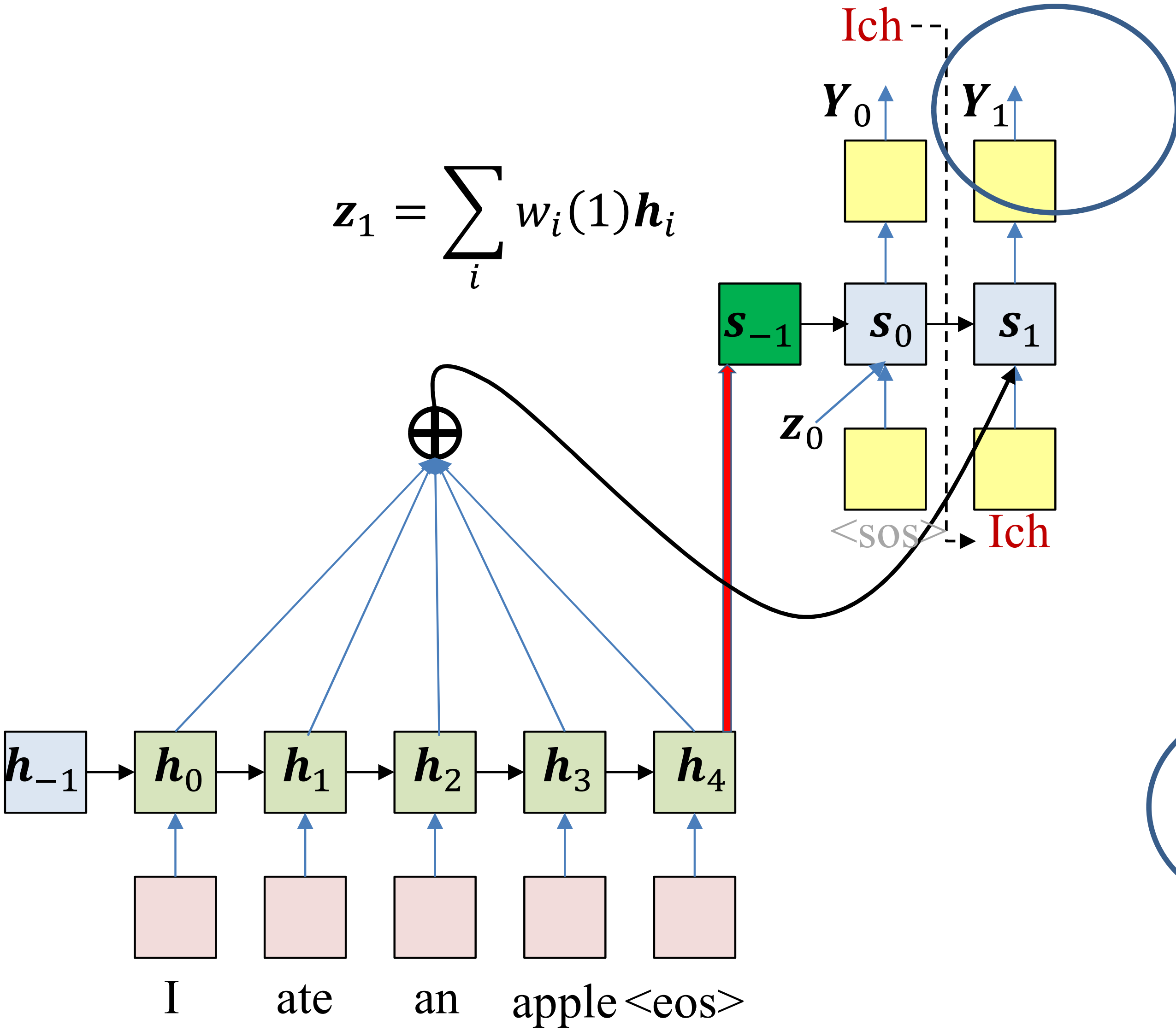
```
# Assuming encoder output H = hin[1]… hin[T] is available
path = <sos>
beam = {path}
pathscore = [path] = 1
state[path] = h[0]  # computed using your favorite method
context[path] = compute_context_with_attention(h[0], H)
do  # Step forward
    nextbeam = {}
    nextpathscore = []
    nextstate = {}
    nextcontext = {}
    for path in beam:
        cfin = path[end]
        hpath = state[path]
        C = context[path]
        y,h = RNN_decode_step(hpath, cfin, C)
        nextC = compute_context_with_attention(h, H)
        for c in Symbolset
            newpath = path + c
            nextstate[newpath] = h
            nextcontext[newpath] = nextC
            nextpathscore[newpath] = pathscore[path]*y[c]
            nextbeam += newpath # Set addition
        end
    end
    beam, pathscore, state, context, bestpath =
                         prune (nextstate, nextpathscore, nextbeam, nextcontext)
until bestpath[end] = <eos>
```

Slightly more efficient.

Does not perform redundant context computation

# What does the attention learn?

$$z_1 = \sum_i w_i(1) h_i$$



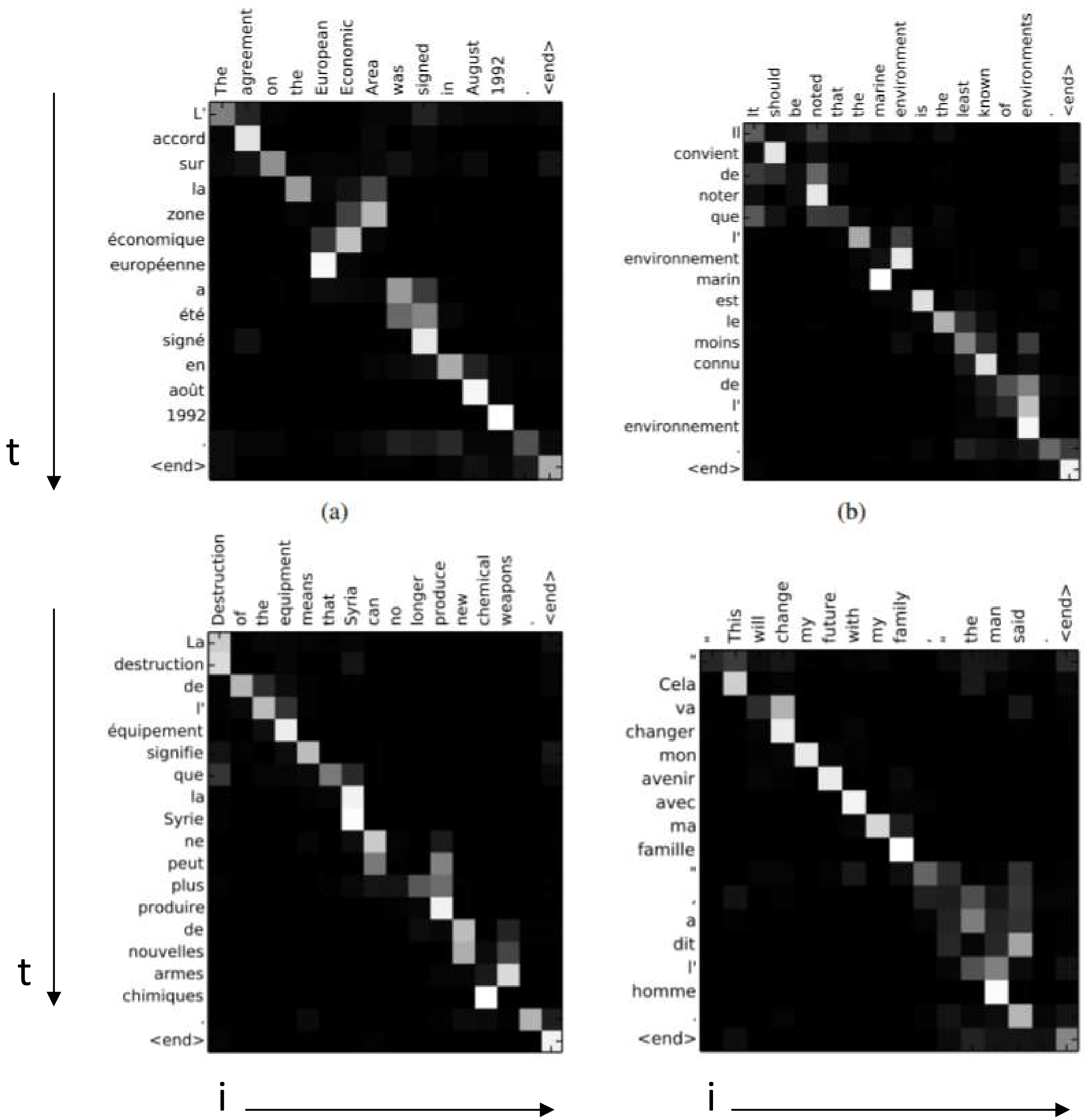

$$g(h_i, s_0) = h_i^T W_g s_0$$

$$e_i(1) = g(h_i, s_0)$$

$$w_i(1) = \frac{\exp(e_i(1))}{\sum_j \exp(e_j(1))}$$

- The key component of this model is the attention weight
  - It captures the relative importance of each position in the input to the current output

# “Alignments” example: Bahdanau et al.




**Plot of $w_i(t)$**

Color shows value (white is larger)

Note how most important input words for any output word get automatically highlighted

The general trend is somewhat linear because word order is roughly similar in both languages

# Translation Examples

| | |
|---|---|
| Source | An admitting privilege is the right of a doctor to admit a patient to a hospital or a medical centre to carry out a diagnosis or a procedure, based on his status as a health care worker at a hospital. |
| Reference | Le privilège d'admission est le droit d'un médecin, en vertu de son statut de membre soignant d'un hôpital, d'admettre un patient dans un hôpital ou un centre médical afin d'y délivrer un diagnostic ou un traitement. |
| RNNenc-50 | Un privilège d'admission est le droit d'un médecin de reconnaître un patient à l'hôpital ou un centre médical d'un diagnostic ou de prendre un diagnostic en fonction de son état de santé. |
| RNNsearch-50 | Un privilège d'admission est le droit d'un médecin d'admettre un patient à un hôpital ou un centre médical pour effectuer un diagnostic ou une procédure, selon son statut de travailleur des soins de santé à l'hôpital. |
| Google Translate | Un privilège admettre est le droit d'un médecin d'admettre un patient dans un hôpital ou un centre médical pour effectuer un diagnostic ou une procédure, fondée sur sa situation en tant que travailleur de soins de santé dans un hôpital. |

| | |
|---|---|
| Source | This kind of experience is part of Disney's efforts to "extend the lifetime of its series and build new relationships with audiences via digital platforms that are becoming ever more important," he added. |
| Reference | Ce type d'expérience entre dans le cadre des efforts de Disney pour "étendre la durée de vie de ses séries et construire de nouvelles relations avec son public grâce à des plateformes numériques qui sont de plus en plus importantes", a-t-il ajouté. |
| RNNenc-50 | Ce type d'expérience fait partie des initiatives du Disney pour "prolonger la durée de vie de ses nouvelles et de développer des liens avec les lecteurs numériques qui deviennent plus complexes. |
| RNNsearch-50 | Ce genre d'expérience fait partie des efforts de Disney pour "prolonger la durée de vie de ses séries et créer de nouvelles relations avec des publics via des plateformes numériques de plus en plus importantes", a-t-il ajouté. |
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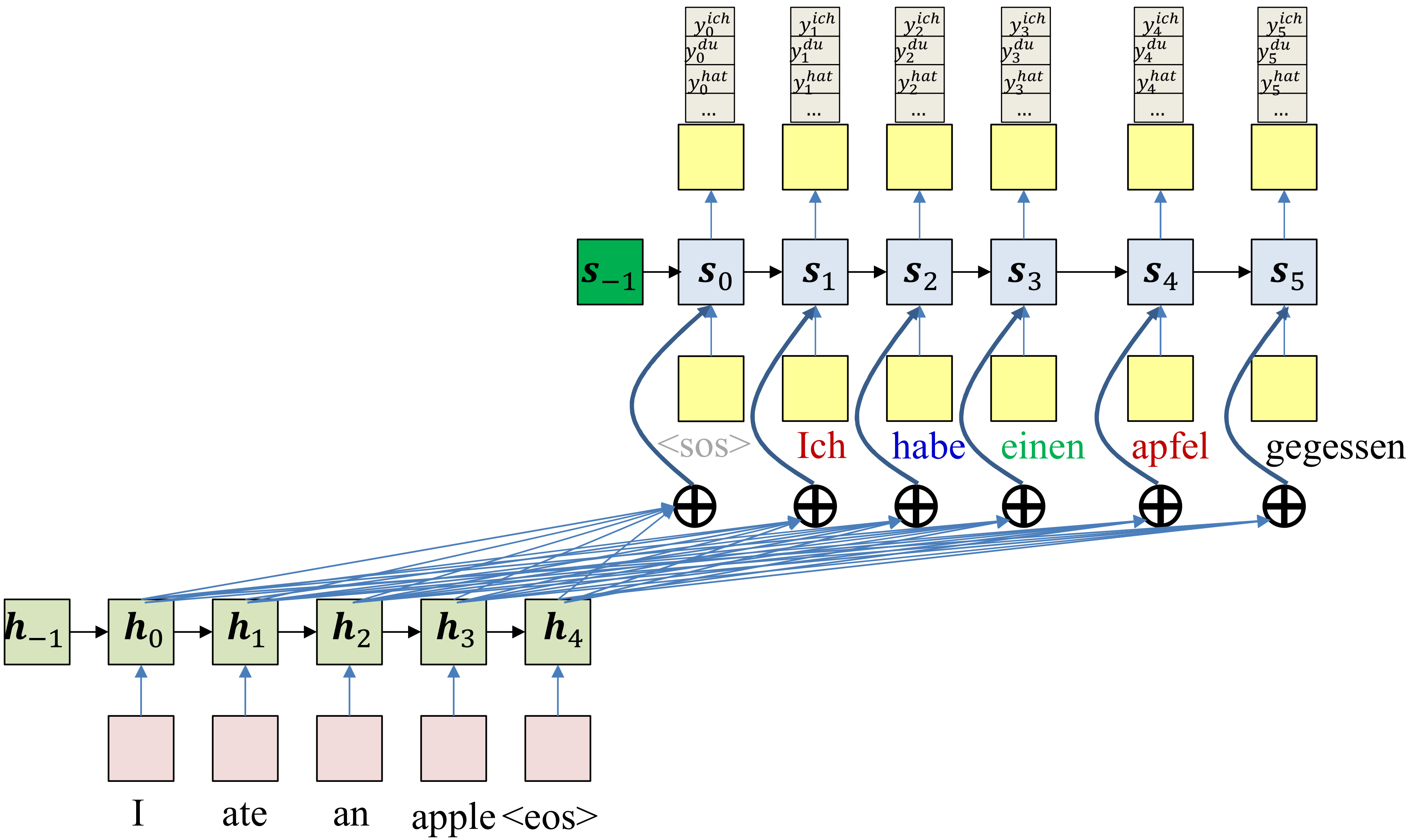
- Bahdanau et al. 2016

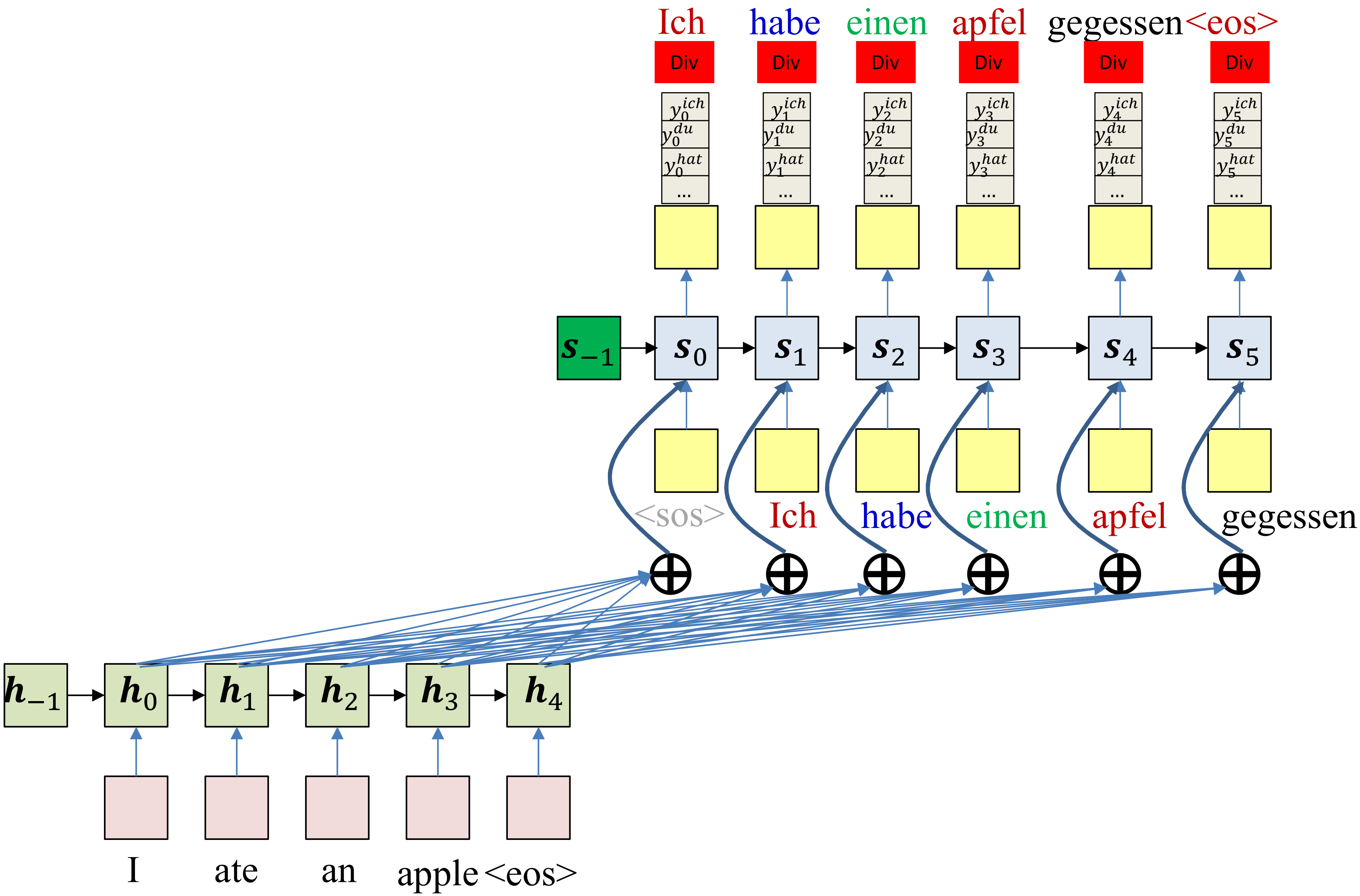
# Training the network

- We have seen how a trained network can be used to compute outputs
  - Convert one sequence to another

- Let’s consider training..

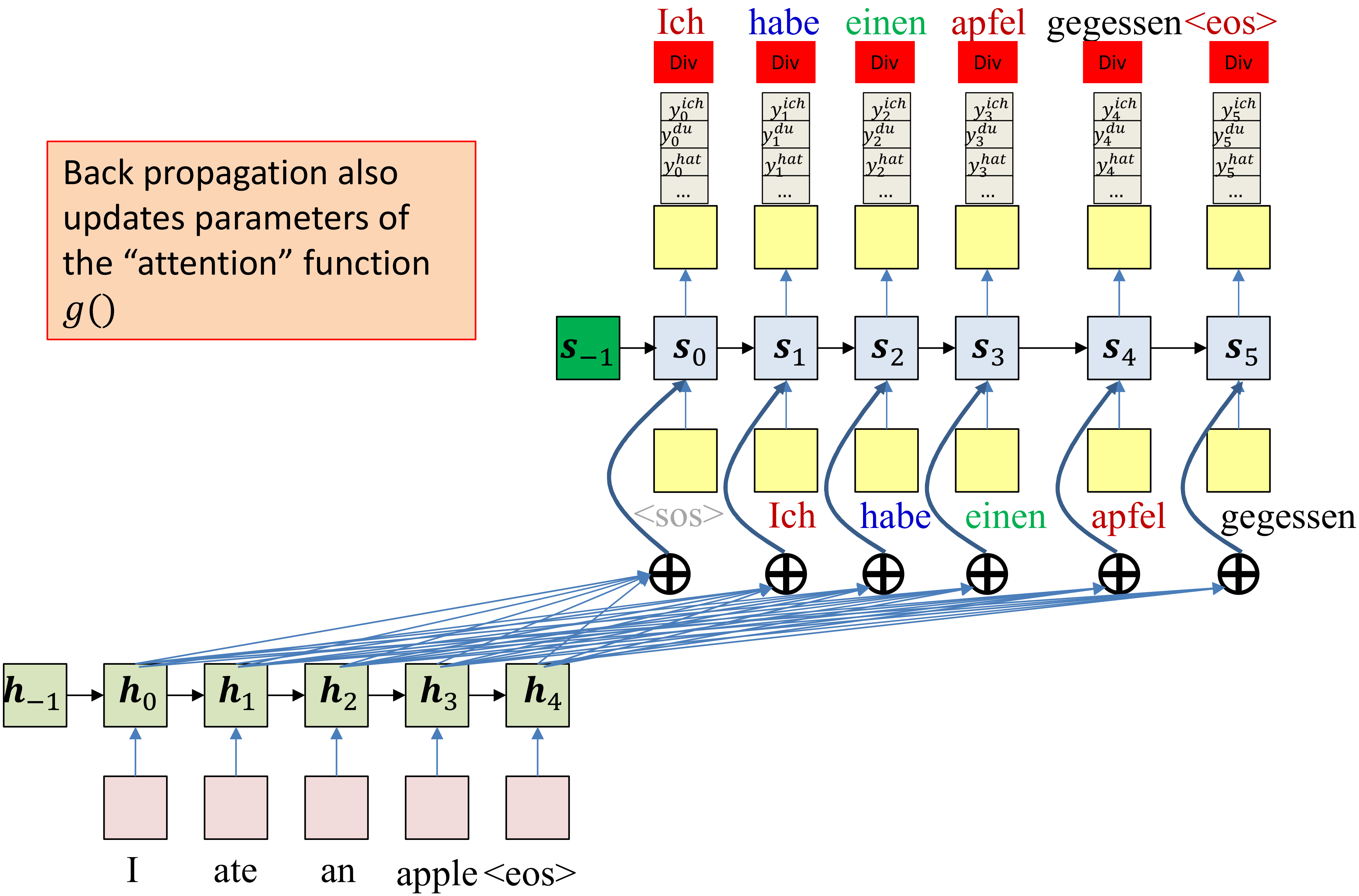



- Given training input (source sequence, target sequence) pairs
- **Forward pass**: Pass the actual Pass the input sequence through the encoder
  - At each time the output is a probability distribution over words




- **Backward pass**: Compute a divergence between target output and output distributions
  - Backpropagate derivatives through the network

Back propagation also updates parameters of the “attention” function $g()$




- **Backward pass**: Compute a divergence between target output and output distributions
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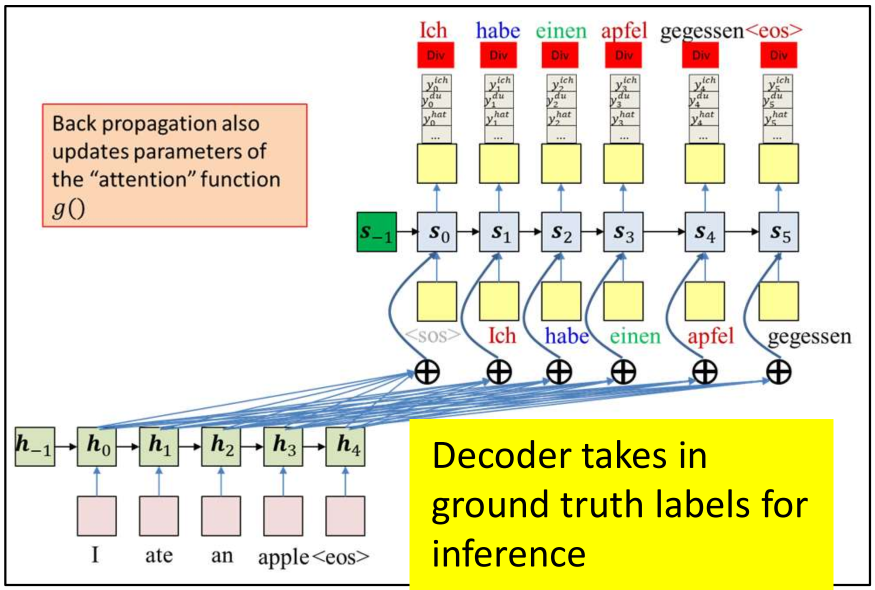
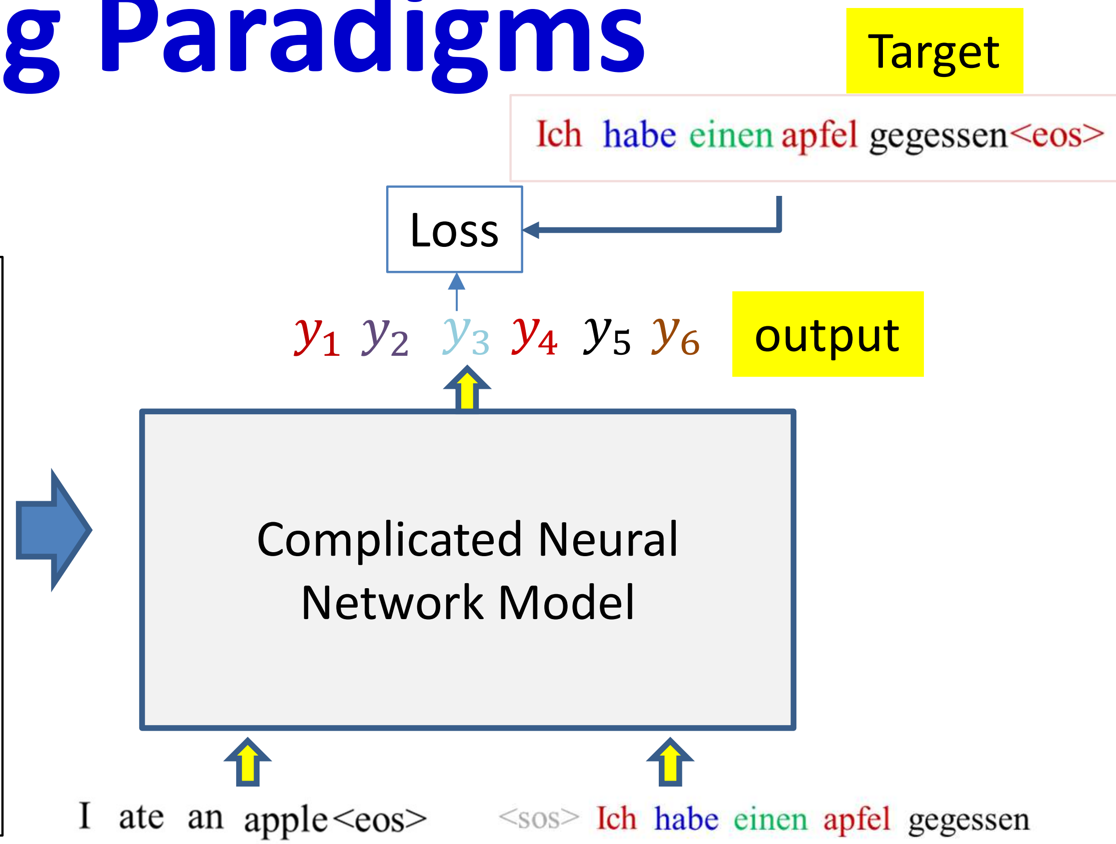
# Training Paradigms

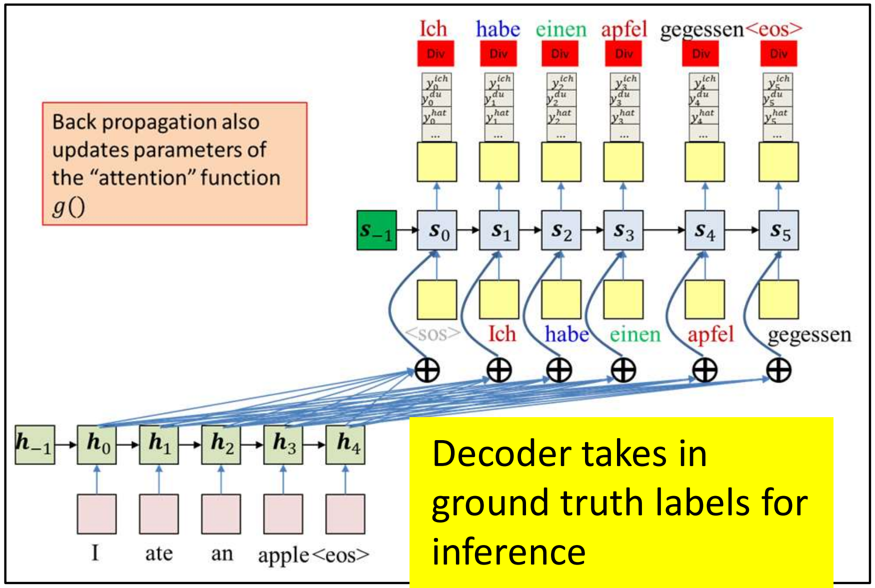


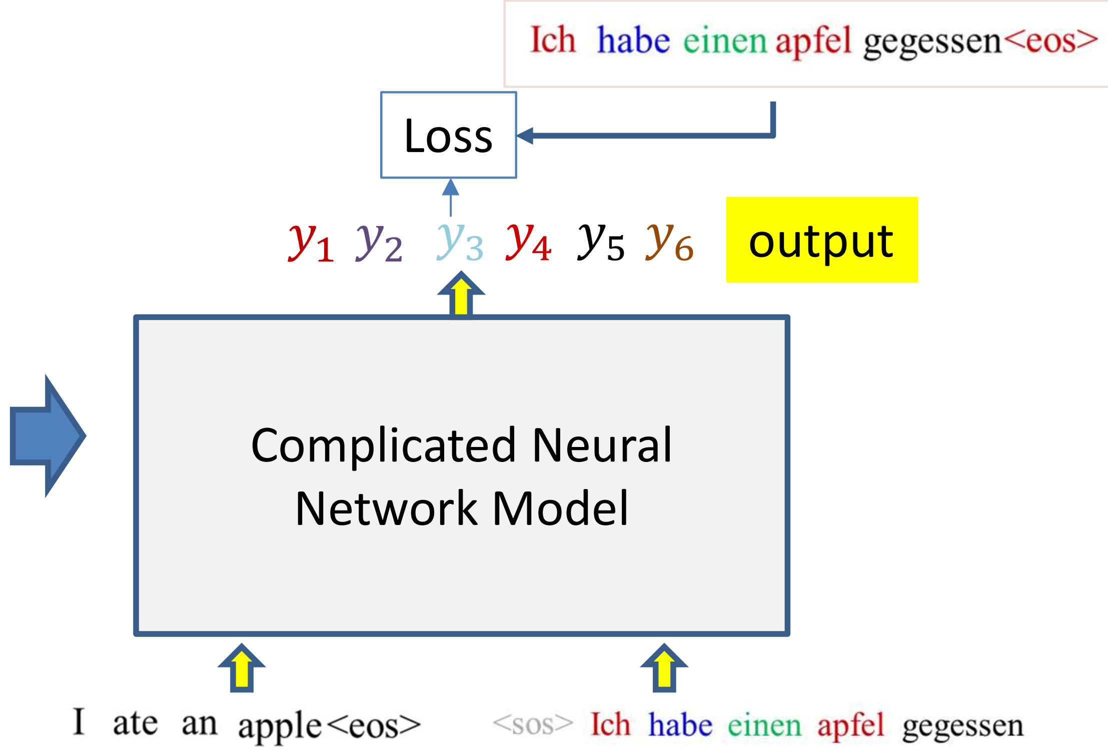




- This approach, in fact, performs maximum-likelihood estimation of the model
  - The forward pass of training takes in both inputs *and ground-truth* output labels for generation
  - This forward pass is *not* like regular inference

# Training Paradigms

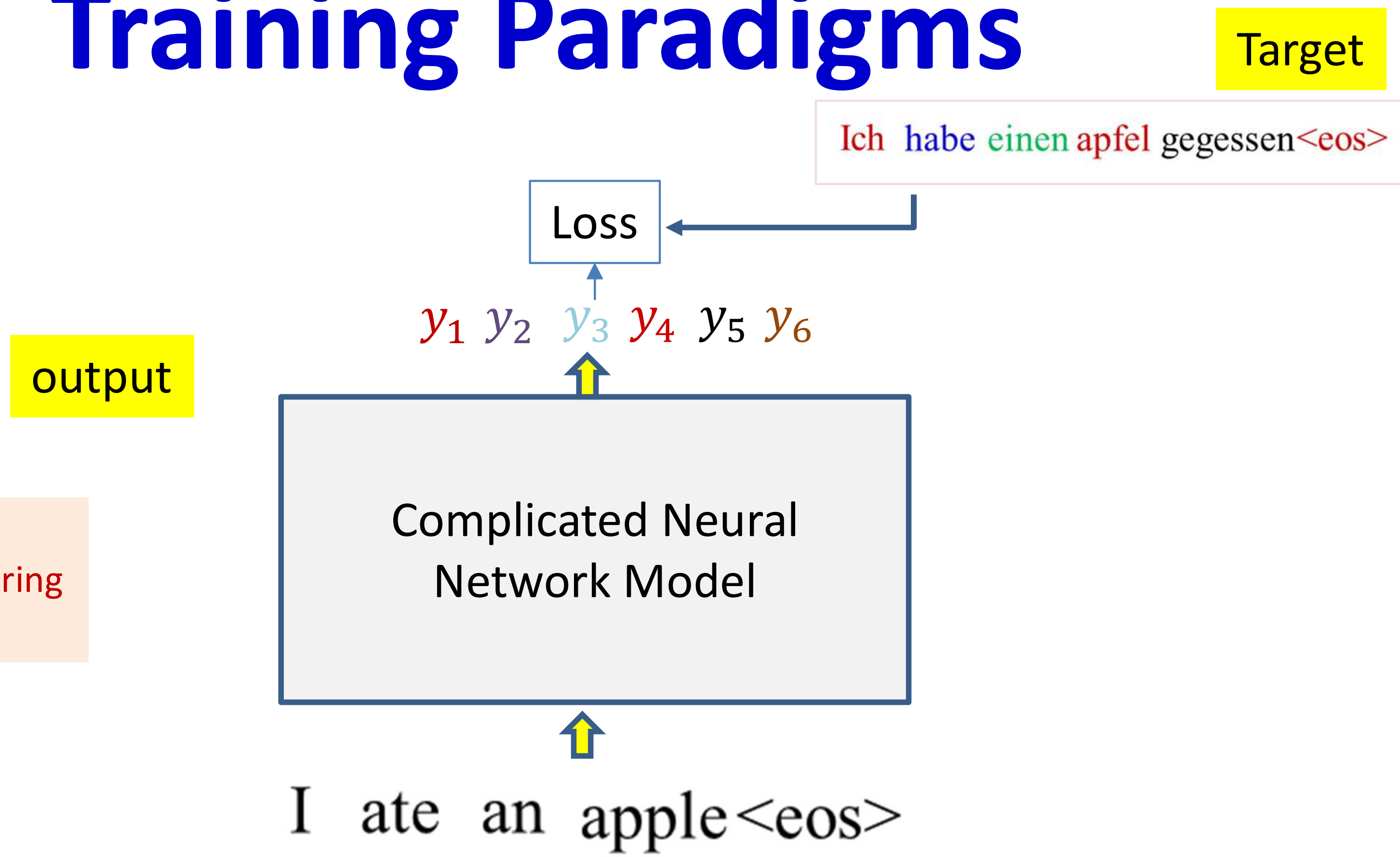
Target




- This approach, in fact, performs maximum-likelihood estimation of the model
  - The forward pass of training takes in both inputs *and ground-truth* output labels for generation
  - This forward pass is *not* like regular inference
- But there's another more traditional mechanism for training
  - Where the model *only* gets the input during the forward pass,
    - The forward pass is identical to inference
  - and the actual ground truth label is *only* used to compute loss…

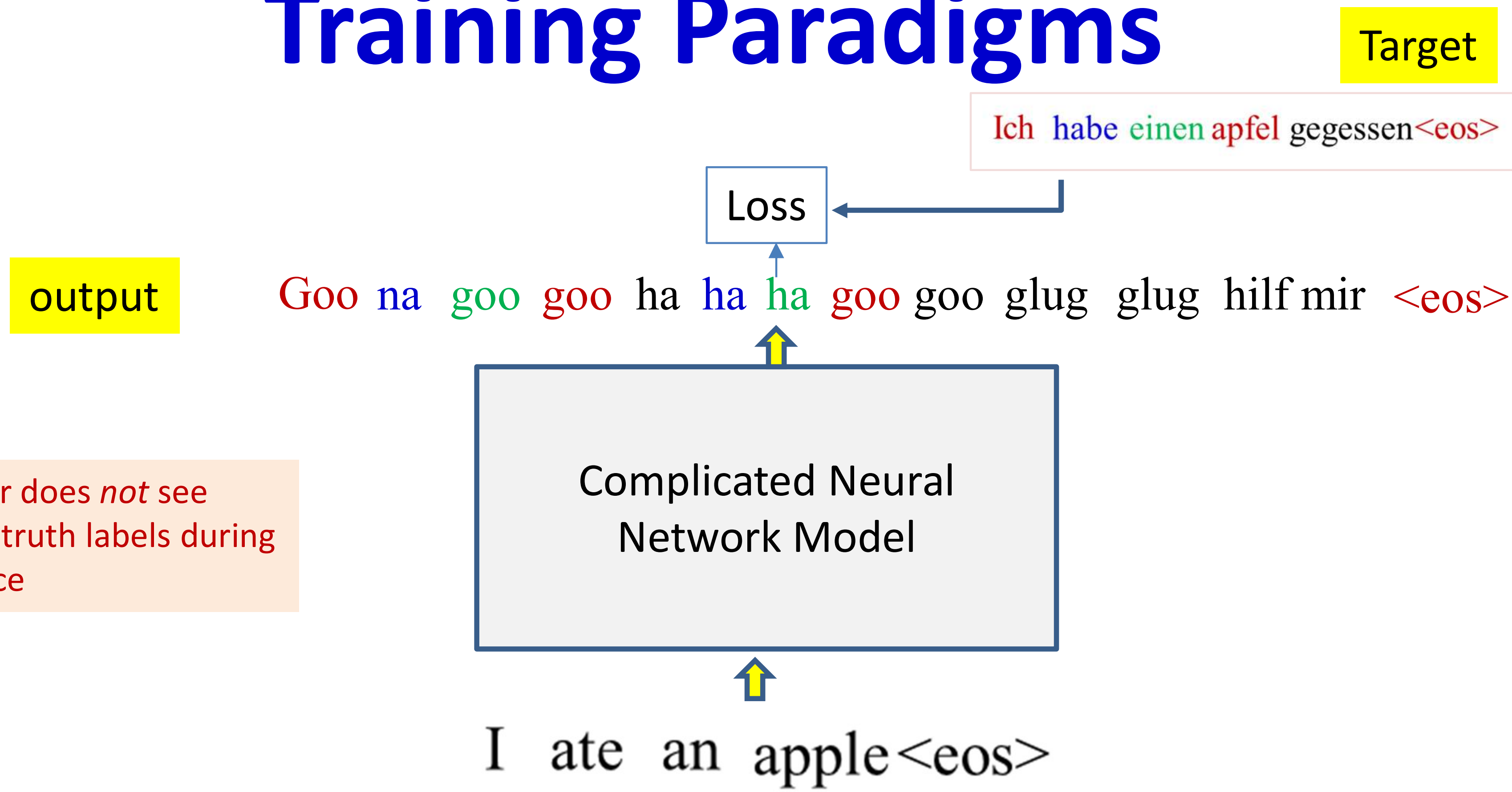
# Training Paradigms




Decoder does *not* see ground truth labels during inference

- The traditional mechanism for training
  - Why would this not work?

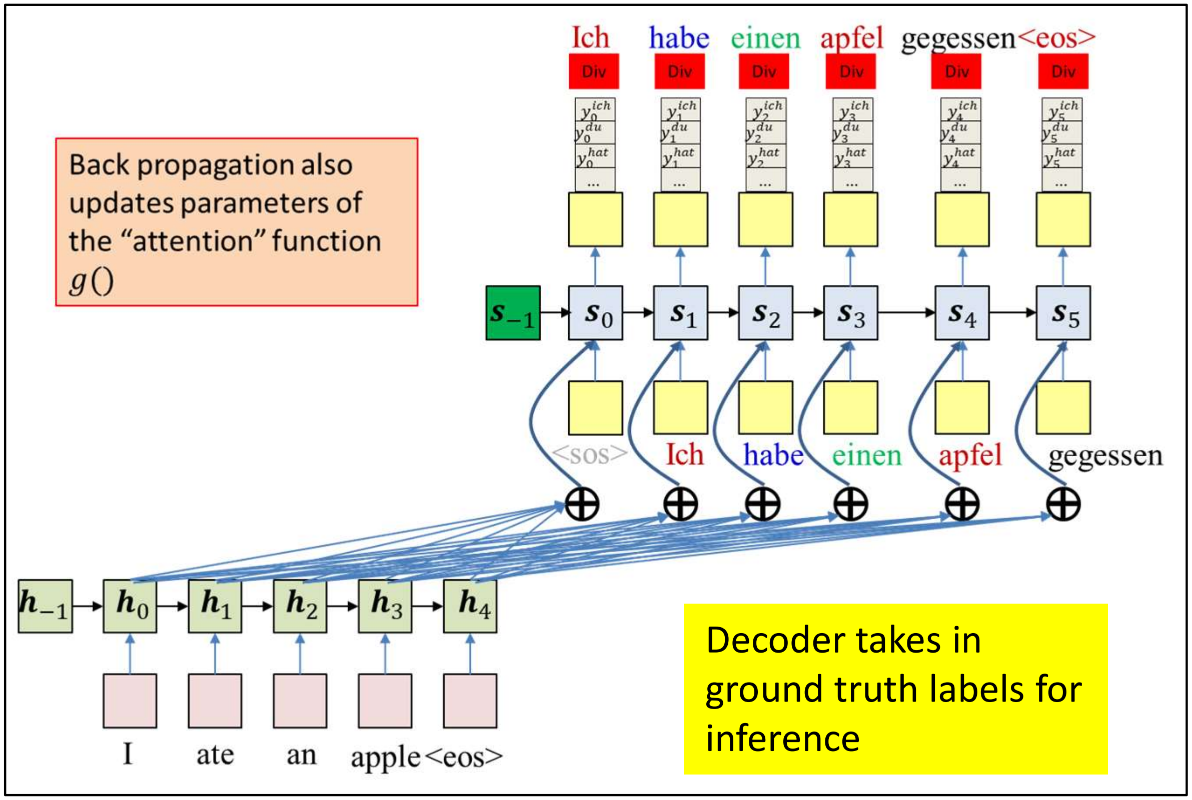
# Training Paradigms




Decoder does *not* see ground truth labels during inference

- The traditional mechanism for training
  - Why would this not work?
- The output could be *anything*
  - And in the early stages of training, *will* be random
  - Cannot really compute a loss

# Training Paradigms

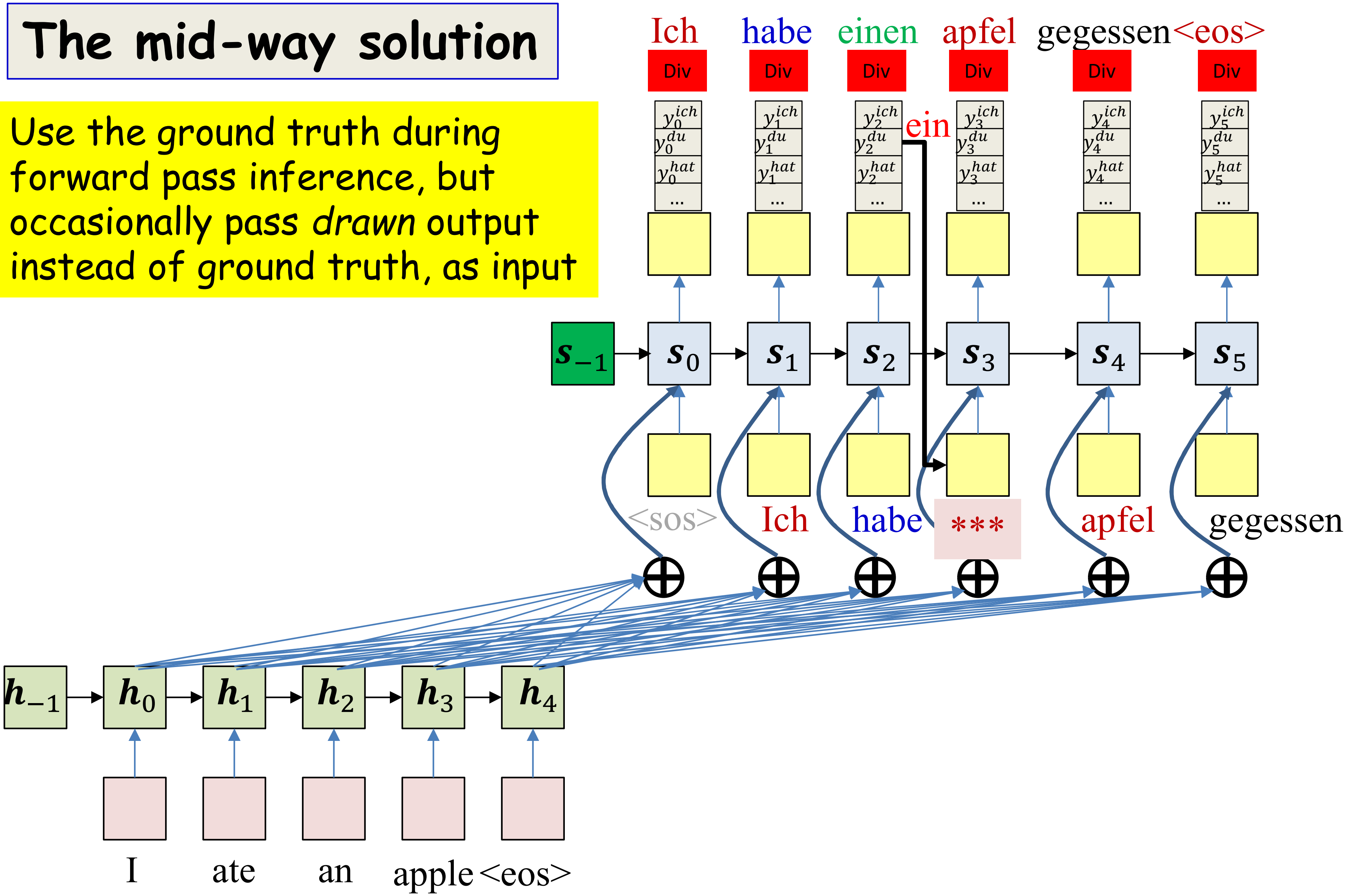



- The maximum-likelihood approach ensures a one-to-one correspondence between outputs and target outputs

- But from the perspective of *traditional* training, it is "cheating" during the inference of the forward pass, where the ground truth is being passed in instead of drawn samples
  - "Teacher forcing"
  - Is there a midway solution?
    - Use drawn words during the forward pass inference while maintaining one-to-one correspondence

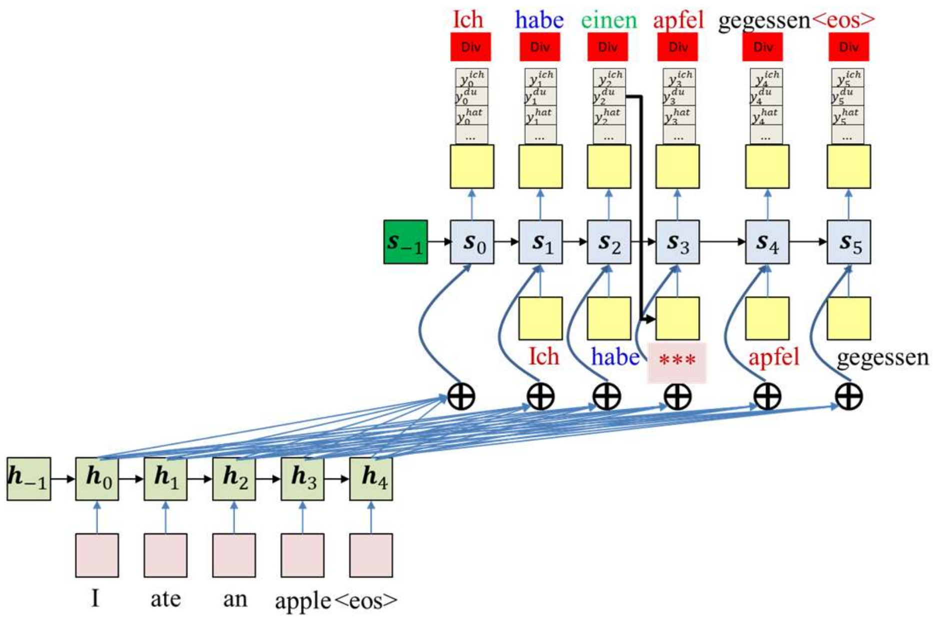
# The mid-way solution

Use the ground truth during forward pass inference, but occasionally pass *drawn* output instead of ground truth, as input




- **Backward pass**: Compute a divergence between target output and output distributions
  - Backpropagate derivatives through the network

# Gumbel Noise trick




- Sampling is not differentiable

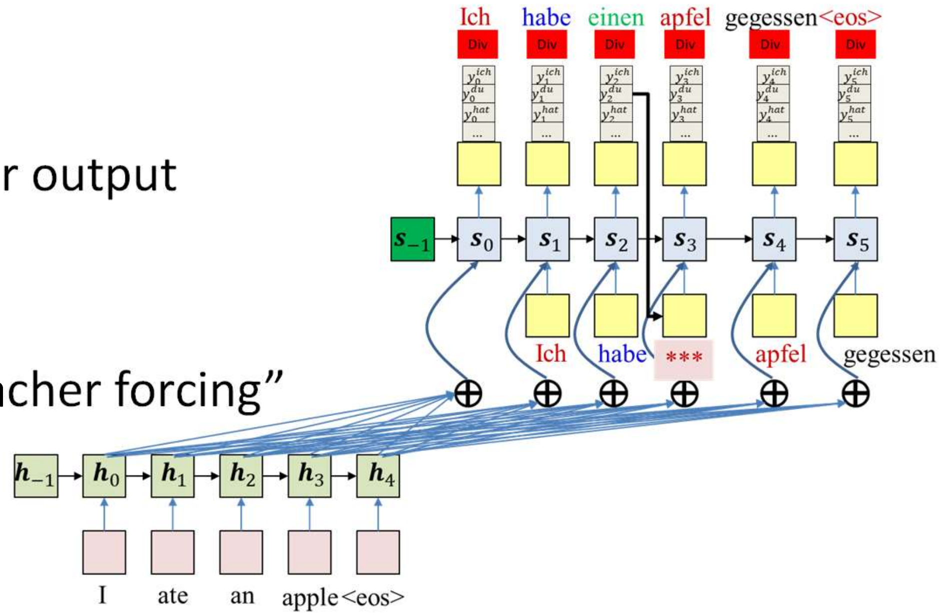
- The "Gumbel noise" trick:
  - "Reparametrization" :

$$RandomSample(Y) = \underset{i}{\operatorname{argmax}}(G_i + \log(Y))$$

  - $G_i$ is drawn from the standard Gumbel distribution $Gumbel(0,1)$
- The "argmax" can be replaced by a "softmax", making the process differentiable w.r.t. network outputs
  - $decoderoutput(t) = softmax(G_i + \log(Y(t)))$
- $\nabla_{Y(t)} decoderouput(t)$ is employed in the chain rule to pass derivatives from t+1 back to $Y(t)$

# Tricks of the trade…

- Teacher forcing:
  - Ideally we would only use the decoder output during inference
  - This will not be stable
  - Passing in ground truth instead is "teacher forcing"



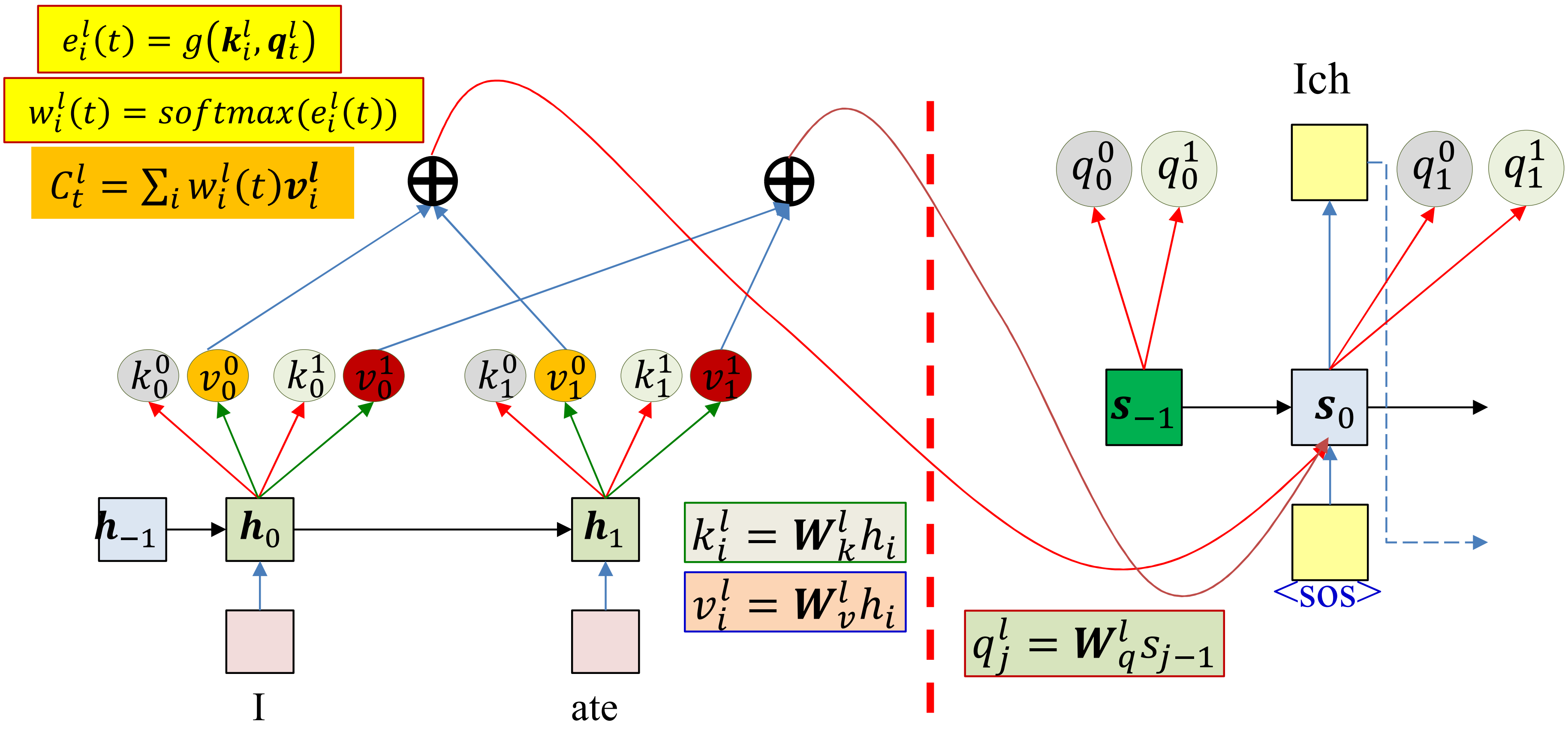

- Sampling the output:
  - Sample the system output and
  - as input during training for only some of the time

- The "Gumbel noise" trick:
  - Sampling is not differentiable, and gradients cannot be passed through it
  - The "Gumbel noise" approach recasts sampling as computing the argmax of a Gumbel distribution, with the network output as parameters
  - The "argmax" can be replaced by a "softmax", making the process differentiable w.r.t. network outputs

# Various extensions

- Bidirectional processing of input sequence
  - Bidirectional networks in encoder
  - E.g. “Neural Machine Translation by Jointly Learning to Align and Translate”, Bahdanau et al. 2016
- Attention: Local attention vs global attention
  - E.g. “Effective Approaches to Attention-based Neural Machine Translation”, Luong et al., 2015
  - Other variants

# Extensions: Multihead attention




- Have multiple query/key/value sets.
  - Each attention "head" uses one of these sets
  - The combined contexts from all heads are passed to the decoder
- Each "attender" focuses on a different aspect of the input that's important for the decode

# Poll 2 (@1213, @1215)

**Which of the following will give you the optimal decode with an attention-based decoder?**

- Full tree search
- Beam search

**Mark all that are true**

- In a query-key-value based attention mechanism, the key and value are used to compute attention weights
- Multi-head attention computes a separate set of keys and values for each head, at each input
- Multi-head attention computes a separate query for each head, at each output
- Training with teacher forcing computes the theoretically correct loss and minimizes it

# Poll 2

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- Training with teacher forcing computes the theoretically correct loss and minimizes it

# Some impressive results..

- Attention-based models are currently responsible for the state of the art in many sequence-conversion systems
  - Machine translation
    - Input: Text in source language
    - Output: Text in target language
  - Speech recognition
    - Input: Speech audio feature vector sequence
    - Output: Transcribed word or character sequence

# Attention models in image captioning

A woman is throwing a frisbee in a park.

A dog is standing on a hardwood floor.


A stop sign is on a road with a mountain in the background.

A little girl sitting on a bed with a teddy bear.

A group of people sitting on a boat in the water.

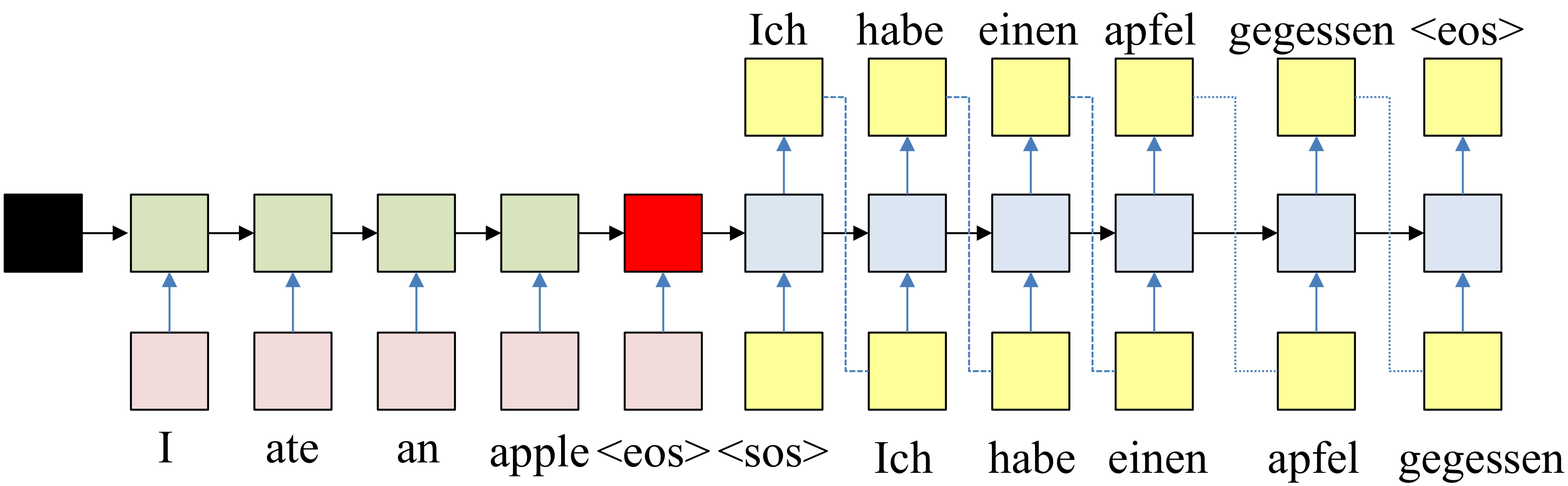
A giraffe standing in a forest with trees in the background.

- "Show attend and tell: Neural image caption generation with visual attention", Xu et al., 2016
- Encoder network is a convolutional neural network
  - Filter outputs at each location are the equivalent of $\boldsymbol{h}_i$ in the regular sequence-to-sequence model

# Recap

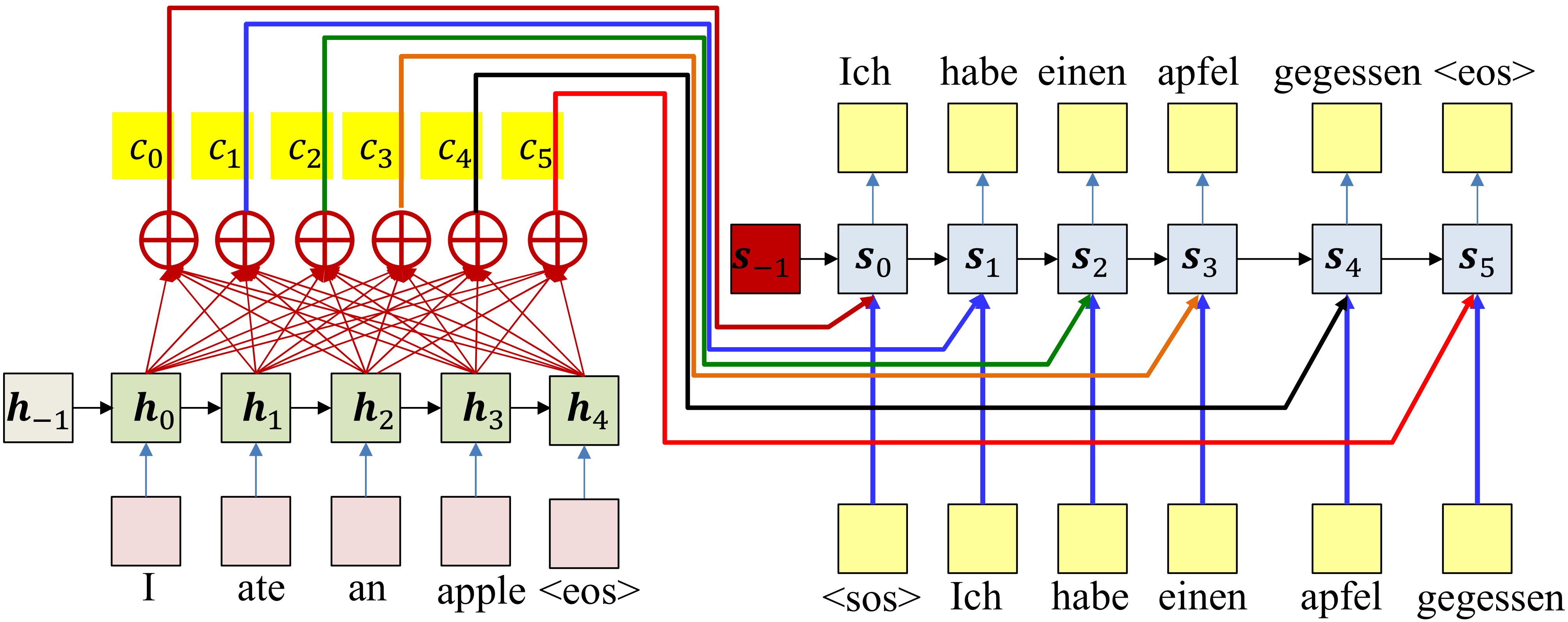
- Have looked at various forms of sequence-to-sequence models
- Generalizations of recurrent neural network formalisms
- For more details, please refer to papers
  - Post on piazza if you have questions
- Will appear in HW4: **Speech recognition with attention models**

# Recap: Seq2Seq models



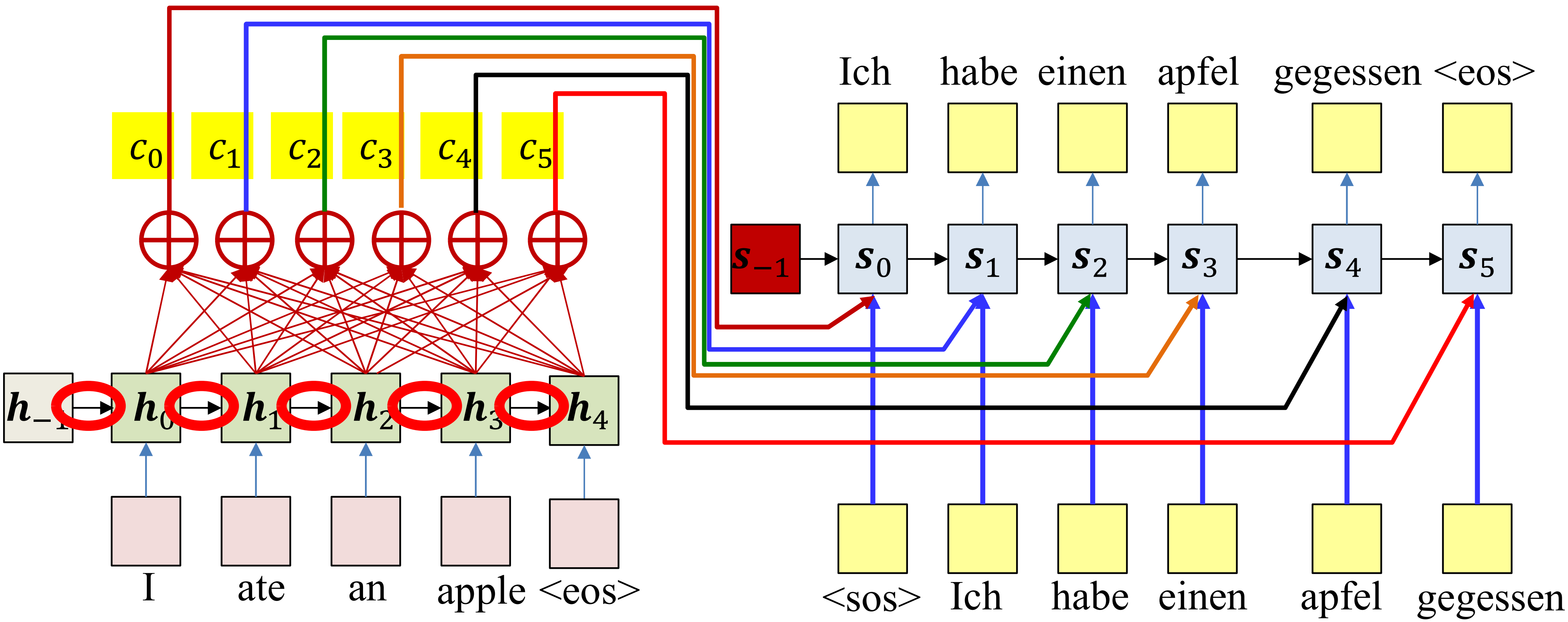

- The input sequence feeds into a recurrent structure
- The input sequence is terminated by an explicit <eos> symbol
  - The hidden activation at the <eos> "stores" all information about the sentence
- Subsequently a *second* RNN uses the hidden activation as initial state to produce a sequence of outputs

# Recap: Attention Models




- Encoder recurrently produces hidden representations of input word sequence
- Decoder recurrently generates output word sequence
  - For each output word the decoder uses a weighted average of the hidden input representations as input "context", along with the recurrent hidden state and the previous output word

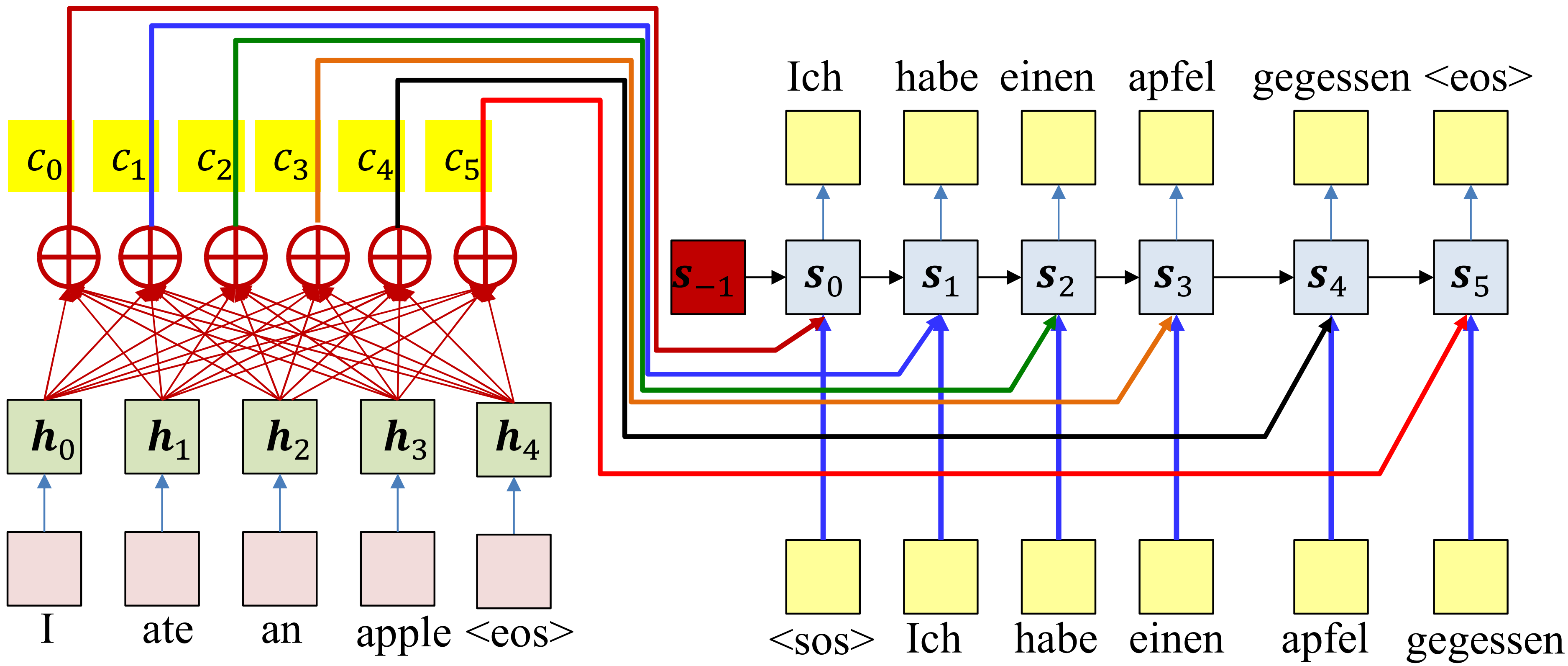
# Recap: Attention Models




- Problem: Because of the recurrence, the hidden representation for any word is also influenced by *all* preceding words
  - The decoder is actually paying attention to the sequence, and not just the word

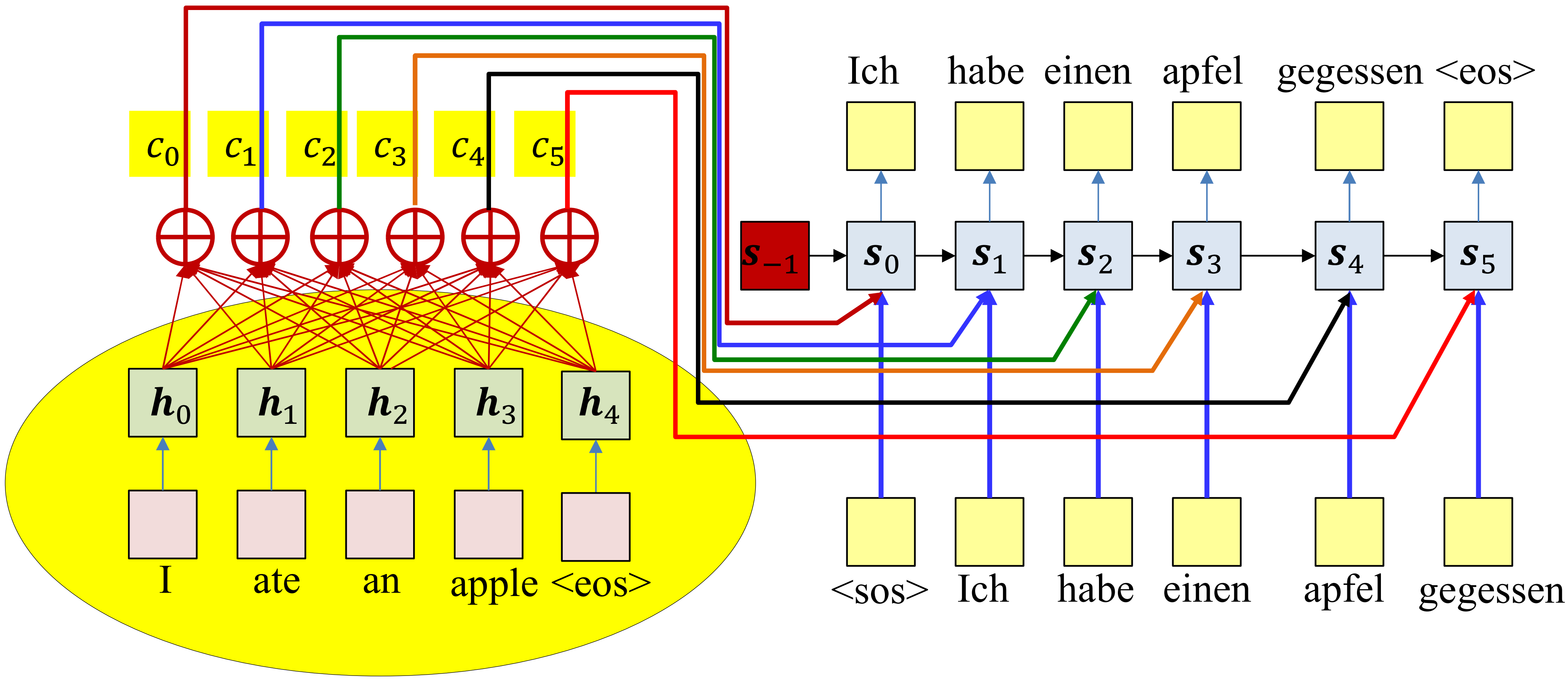
- If the decoder is automatically figuring out which words of the input to attend to at each time, is recurrence in the input even necessary?

# Non-recurrent encoder



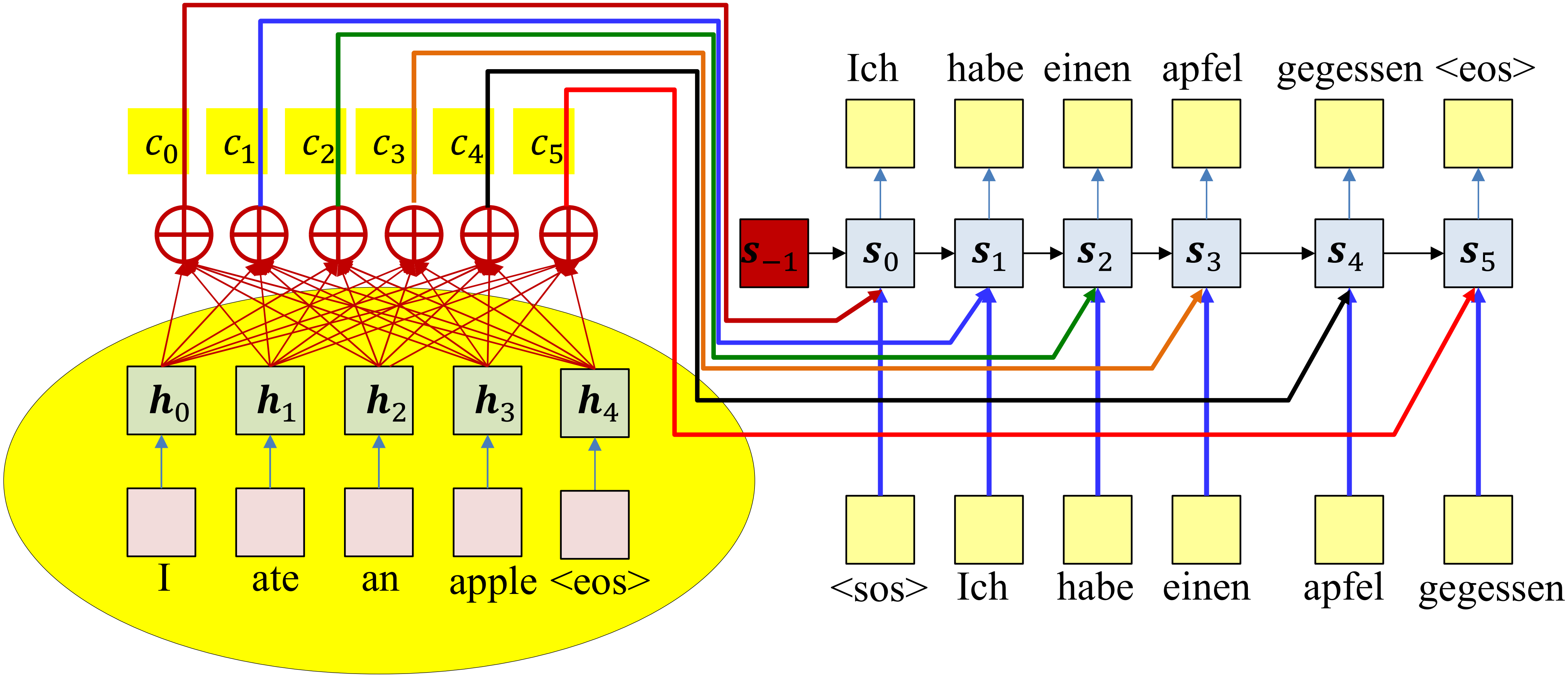

- Modification: Let us eliminate the recurrence in the encoder

# Non-recurrent encoder



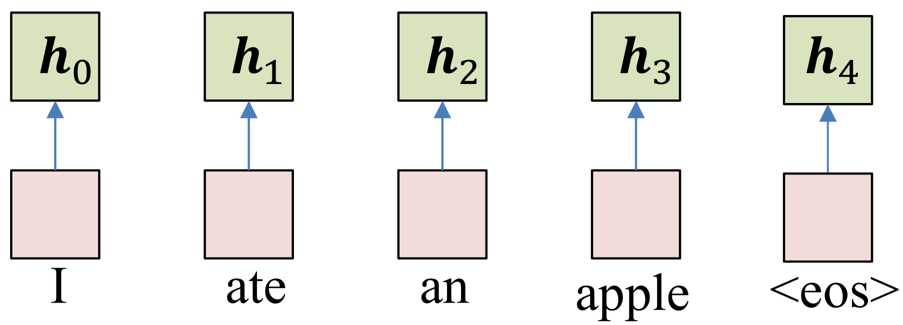

- But this will eliminate *context-specificity* in the encoder embeddings
  - The embedding for "an" must really depend on the remaining words
    - It could be translated to "ein", "einer", or "eines" depending on the context.
- Solution: Use the attention framework itself to introduce context-specificity in embeddings

# Recap: Non-recurrent encoder




- The encoder in a sequence-to-sequence model can be composed without recurrence.
- Use the attention framework itself to introduce context-specificity in embeddings
  - "Self" attention

# Self attention



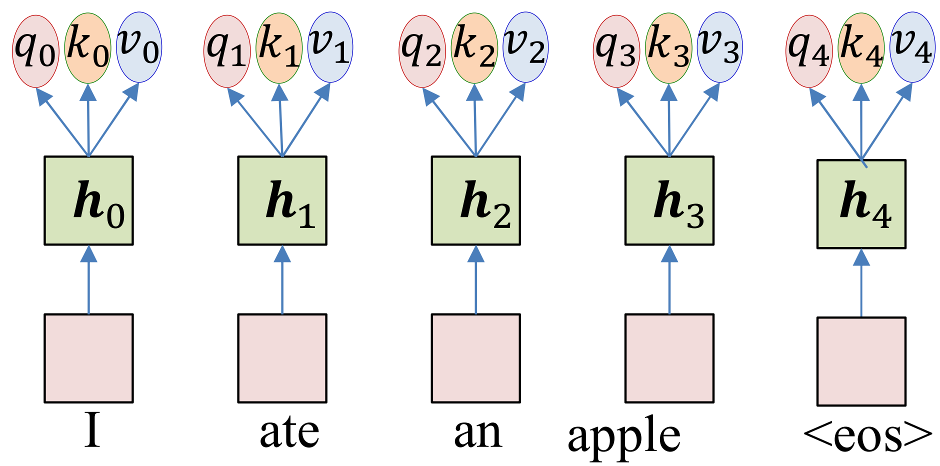

- First, for every word in the input sequence we compute an initial representation
  - E.g. using a single MLP layer

# Self attention

$q_i = \boldsymbol{W}_q h_i$

$k_i = \boldsymbol{W}_k h_i$

$v_i = \boldsymbol{W}_v h_i$




- Then, from each of the hidden representations, we compute a query, a key, and a value.
  - Using separate linear transforms
  - The weight matrices $W_q$, $W_k$ and $W_v$ are learnable parameters

# Self Attention

$q_i = \boldsymbol{W}_q h_i$

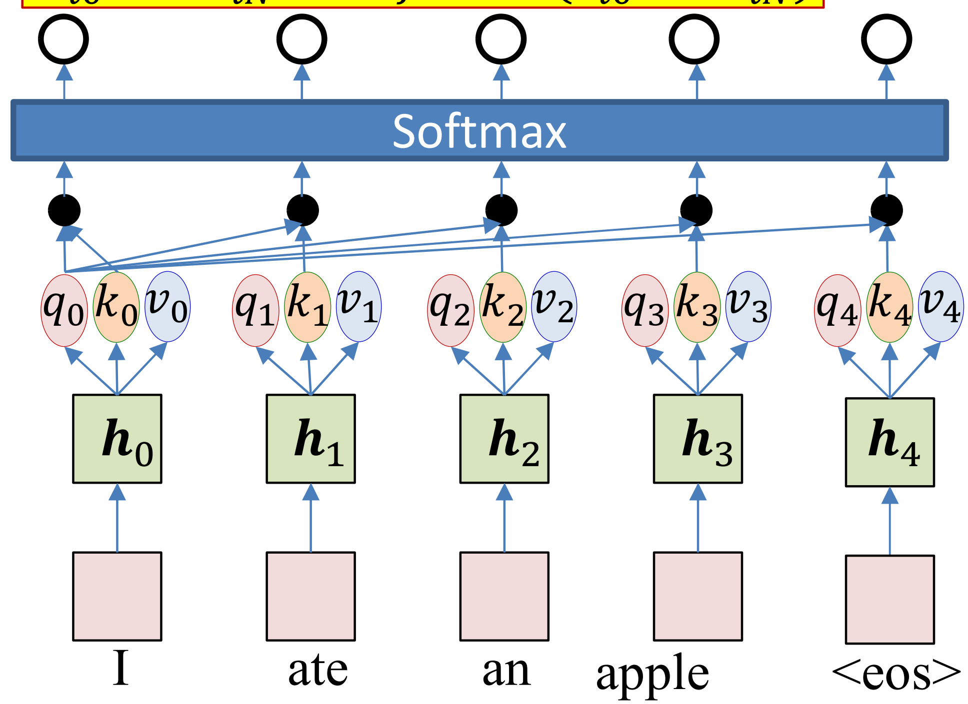
$k_i = \boldsymbol{W}_k h_i$

$v_i = \boldsymbol{W}_v h_i$

$w_{ij} = attn(q_i, k_{0:N})$

$e_{ij} = q_i^T k_j$

$w_{i0}, \dots, w_{iN} = softmax(e_{i0}, \dots, e_{iN})$




- For each word, we compute an attention weight between that word and all other words
  - The raw attention of the $i$th word to the $j$th word is a function of query $q_i$ and key $k_j$
  - The raw attention values are put through a softmax to get the final attention weights

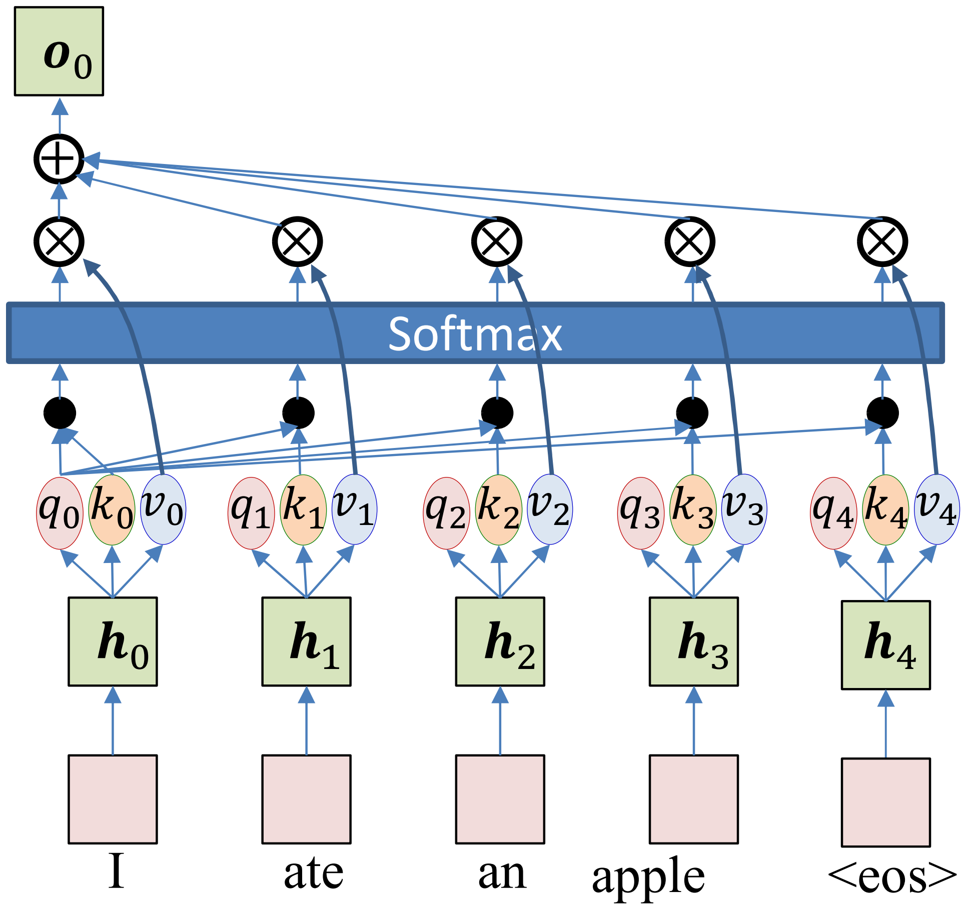
$$\boxed{q_i = \boldsymbol{W}_q h_i}$$

$$\boxed{k_i = \boldsymbol{W}_k h_i}$$

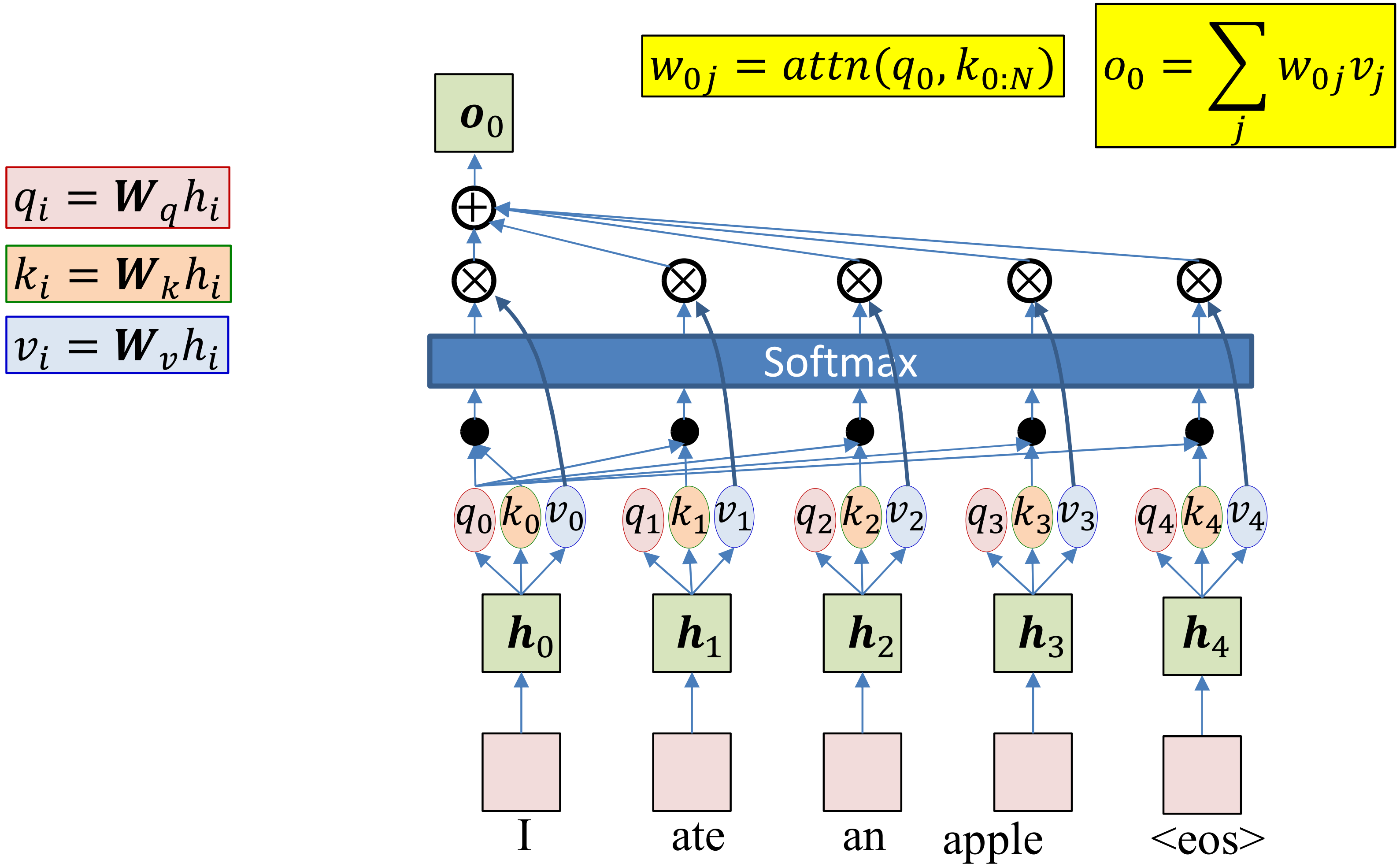
$$\boxed{v_i = \boldsymbol{W}_v h_i}$$

$$w_{ij} = attn(q_i, k_{0:N})$$

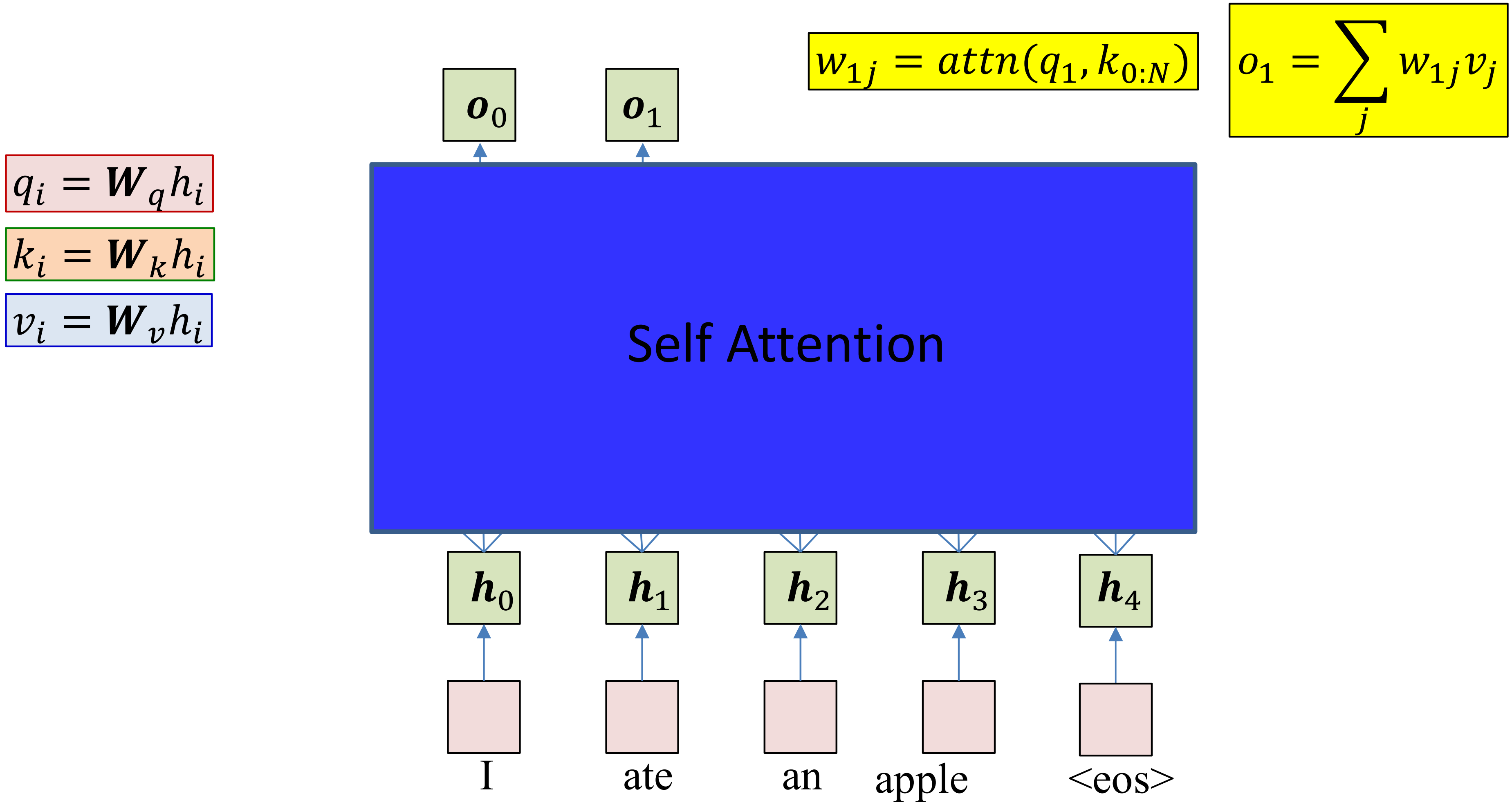
$$\boxed{o_i = \sum_j w_{ij} v_j}$$




- The updated representation for the word is the attention-weighted sum of the values for all words
  - Including itself




- Compute query-key-value sets for every word
- For each word
  - Using the query for that word, compute attention weights for all words using their keys
  - Compute updated representation for the word as attention-weighted sum of values of all words




- Compute query-key-value sets for every word
- For each word
  - Using the query for that word, compute attention weights for all words using their keys
  - Compute updated representation for the word as attention-weighted sum of values of all words

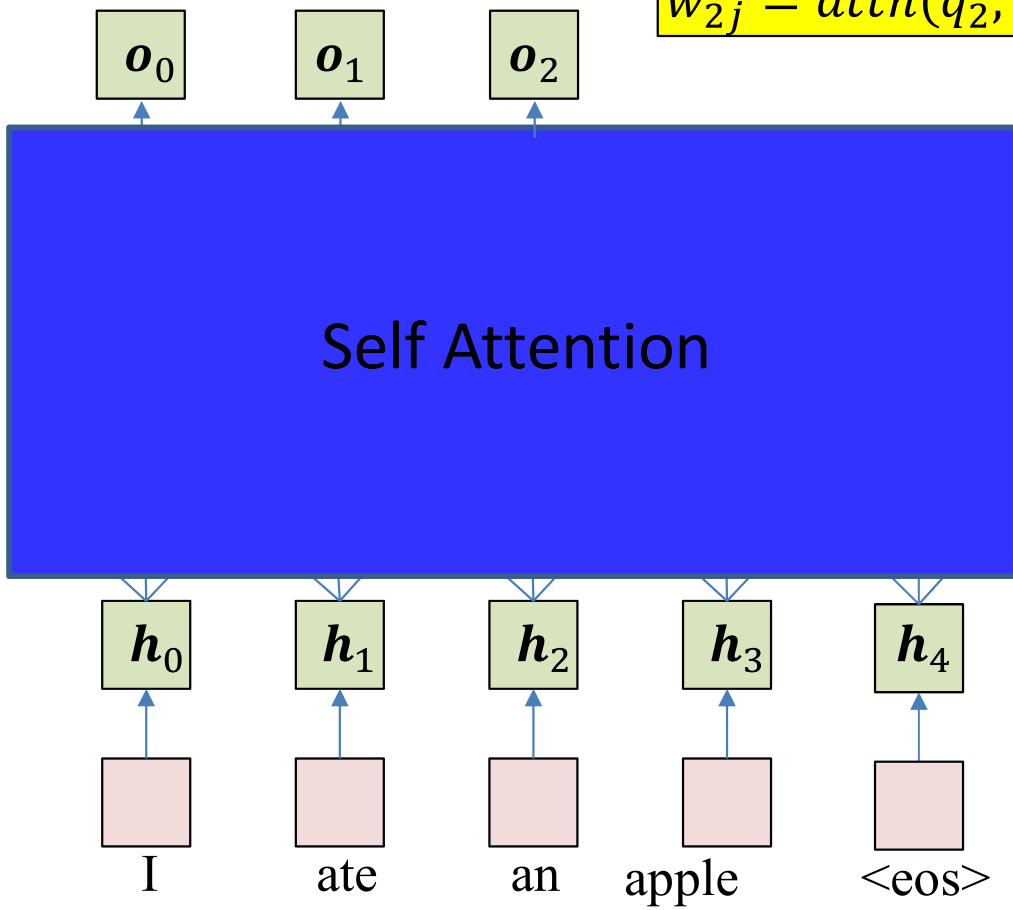
$q_i = \boldsymbol{W}_q h_i$

$k_i = \boldsymbol{W}_k h_i$

$v_i = \boldsymbol{W}_v h_i$

$w_{2j} = attn(q_2, k_{0:N})$

$o_2 = \sum_j w_{2j} v_j$




- Compute query-key-value sets for every word
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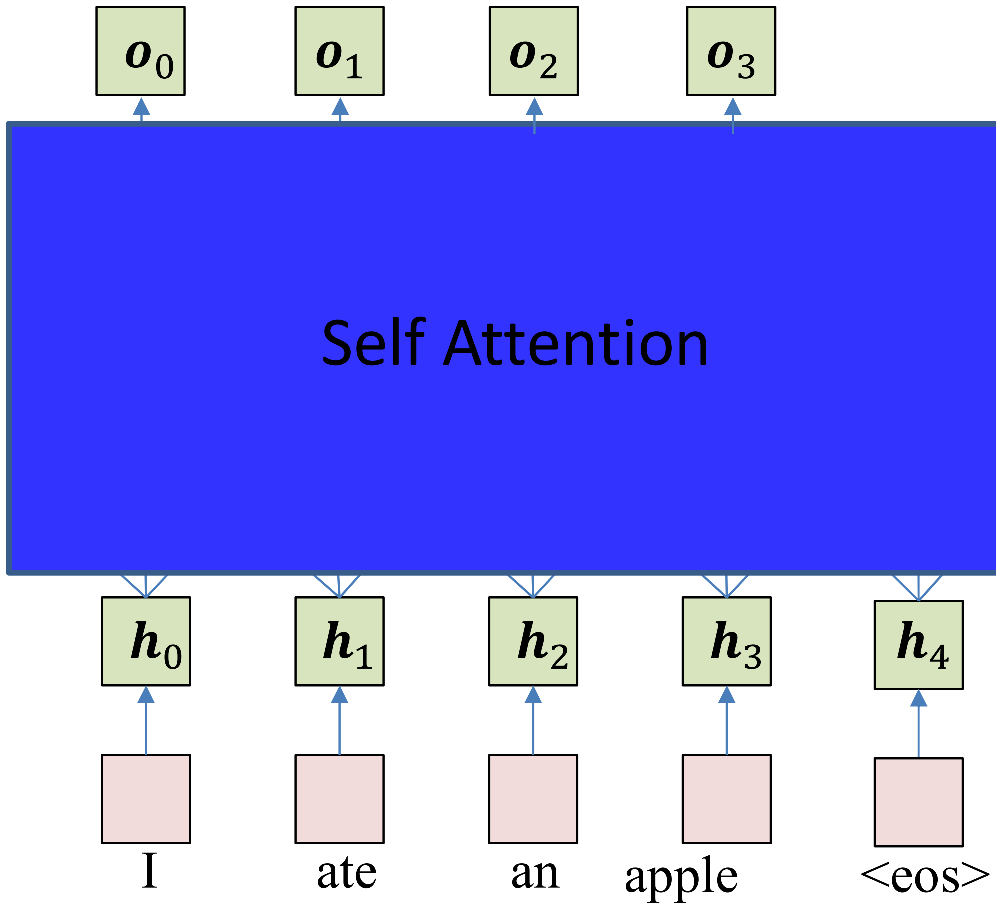


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- For each word
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$$q_i = \boldsymbol{W}_q h_i$$

$$k_i = \boldsymbol{W}_k h_i$$

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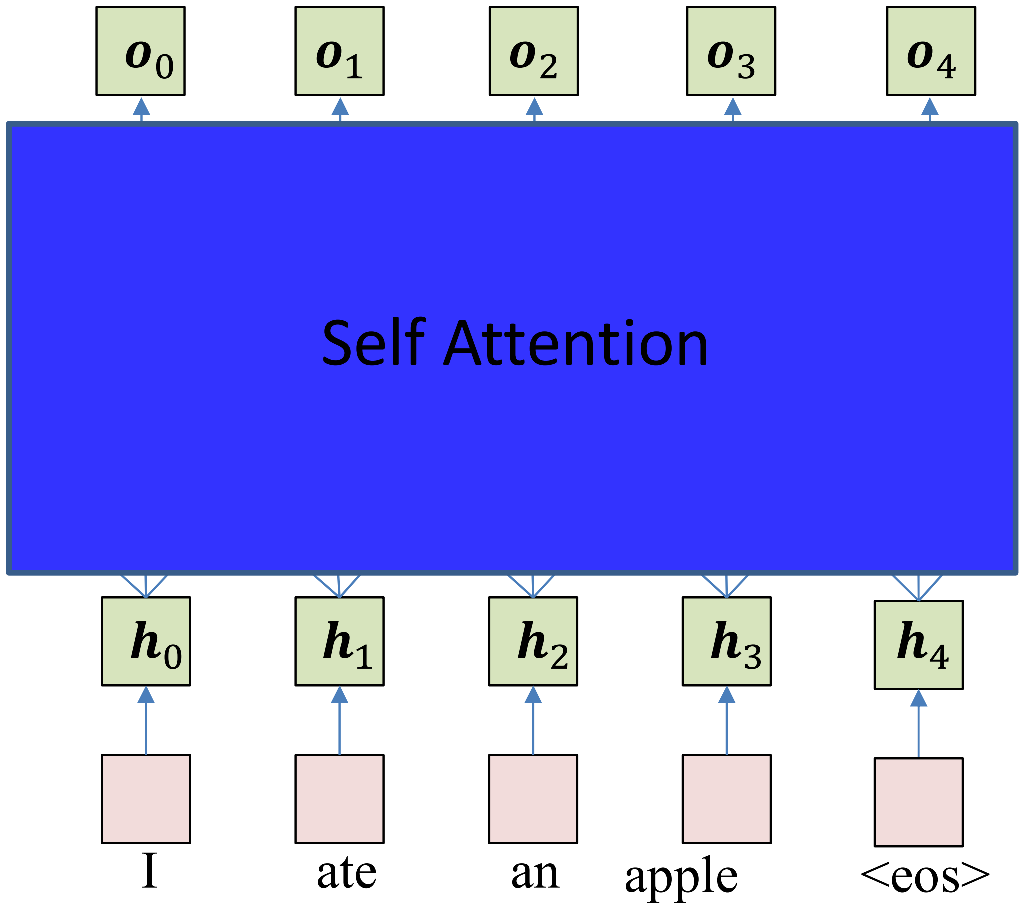



- Compute query-key-value sets for every word
- For each word
  - Using the query for that word, compute attention weights for all words using their keys
  - Compute updated representation for the word as attention-weighted sum of values of all words

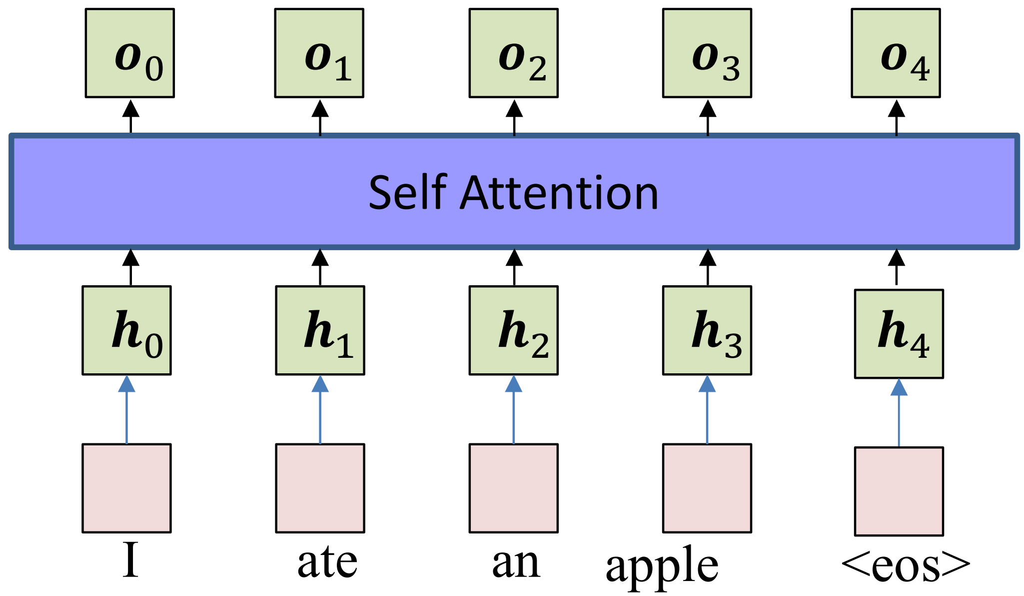
$$w_{ij} = attn(q_i, k_{0:N})$$

$$q_i = \boldsymbol{W}_q h_i$$

$$k_i = \boldsymbol{W}_k h_i$$

$$v_i = \boldsymbol{W}_v h_i$$

$$o_i = \sum_j w_{ij} v_j$$

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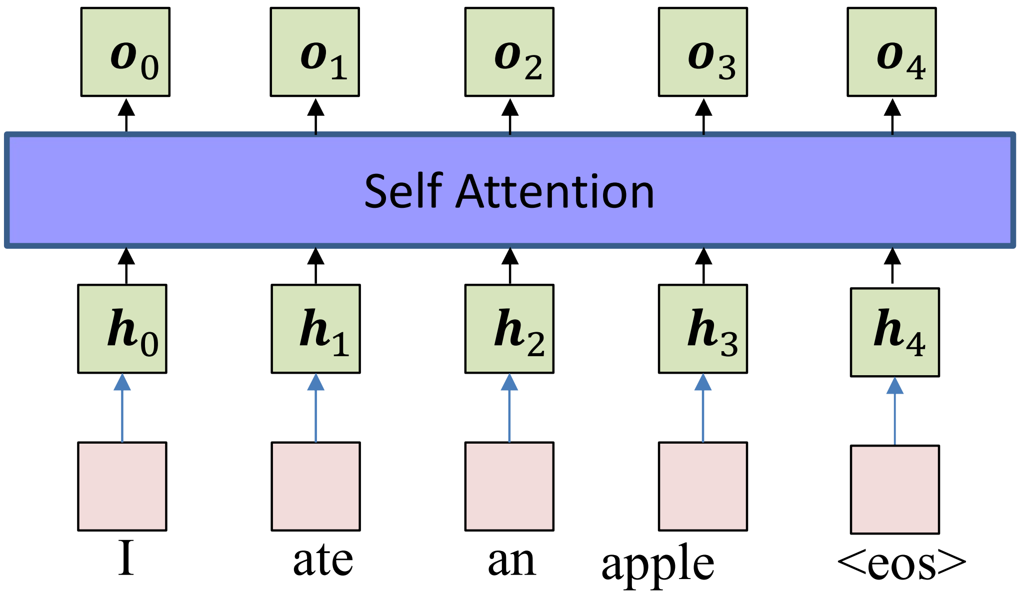



This is a "single-head" self-attention block

$$q_i^a = \boldsymbol{W}_q^a h_i$$

$$k_i^a = \boldsymbol{W}_k^a h_i$$

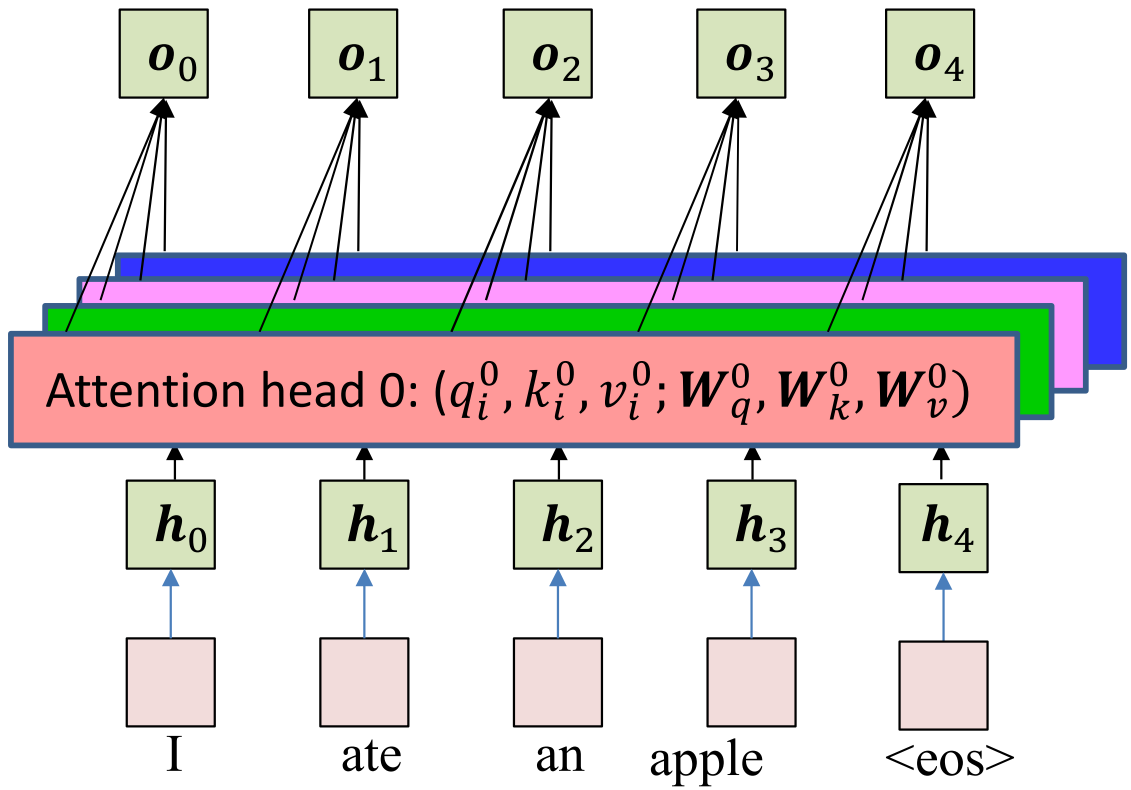
$$v_i^a = \boldsymbol{W}_v^a h_i$$

$$w_{ij}^a = attn(q_i^a, k_{0:N}^a)$$

$$o_i^a = \sum_j w_{ij}^a v_j^a$$

Concatenate

$$o_i = [o_i^1; o_i^2; o_i^3; \ldots; o_i^H]$$




- We can have *multiple* such attention "heads"
  - Each will have an independent set of queries, keys and values
  - Each will obtain an independent set of attention weights
    - Potentially focusing on a different aspect of the input than other heads
  - Each computes an independent output
- The final output is the concatenation of the outputs of these attention heads
- "MULTI-HEAD ATTENTION" (actually Multi-head *self* attention)

$q_i^a = \boldsymbol{W}_q^a h_i$

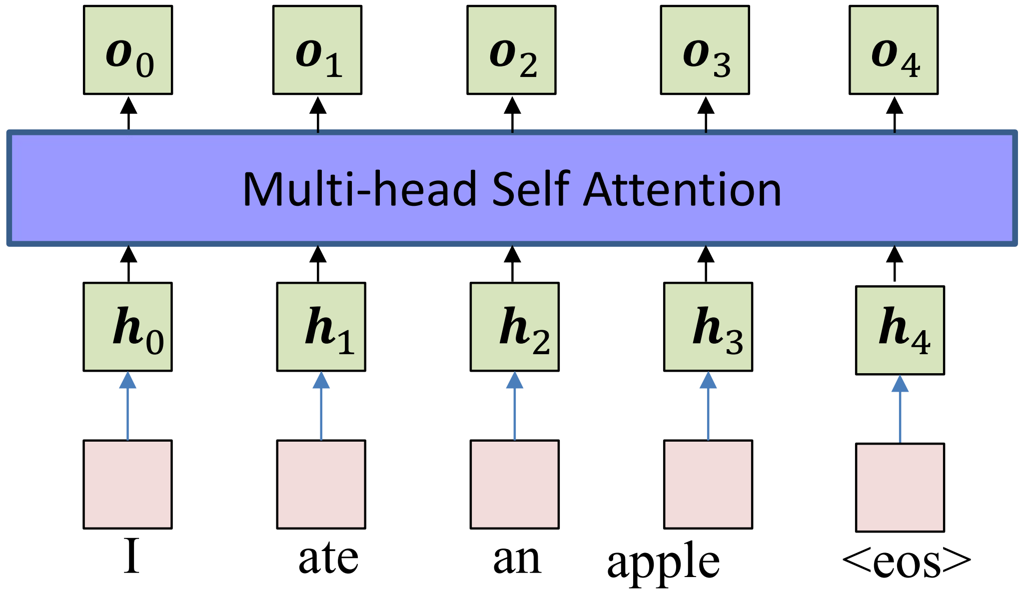
$k_i^a = \boldsymbol{W}_k^a h_i$

$v_i^a = \boldsymbol{W}_v^a h_i$

$w_{ij}^a = attn(q_i^a, k_{0:N}^a)$

$$o_i^a = \sum_j w_{ij}^a v_j^a$$

$o_i = [o_i^1; o_i^2; o_i^3; \ldots; o_i^H]$




- Multi-head self attention
  - Multiple self-attention modules in parallel

$$q_i^a = \boldsymbol{W}_q^a h_i$$

$$k_i^a = \boldsymbol{W}_k^a h_i$$

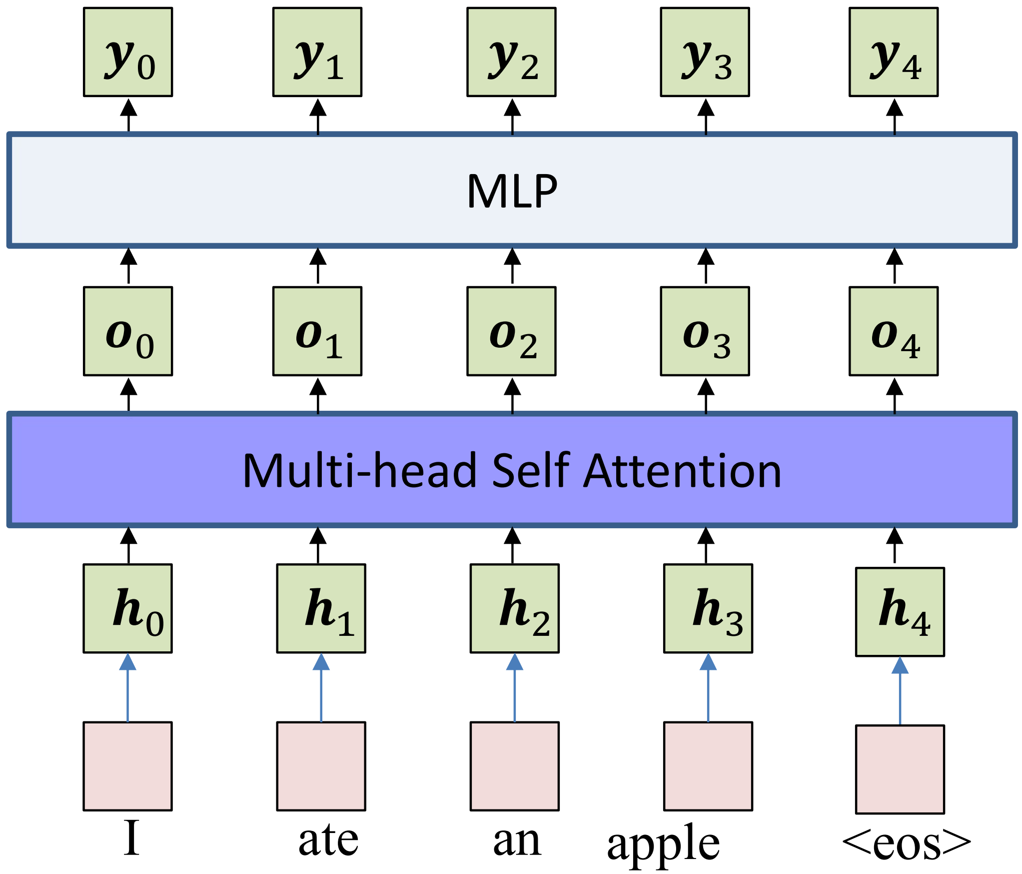
$$v_i^a = \boldsymbol{W}_v^a h_i$$

$$w_{ij}^a = attn(q_i^a, k_{0:N}^a)$$

$$o_i^a = \sum_j w_{ij}^a v_j^a$$

$$o_i = [o_i^1; o_i^2; o_i^3; \dots; o_i^H]$$

$$y_i = MLP(o_i)$$




- Typically, the output of the multi-head self attention is passed through one or more regular feedforward layers
  - Affine layer followed by a non-linear activation such as ReLU

$$q_i^a = \boldsymbol{W}_q^a h_i$$

$$k_i^a = \boldsymbol{W}_k^a h_i$$

$$v_i^a = \boldsymbol{W}_v^a h_i$$

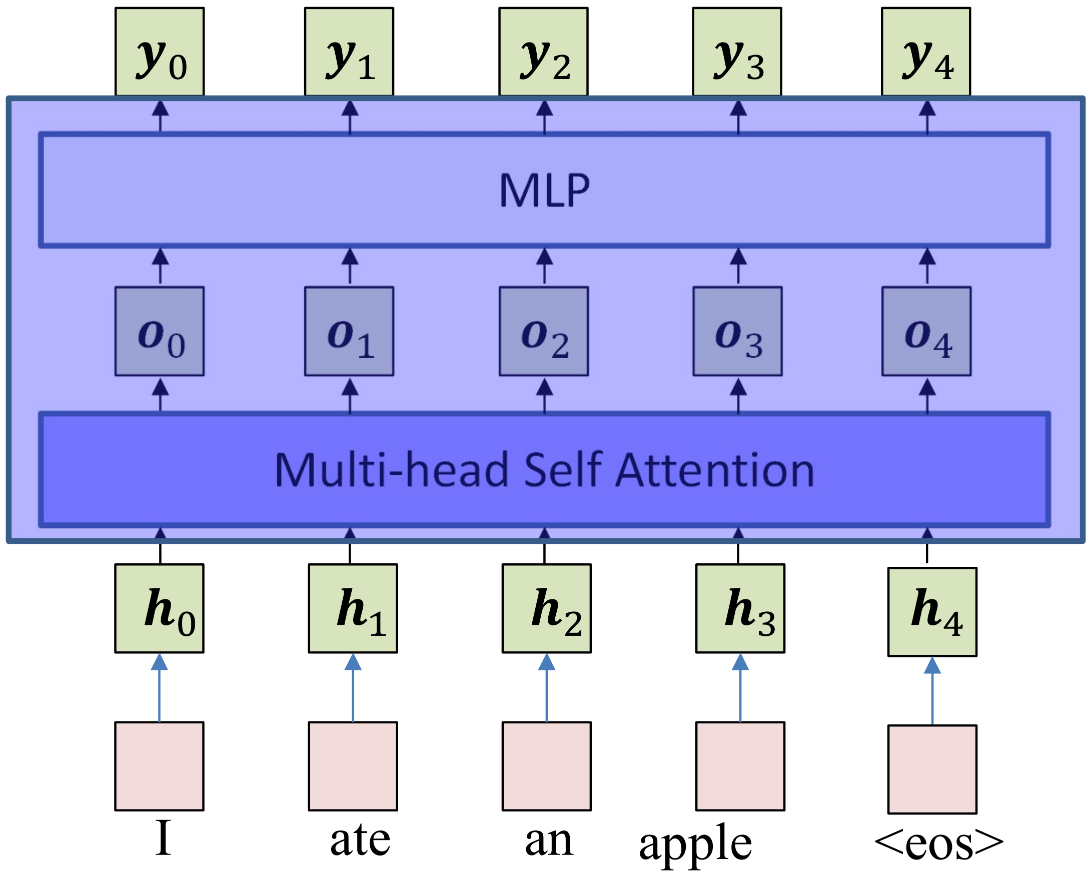
$$w_{ij}^a = attn(q_i^a, k_{0:N}^a)$$

$$o_i^a = \sum_j w_{ij}^a v_j^a$$

$$o_i = [o_i^1; o_i^2; o_i^3; \dots; o_i^H]$$

$$y_i = MLP(o_i)$$

MULTI-HEAD SELF ATTENTION BLOCK




- The entire unit, including multi-head self-attention module followed by MLP is a ***multi-head self-attention block***

$q_i^a = \boldsymbol{W}_q^a h_i$

$k_i^a = \boldsymbol{W}_k^a h_i$

$v_i^a = \boldsymbol{W}_v^a h_i$

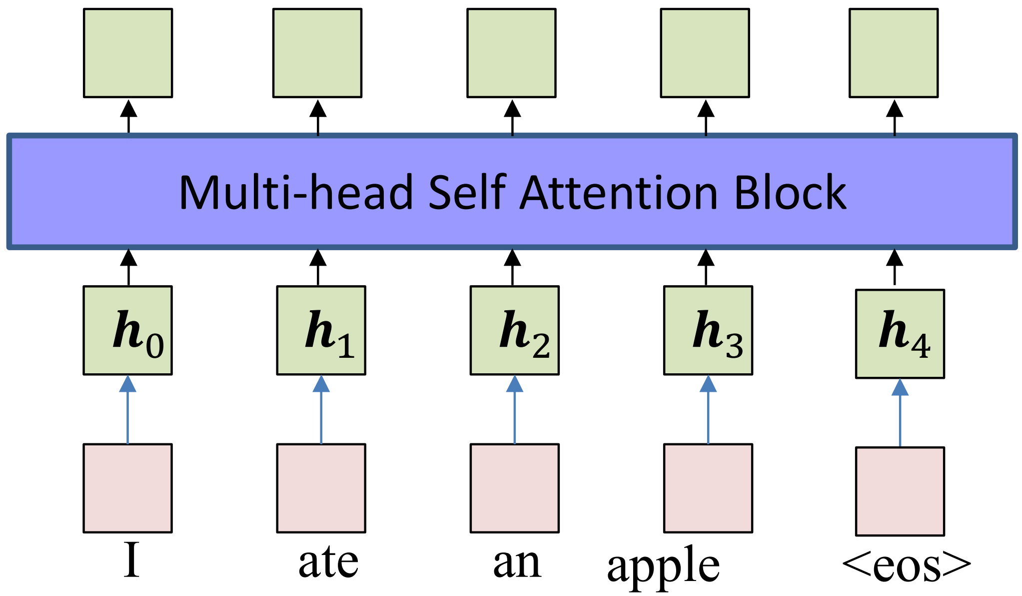
## MULTI-HEAD SELF ATTENTION BLOCK

$w_{ij}^a = attn(q_i^a, k_{0:N}^a)$

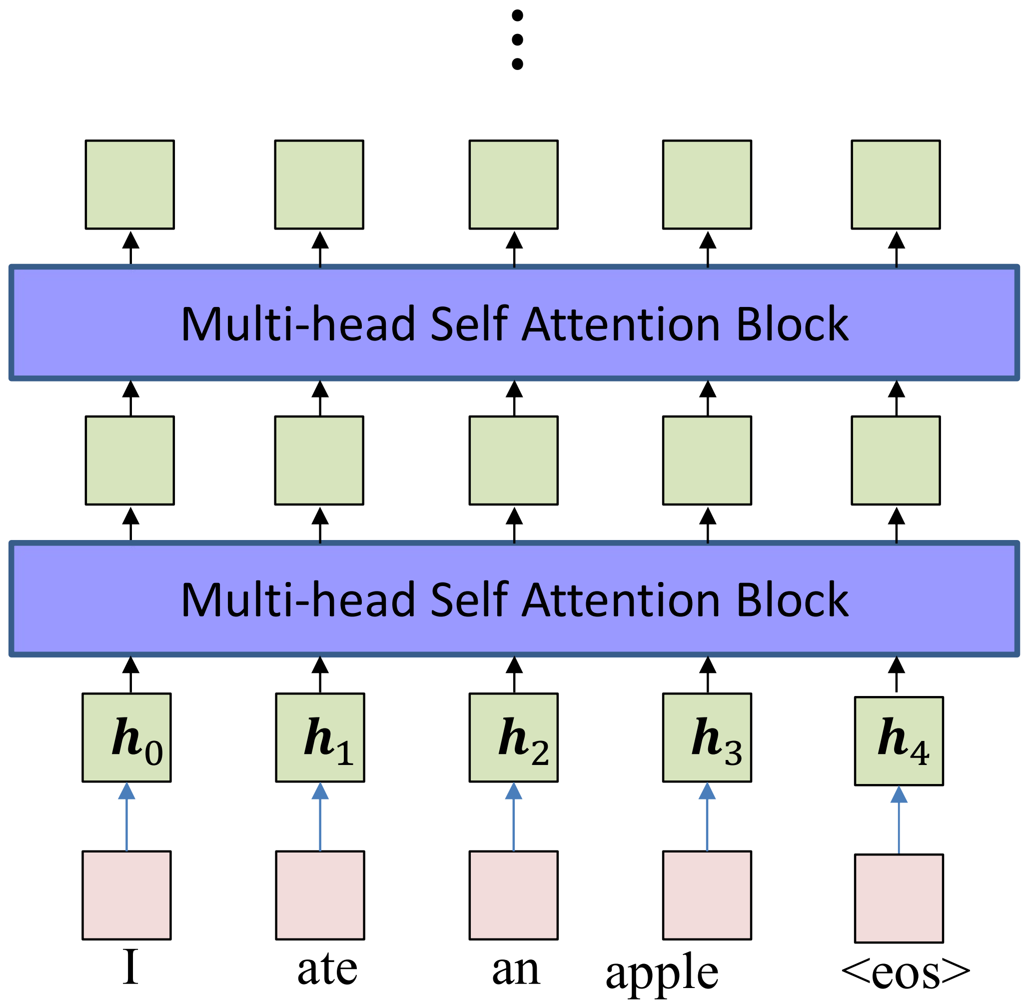
$$o_i^a = \sum_j w_{ij}^a v_j^a$$

$o_i = [o_i^1; o_i^2; o_i^3; \dots; o_i^H]$

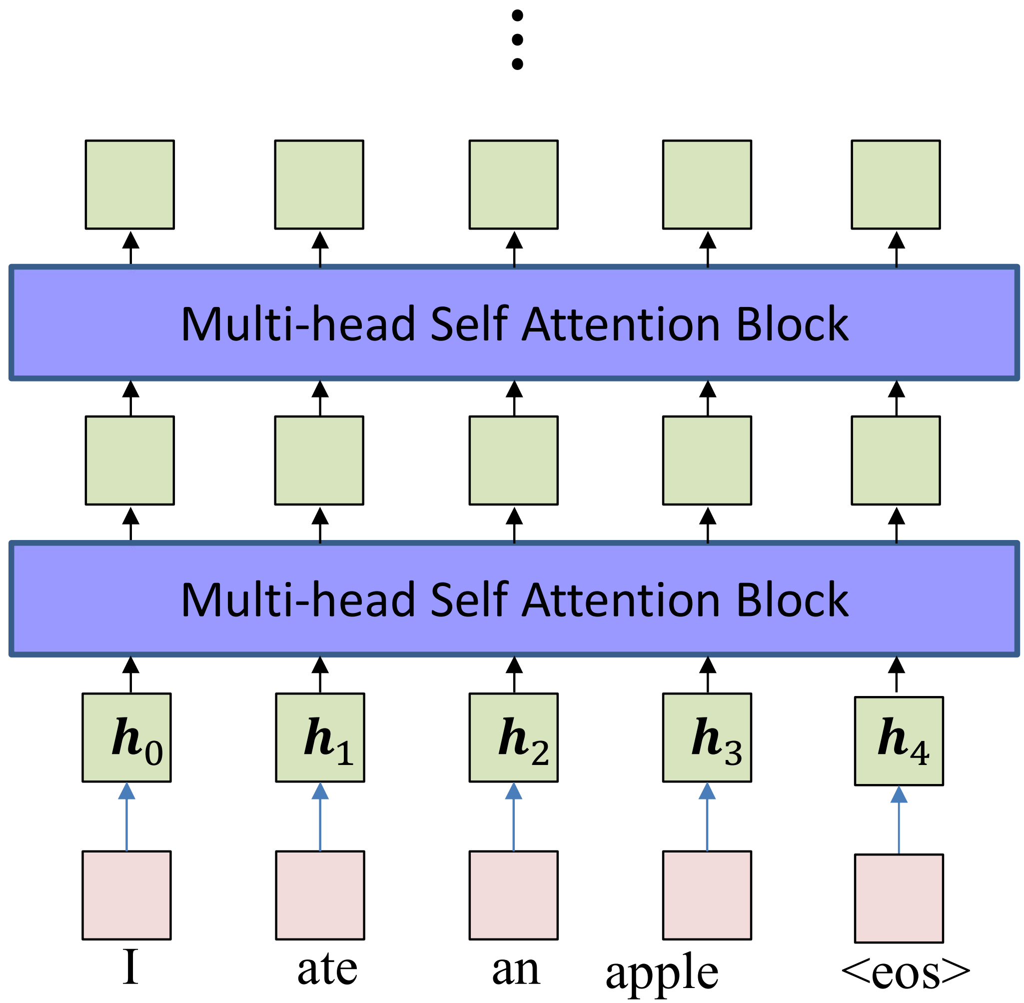
$y_i = MLP(o_i)$



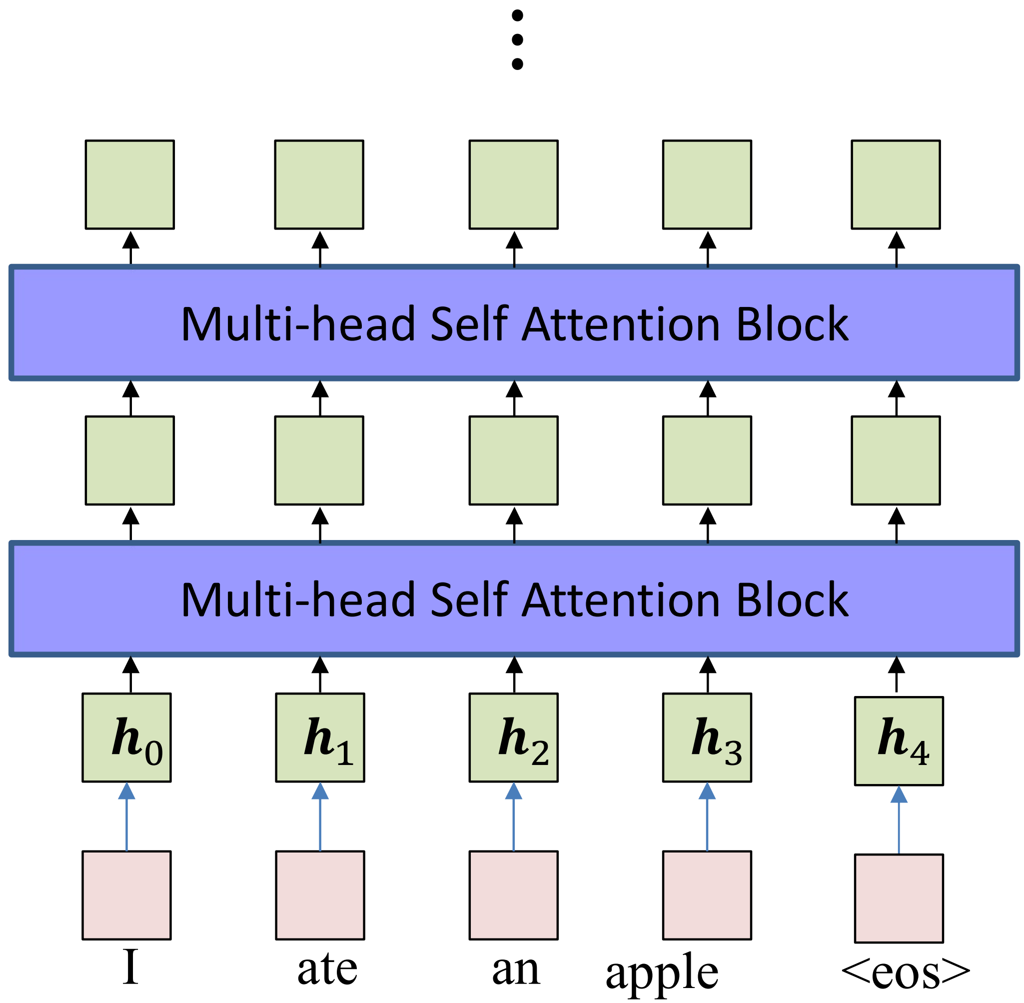

- The entire unit, including multi-head self-attention module followed by MLP is a ***multi-head self-attention block***



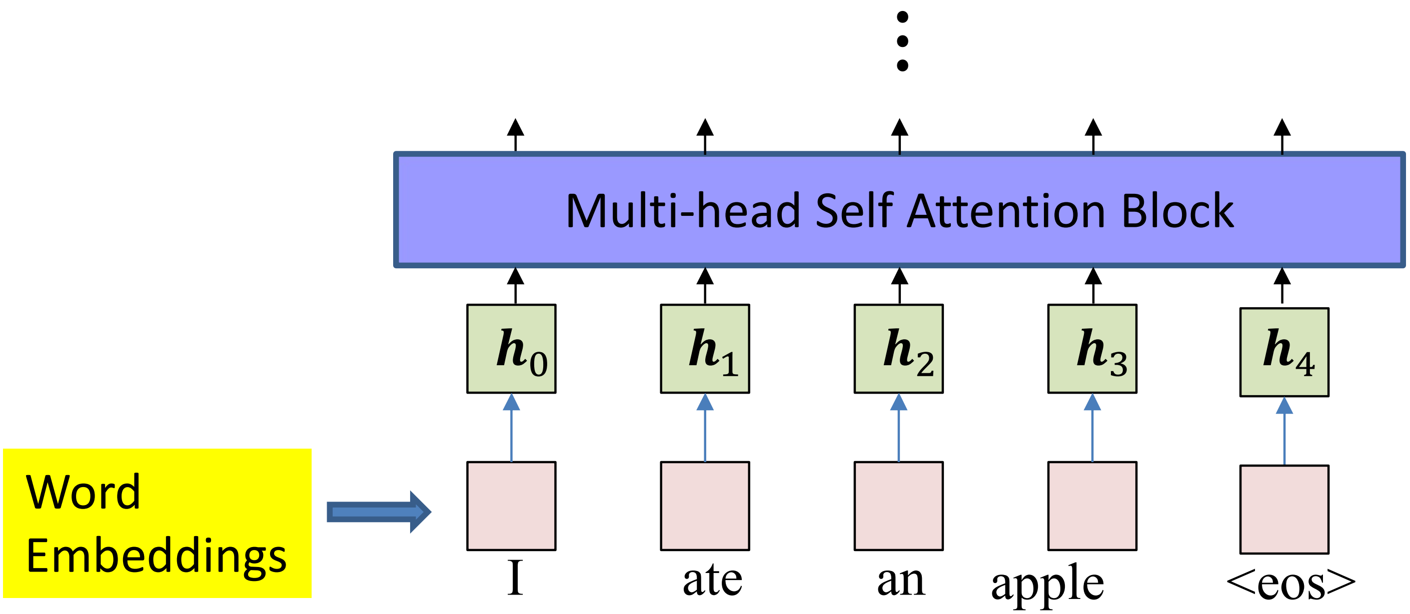

- The encoder can include many layers of such blocks
- No need for recurrence…



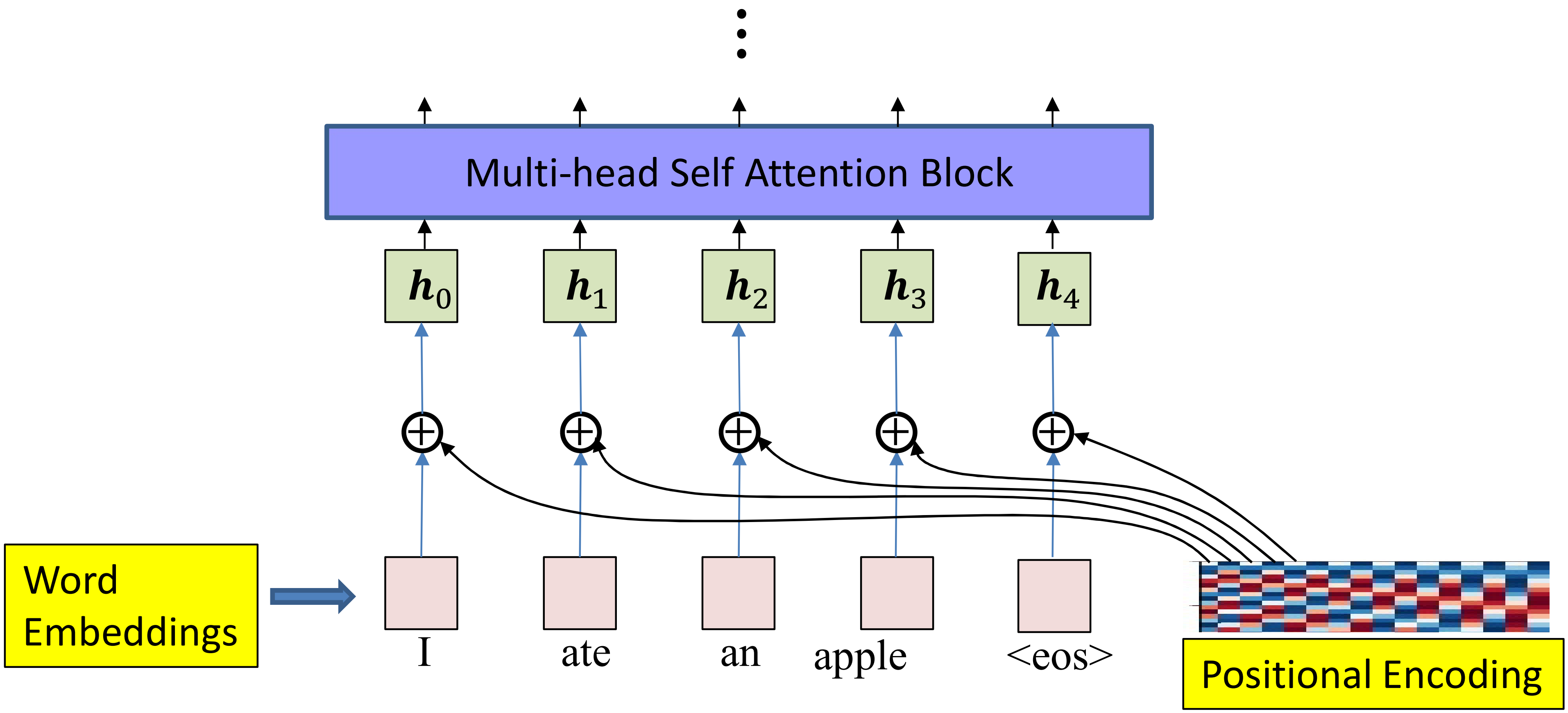

- Recap: The encoder in a sequence-to-sequence model can replace recurrence through a series of "multi-head self attention" blocks



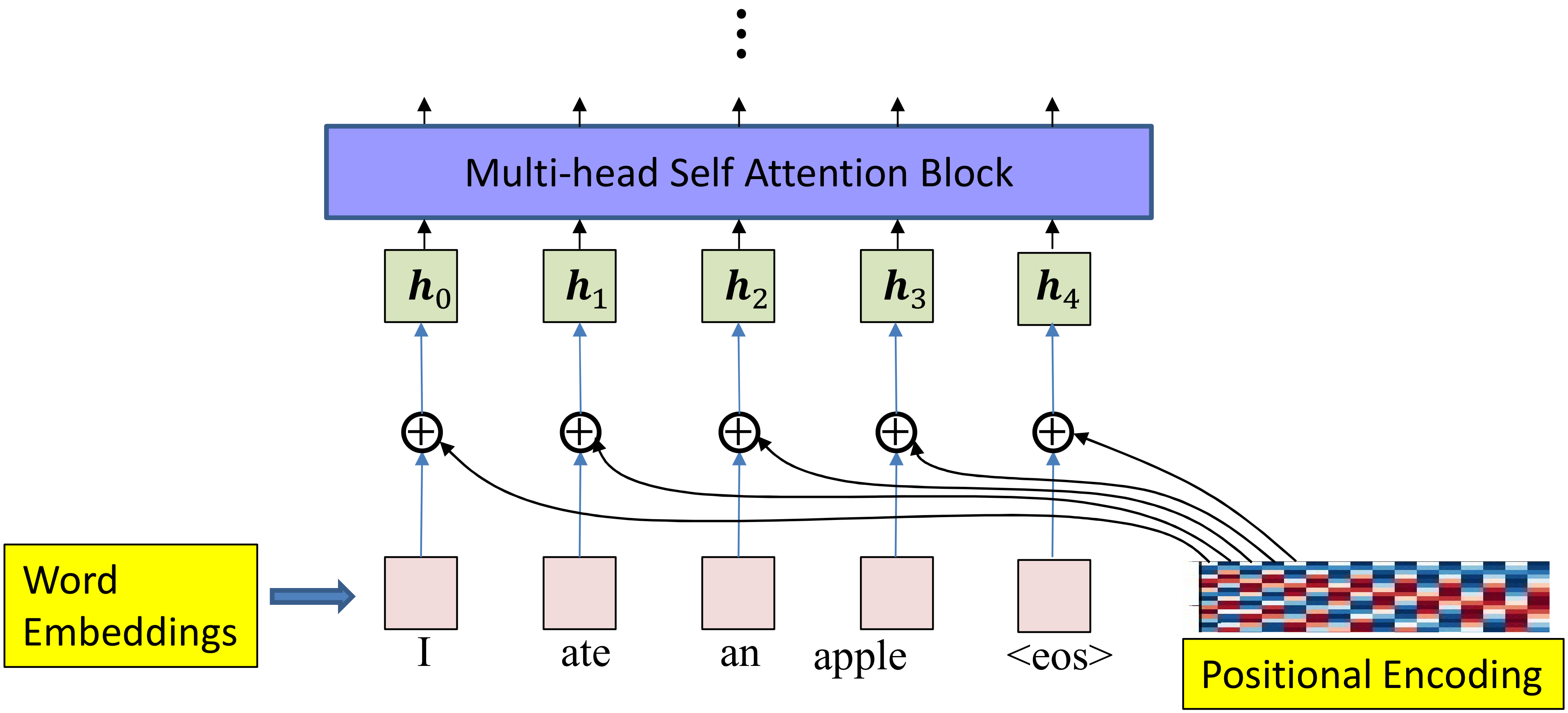

- Recap: The encoder in a sequence-to-sequence model can replace recurrence through a series of “multi-head self attention” blocks
- But this still ignores *relative position*
  - A context word one word away is different from one 10 words away
  - The attention framework does not take distance into consideration




- Note that the inputs are actually word *embeddings*




- Note that the inputs are actually word *embeddings*
- We add a “positional” encoding to them to capture the relative distance from one another



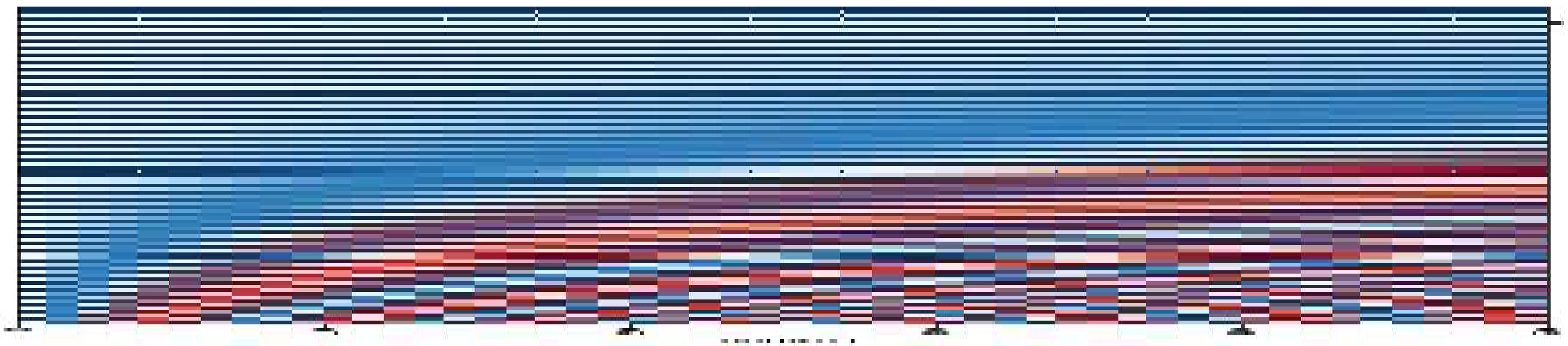

- **Positional Encoding:** A sequence of vectors $P_0, \dots, P_N$, to encode position
  - Every vector is unique (and uniquely represents time)
  - Relationship between $P_t$ and $P_{t+\tau}$ only depends on the distance between them

$$P_{t+\tau} = M_\tau P_t$$

- The linear relationship between $P_t$ and $P_{t+\tau}$ enables the net to learn shift-invariant "gap" dependent relationships

# Positional Encoding

regenerate



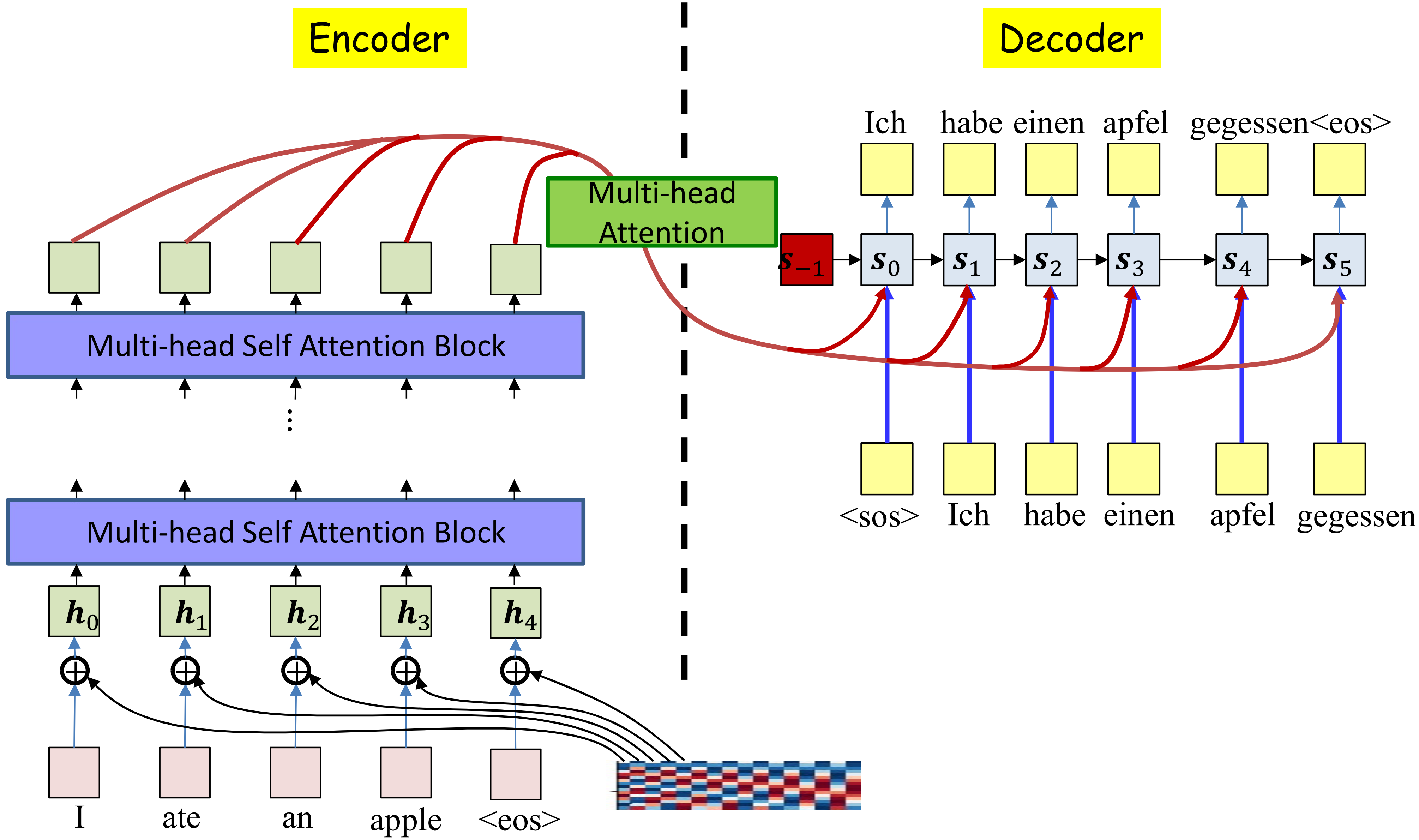
$$P_t = \begin{bmatrix} \sin \omega_1 t \\ \cos \omega_1 t \\ \sin \omega_2 t \\ \cos \omega_2 t \\ \vdots \\ \sin \omega_{d/2} t \\ \sin \omega_{d/2} t \end{bmatrix}$$

$$\omega_l = \frac{1}{10000^{2l/d}}$$

$$P_{t+\tau} = M_\tau P_t$$

$$M_\tau = diag\left(\begin{bmatrix} \cos \omega_l \tau & \sin \omega_l \tau \\ -\sin \omega_l \tau & \cos \omega_l \tau \end{bmatrix}, l = 1 \dots d/2\right)$$

- A vector of sines and cosines of a harmonic series of frequencies
  - Every $2l$-th component of $P_t$ is $\sin \omega_l t$
  - Every $2l + 1$-th component of $P_t$ is $\cos \omega_l t$
- Never repeats
- Has the linearity property required

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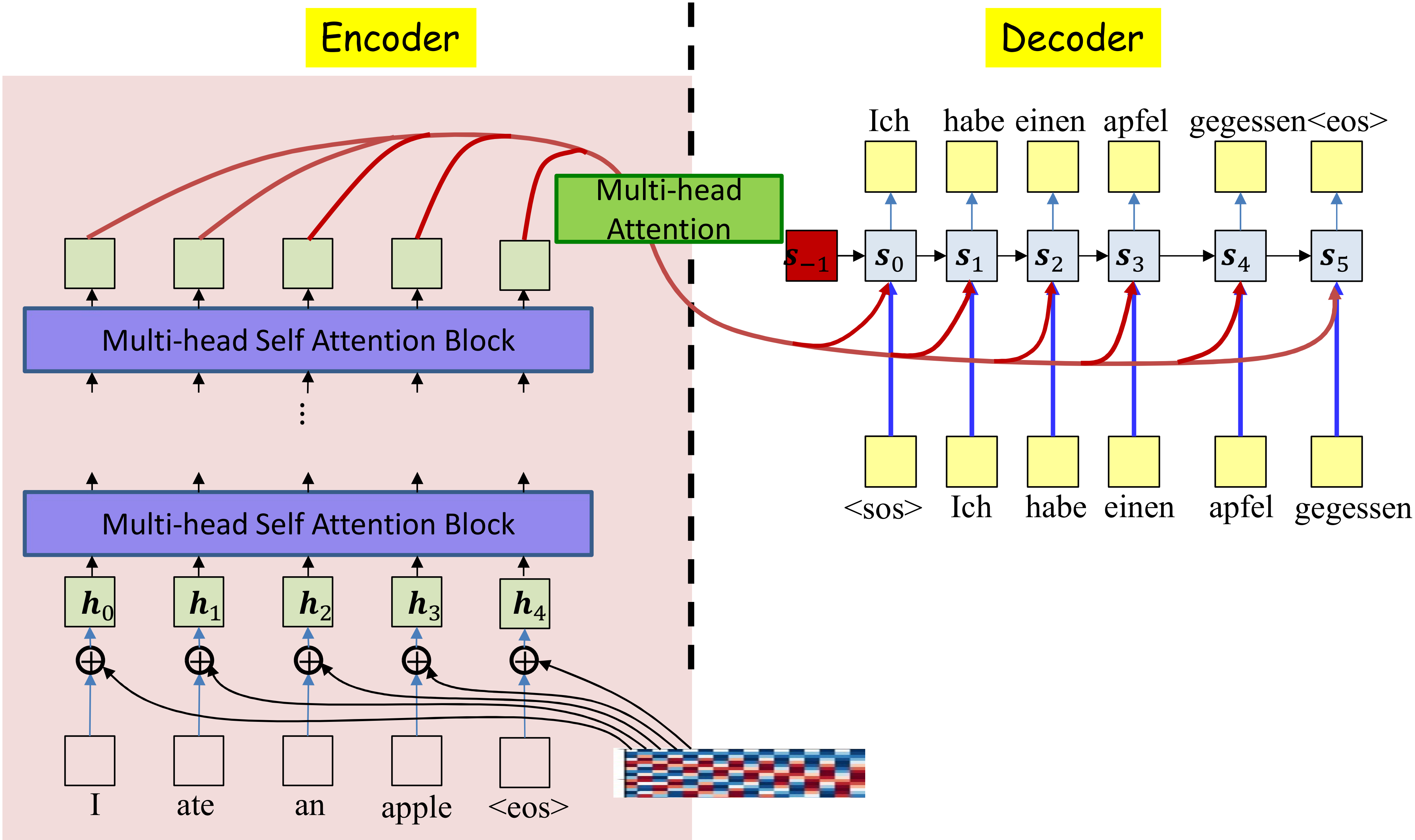
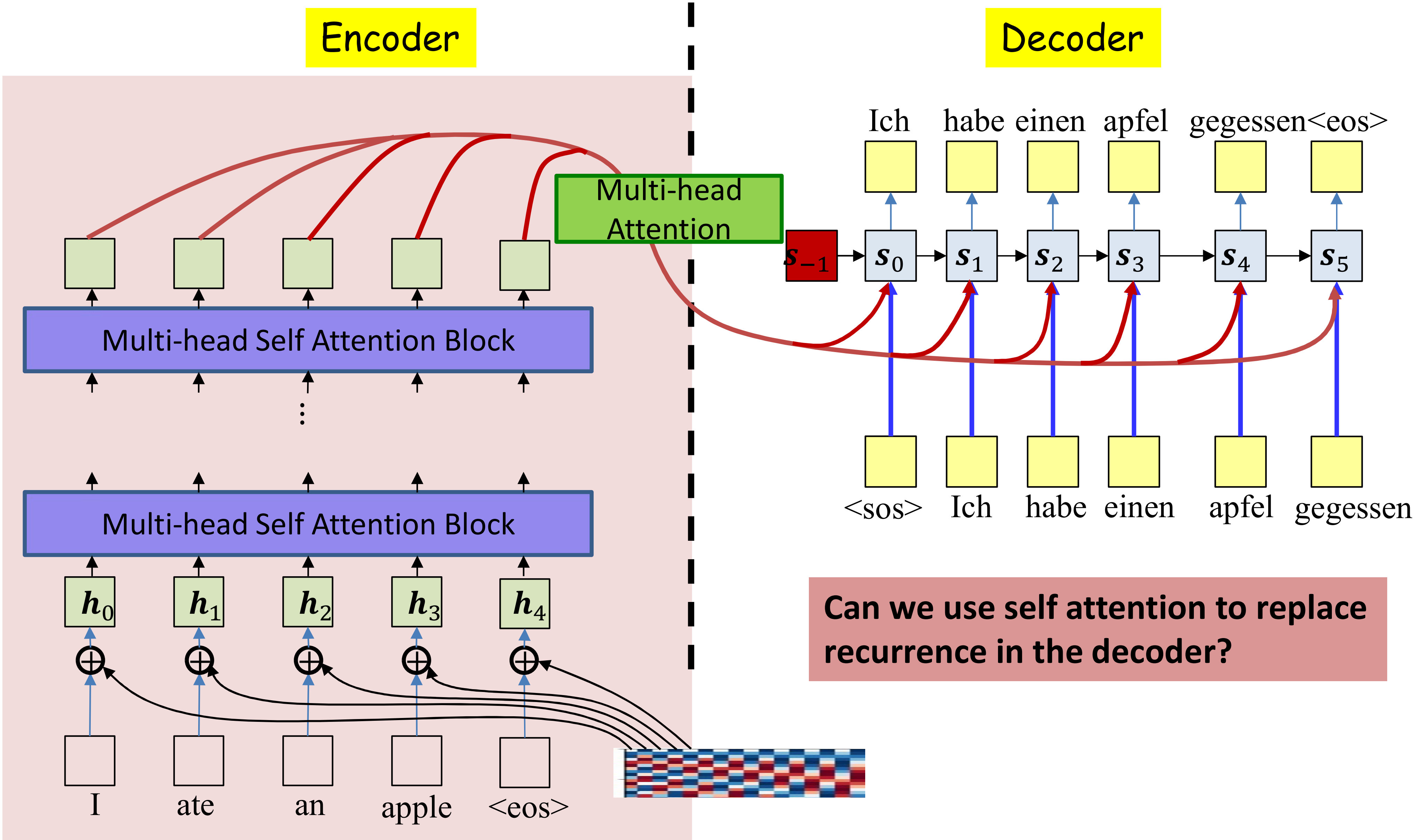
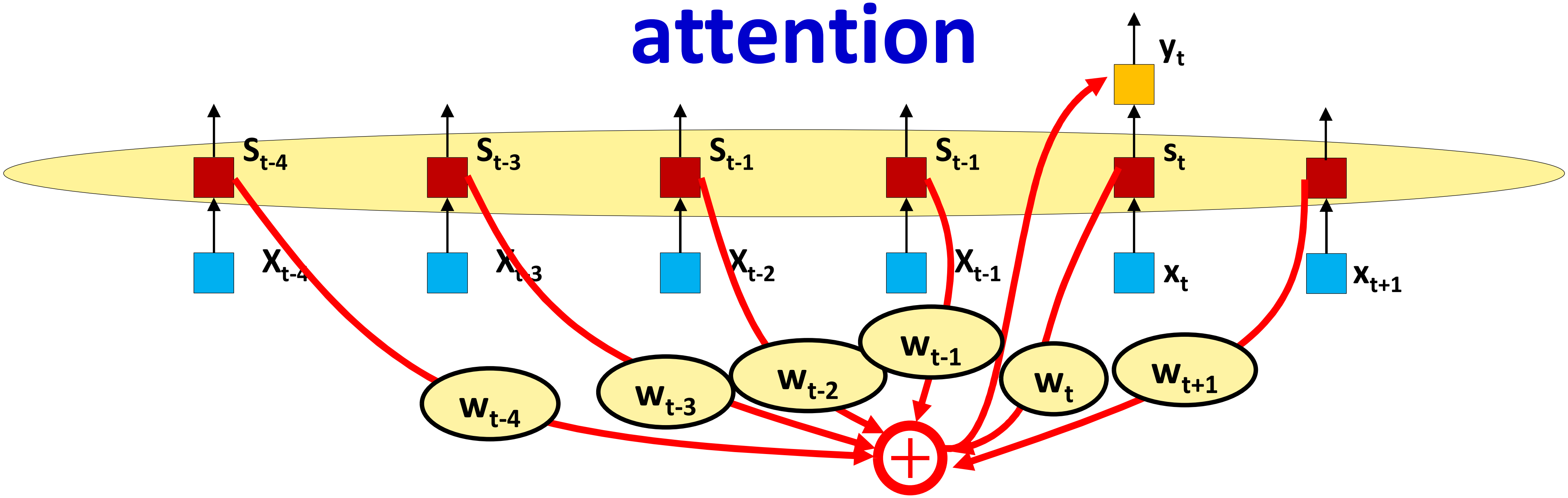



- The self-attending encoder!!




- The self-attending encoder!!

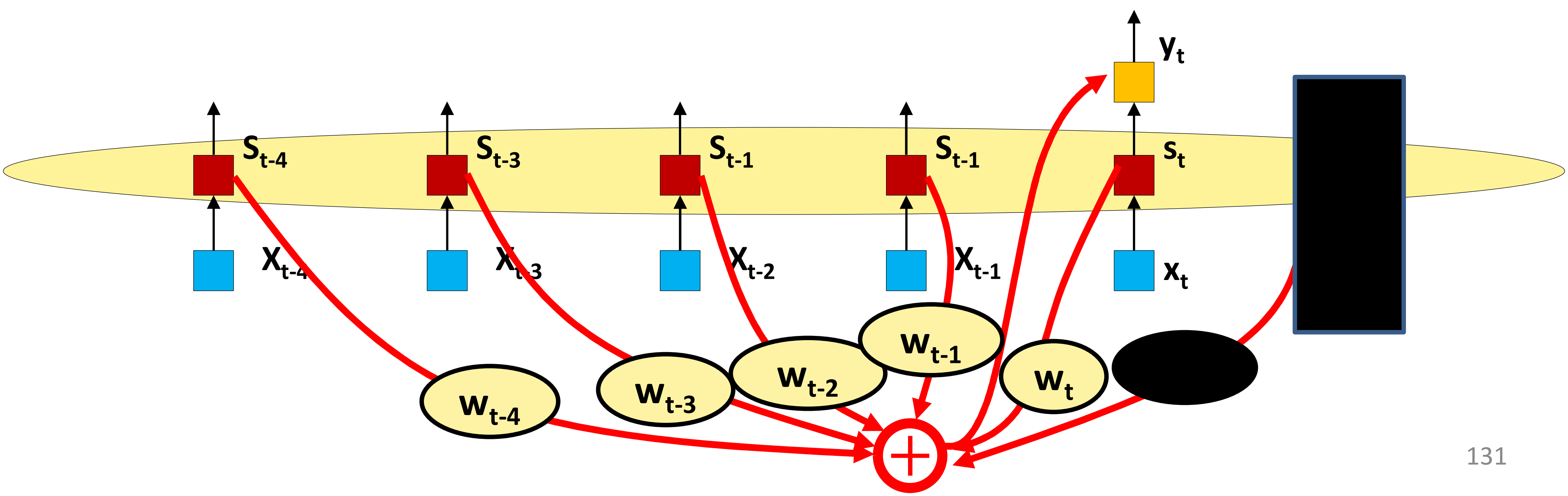
# Self attention and masked self attention




- **Self attention in encoder:** Can use input embedding at time t+1 and further to compute output at time t, because all inputs are available

# Self attention and masked self attention

- **Self attention in decoder:** Decoder is sequential
  - Each word is produced using the previous word as input
  - Only embeddings until time t are available to compute the output at time t
- The attention will have to be "masked", forcing attention weights for t+1 and later to 0



# Masked self-attention block

$q_i = \boldsymbol{W}_q h_i$

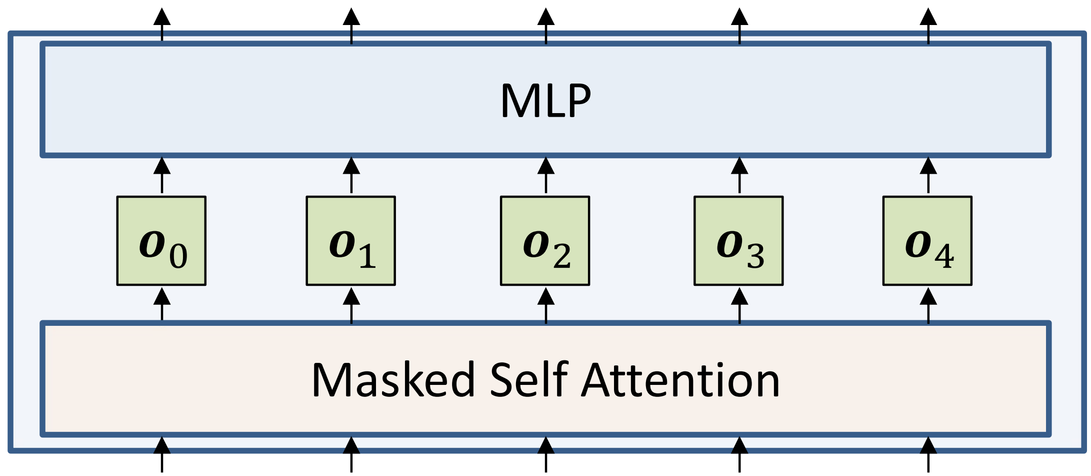
$k_i = \boldsymbol{W}_k h_i$

$v_i = \boldsymbol{W}_v h_i$

$e_{ij} = q_i^T k_j$

$w_{i0}, \dots, w_{ii} = softmax(e_{i0}, \dots, e_{ii})$

$$o_i = \sum_{j=0}^{i-1} w_{ij} v_j$$




- The "masked self attention ***block***" includes an MLP after the masked self attention
  - Like in the encoder

# Masked self-attention block

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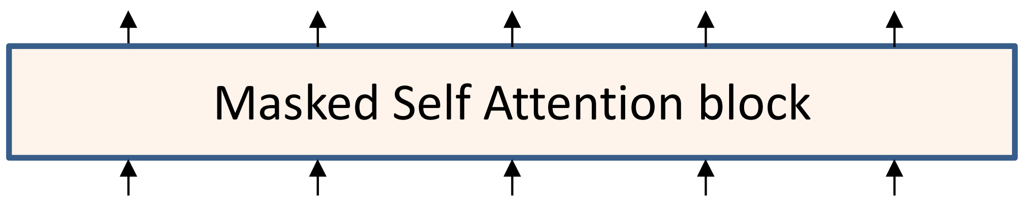


- The "masked self attention ***block***" sequentially computes outputs begin to end
  - Sequential nature of decoding prevents outputs from being computed in parallel
  - Unlike in an encoder

# Masked multi-head self-attention

$q_i^a = \boldsymbol{W}_q^a h_i$

$k_i^a = \boldsymbol{W}_k^a h_i$

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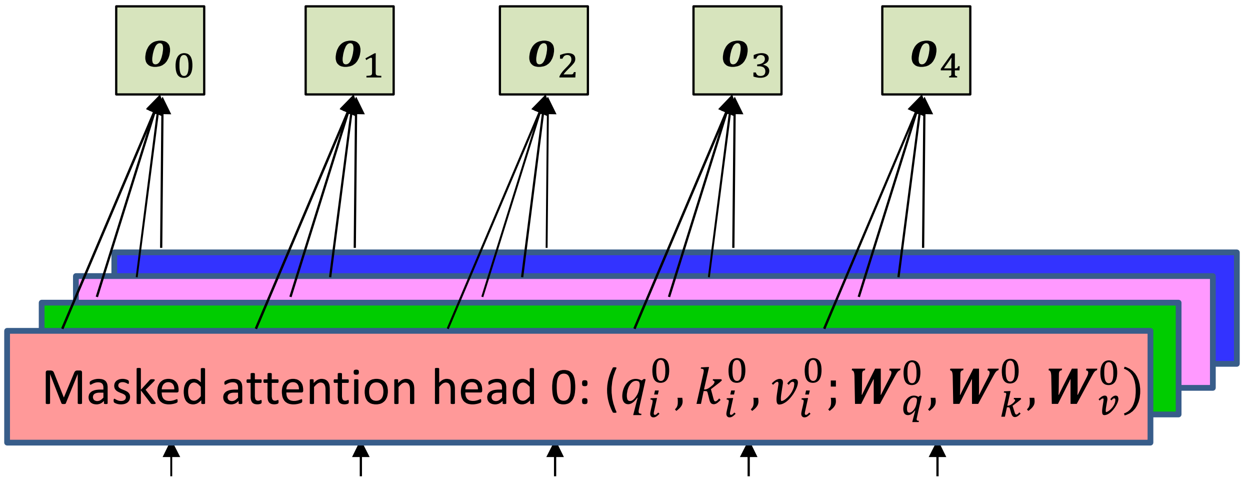
$w_{ij}^a = attn(q_i^a, k_{0:i-1}^a)$

$o_i^a = \sum_j w_{ij}^a v_j^a$




- The "masked ***multi-head*** self attention ***block***" includes multiple masked attention heads
  - Like in the encoder

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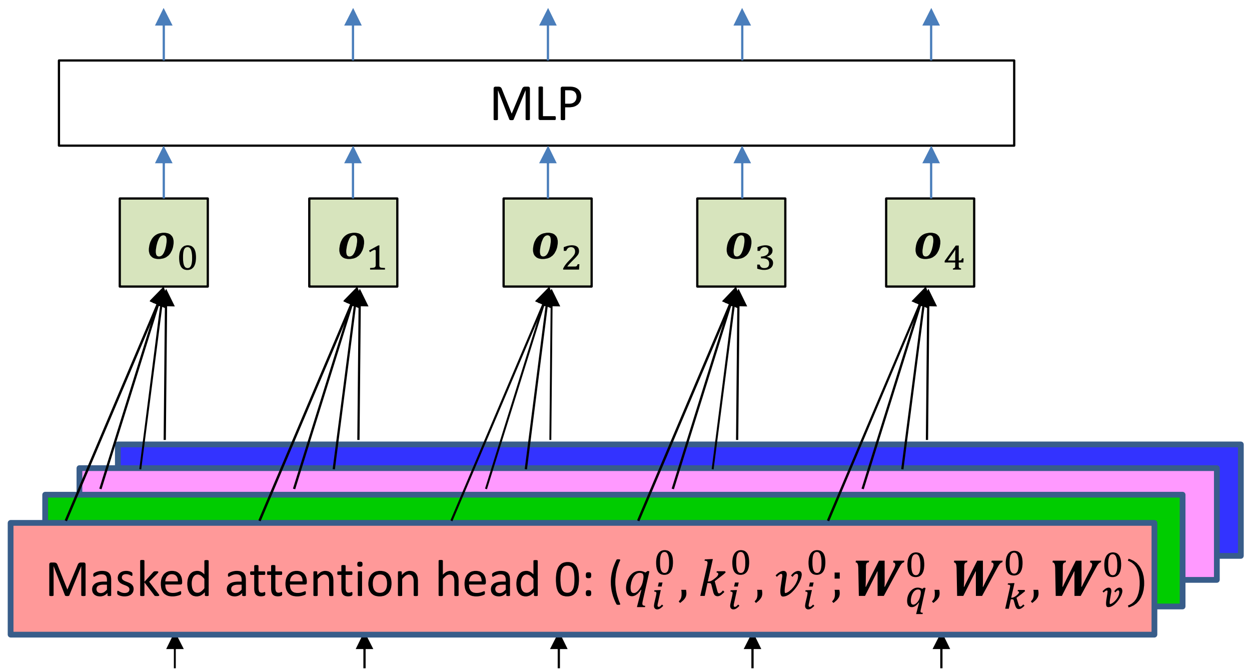


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# Masked multi-head self-attention block

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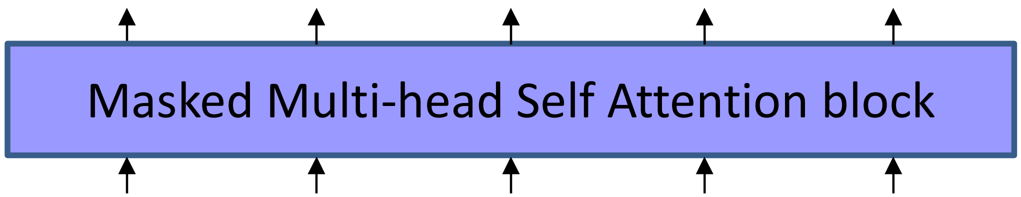
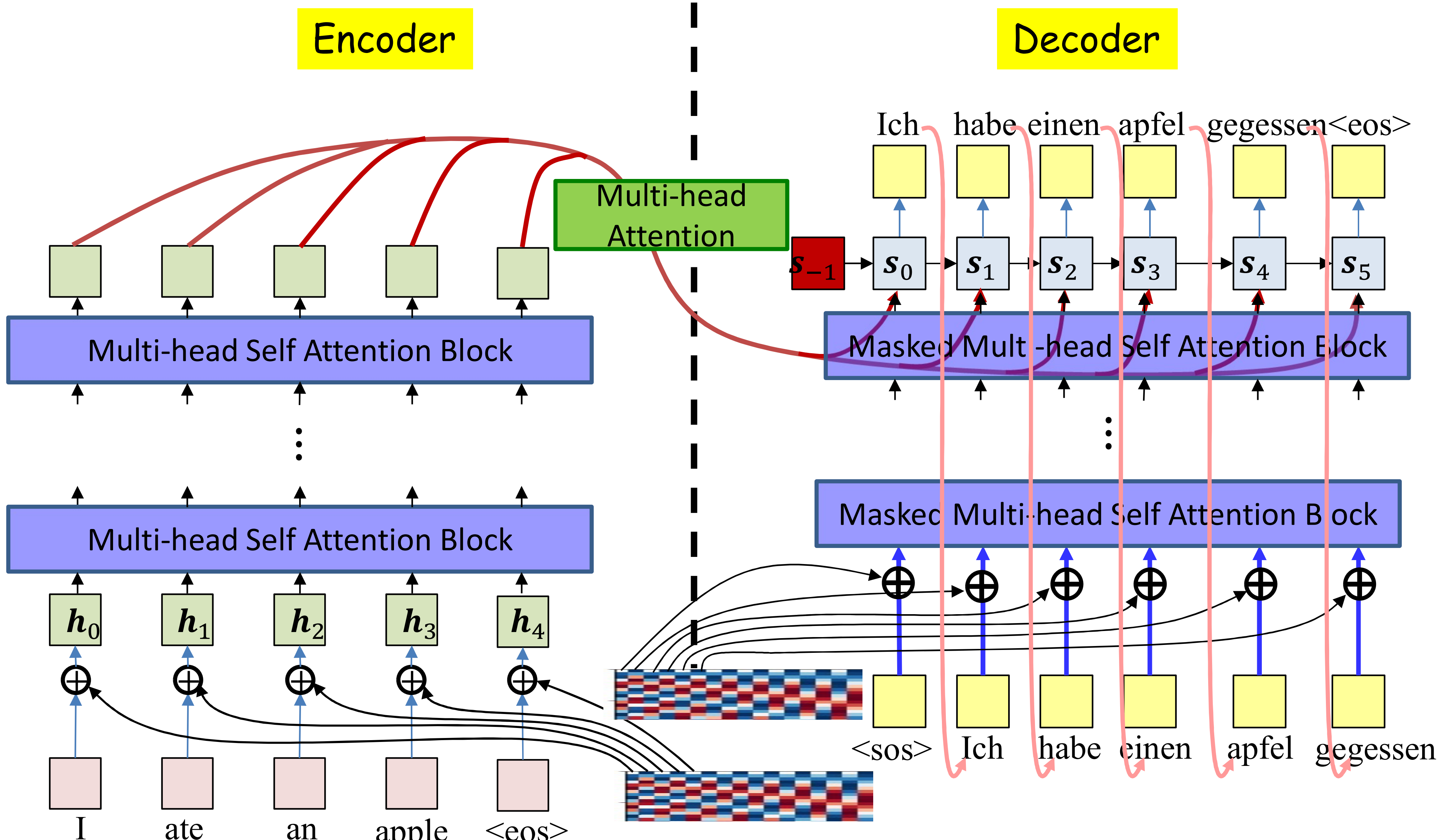



- The "masked ***multi-head*** self attention ***block***" includes multiple masked attention heads, followed by an MLP
  - Like in the encoder



Encoder
Decoder
Multi-head Attention
Multi-head Self Attention Block
Multi-head Self Attention Block
Masked Multi-head Self Attention Block
Masked Multi-head Self Attention Block
$h_0$ $h_1$ $h_2$ $h_3$ $h_4$
$s_{-1}$ $s_0$ $s_1$ $s_2$ $s_3$ $s_4$ $s_5$
I ate an apple <eos>
Ich habe einen apfel gegessen <eos>
<sos> Ich habe einen apfel gegessen

# Poll 3 (@1216)

**Mark all that are true**

- Self attention computed for an N-length input requires the computation of an N x N attention weight matrix for each head
- Masked self attention is only required in the first layer of the decoder. Subsequent layers see the entire output of the previous layers and can use full self attention
- We cannot combine recurrent layers with self attention layers
- Positional encodings are different in the encoder and decoder because the self attention in the decoder is masked.

# Poll 3

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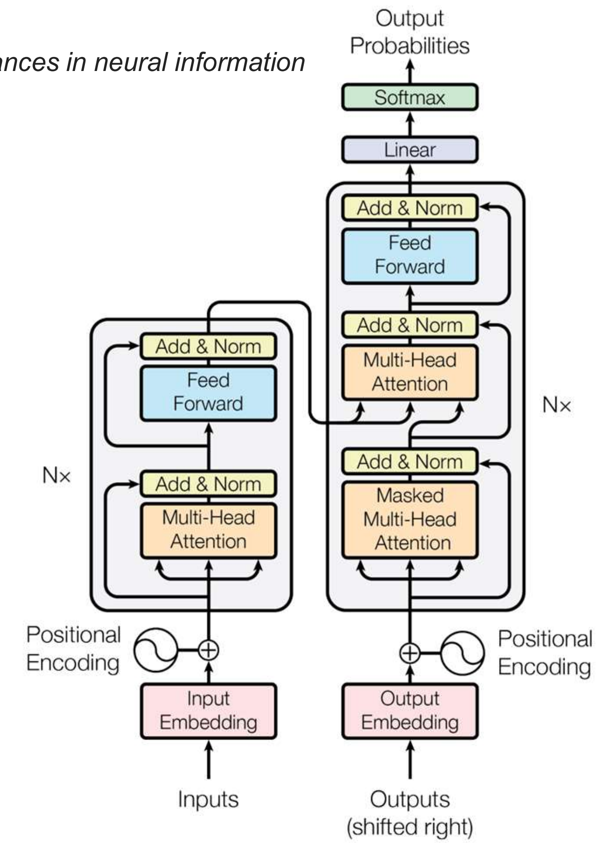
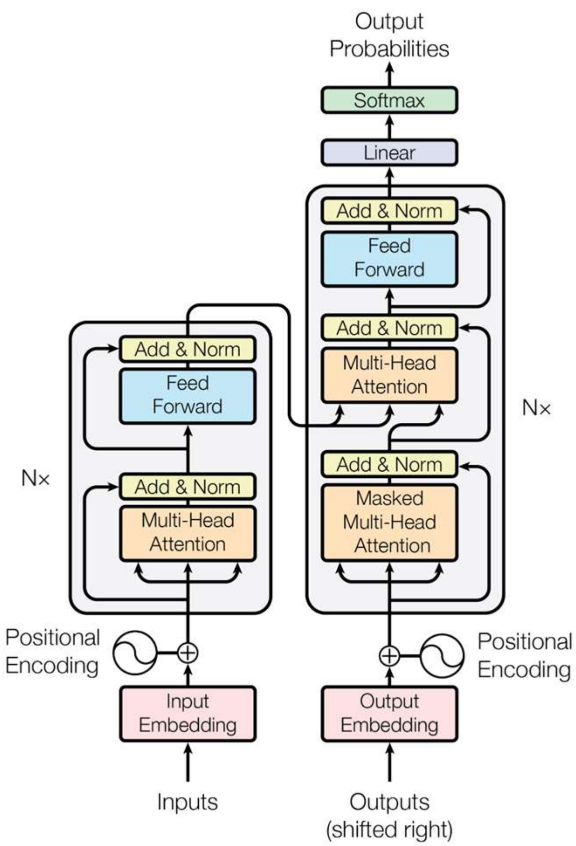
# Transformer: Attention is all you need

Vaswani, Ashish, et al. "Attention is all you need." *Advances in neural information processing systems*. 2017.




- Transformer: A sequence-to-sequence model that replaces recurrence with positional encoding and multi-head self attention
  - "Attention is all you need"

# Transformer




From "Attention is all you need"

Table 2: The Transformer achieves better BLEU scores than previous state-of-the-art models on the English-to-German and English-to-French newstest2014 tests at a fraction of the training cost.

| Model | BLEU | | Training Cost (FLOPs) | |
|---|---|---|---|---|
| | EN-DE | EN-FR | EN-DE | EN-FR |
| ByteNet [18] | 23.75 | | | |
| Deep-Att + PosUnk [39] | | 39.2 | | $1.0 \cdot 10^{20}$ |
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- Transformer: tremendous decrease in model computation for similar performance as state-of-art translation models
- The last row in the table shows transformer performance
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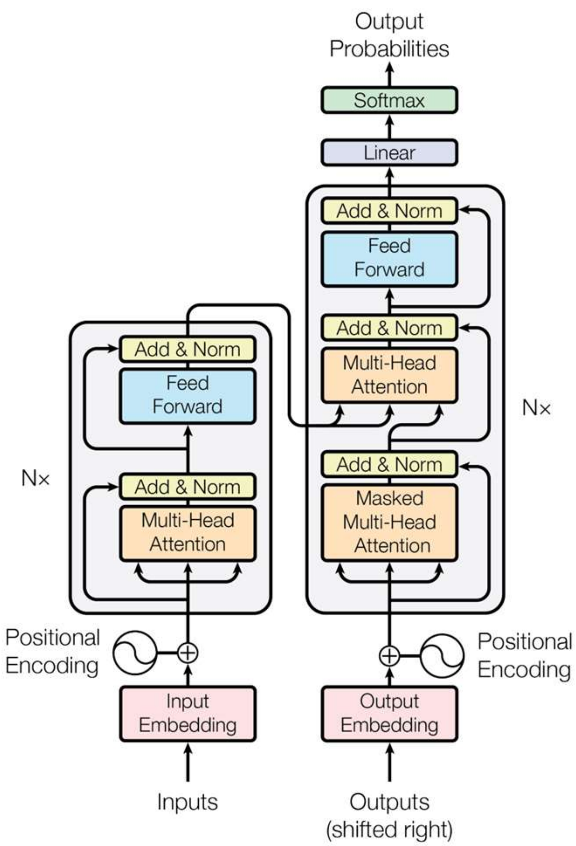
# Transformer

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Why so good?

Why so fast?

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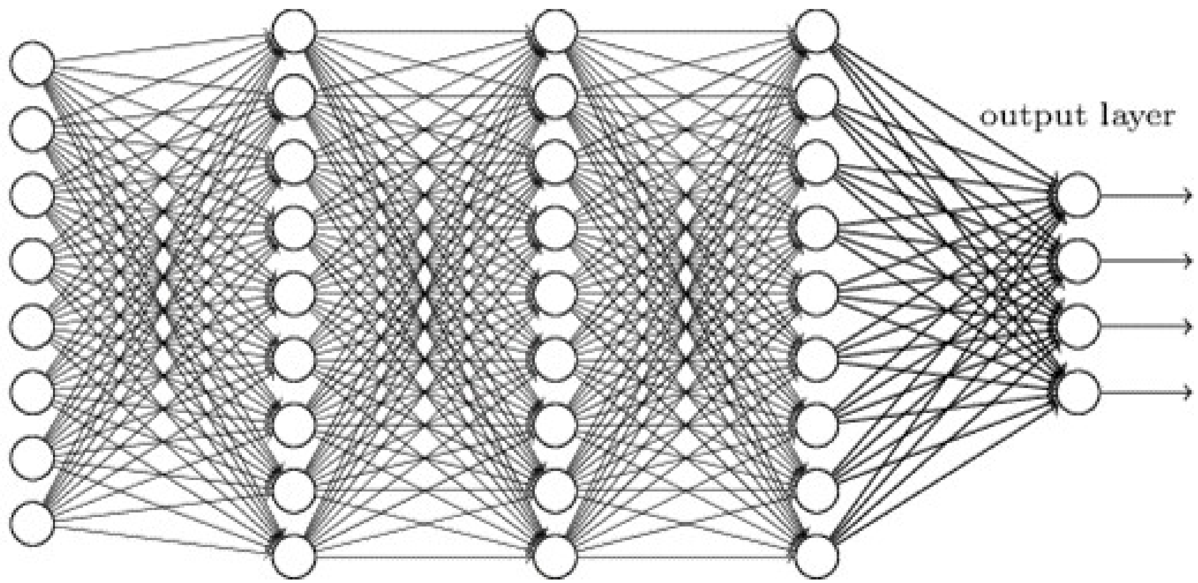
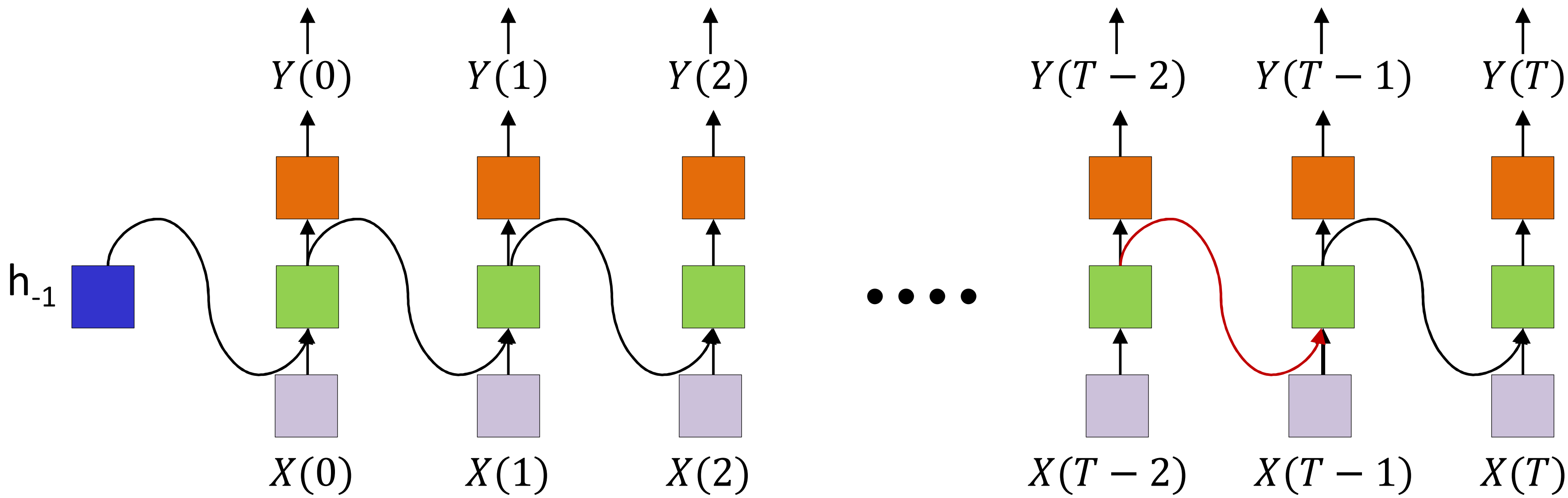
# Recap: Vanishing/exploding gradients




$$\nabla_{f_k} Div = \nabla D . \nabla f_N . W_N . \nabla f_{N-1} . W_{N-1} \dots \nabla f_{k+1} W_{k+1}$$

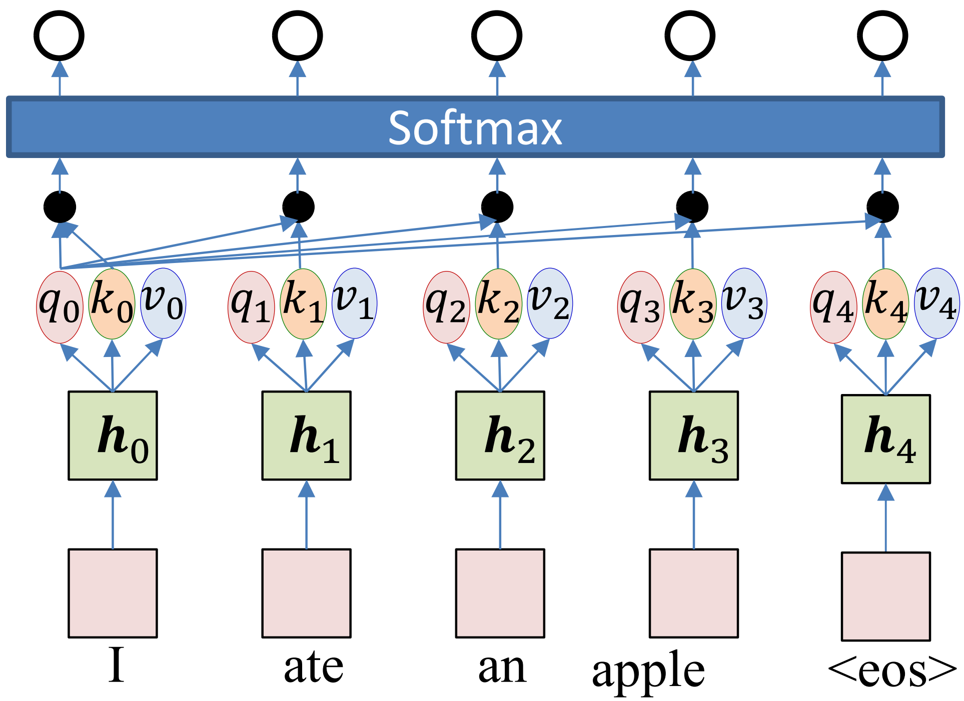
- RNNs are just very deep networks
- LSTMs mitigate the problem at the cost of 3x more matrix multiplications
- Transformers get rid of it! To encode a full sentence, they have way fewer layers than an unrolled RNN.
- The same goes with the vanishing memory issue to an extent.

# Processing order



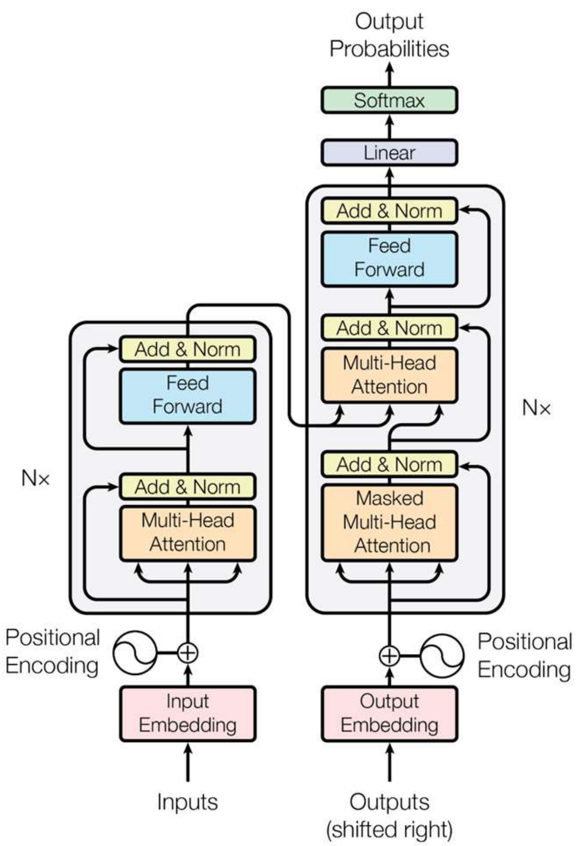

- Computing $Y(T)$ requires $Y(T-1)$...
- Which requires $Y(T-2)$, etc...
- RNN inputs must be processed *in order* → slow implementation

# Processing order




- $q_n, k_n, v_n$ can be computed separately.
- $n^2$ $< q_n, k_n >$ dot products to compute.
- Self attention is easy to compute in parallel → Faster implementations

# Transformer

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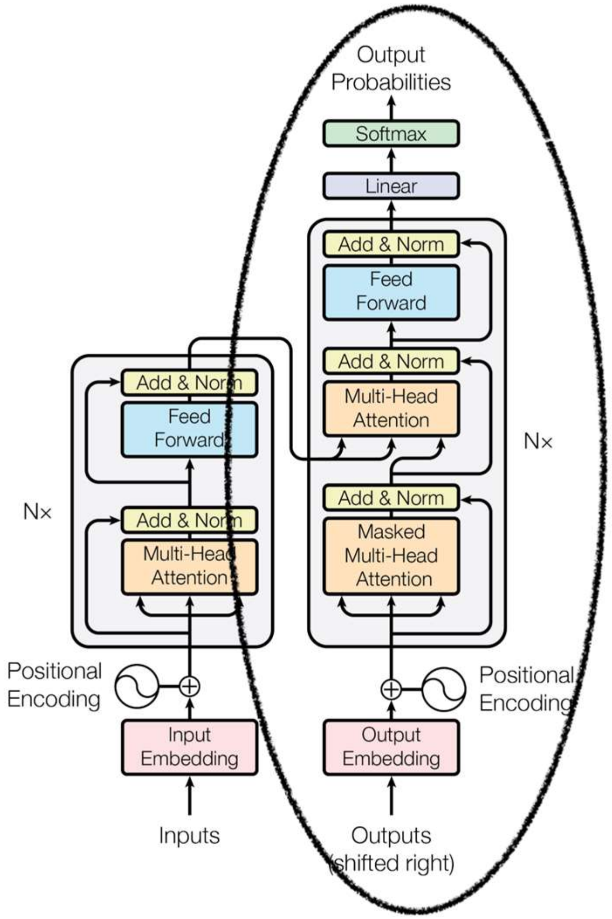
# GPT



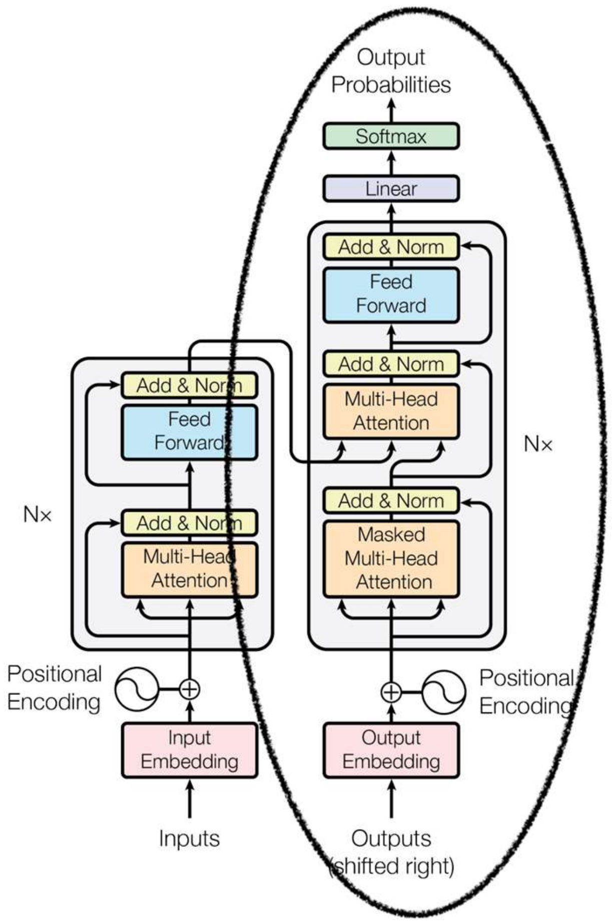

Alec Radford et. al., Improving Language Understanding by Generative Pre-Training

Table 5: Analysis of various model ablations on different tasks. Avg. score is a unweighted average of all the results. (*mc*= Mathews correlation, *acc*=Accuracy, *pc*=Pearson correlation)

| Method | Avg. Score | CoLA (mc) | SST2 (acc) | MRPC (F1) | STSB (pc) | QQP (F1) | MNLI (acc) | QNLI (acc) | RTE (acc) |
|---|---|---|---|---|---|---|---|---|---|
| Transformer w/ aux LM (full) | 74.7 | 45.4 | 91.3 | 82.3 | 82.0 | **70.3** | **81.8** | **88.1** | **56.0** |
| Transformer w/o pre-training | 59.9 | 18.9 | 84.0 | 79.4 | 30.9 | 65.5 | 75.7 | 71.2 | 53.8 |
| Transformer w/o aux LM | **75.0** | **47.9** | **92.0** | **84.9** | **83.2** | 69.8 | 81.1 | 86.9 | 54.4 |
| LSTM w/ aux LM | 69.1 | 30.3 | 90.5 | 83.2 | 71.8 | 68.1 | 73.7 | 81.1 | 54.6 |

- GPT uses only the decoder of the transformer as an LM
  - “Transformer w/o aux LM”
- Large performance improvement in many tasks

# GPT

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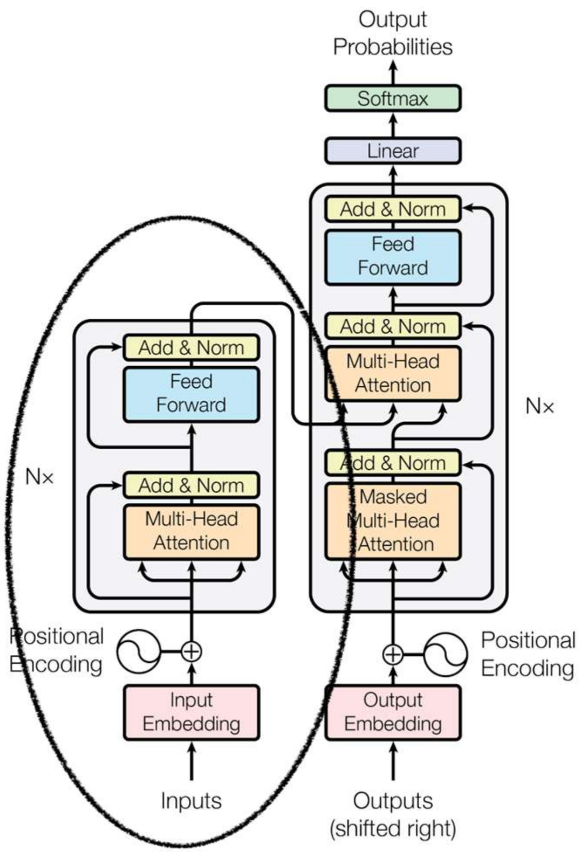
- Add *Task conditioning*: put the nature of your task in the input (not just LM)
- Parameters x1000

→ **GPT-3** : Generalizes to **more tasks**, not just more inputs!

# BERT




| System | MNLI-(m/mm) 392k | QQP 363k | QNLI 108k | SST-2 67k | CoLA 8.5k | STS-B 5.7k | MRPC 3.5k | RTE 2.5k | **Average** - |
|---|---|---|---|---|---|---|---|---|---|
| Pre-OpenAI SOTA | 80.6/80.1 | 66.1 | 82.3 | 93.2 | 35.0 | 81.0 | 86.0 | 61.7 | 74.0 |
| BiLSTM+ELMo+Attn | 76.4/76.1 | 64.8 | 79.8 | 90.4 | 36.0 | 73.3 | 84.9 | 56.8 | 71.0 |
| OpenAI GPT | 82.1/81.4 | 70.3 | 87.4 | 91.3 | 45.4 | 80.0 | 82.3 | 56.0 | 75.1 |
| $\text{BERT}_{\text{BASE}}$ | 84.6/83.4 | 71.2 | 90.5 | 93.5 | 52.1 | 85.8 | 88.9 | 66.4 | 79.6 |
| $\text{BERT}_{\text{LARGE}}$ | **86.7/85.9** | **72.1** | **92.7** | **94.9** | **60.5** | **86.5** | **89.3** | **70.1** | **82.1** |

Table 1: GLUE Test results, scored by the evaluation server (`https://gluebenchmark.com/leaderboard`). The number below each task denotes the number of training examples. The "Average" column is slightly different than the official GLUE score, since we exclude the problematic WNLI set.[8] BERT and OpenAI GPT are single-model, single task. F1 scores are reported for QQP and MRPC, Spearman correlations are reported for STS-B, and accuracy scores are reported for the other tasks. We exclude entries that use BERT as one of their components.

- Bert: Only uses encoder of transformer to derive word and sentence embeddings
- Trained to "fill in the blanks"
- This is *representation learning* (more next lecture)

# Attention is all you need

- Self-attention can effectively replace recurrence in sequence-to-sequence models
  - "Transformers"
  - Requires "positional encoding" to capture positional information

- Can also be used in regular sequence analysis settings as a substitute for recurrence

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# Poll 4 (@1217)

**Mark all that are true**

- BERT is essentially the encoder of a transformer model
- GPT is essentially the encoder of a transformer model
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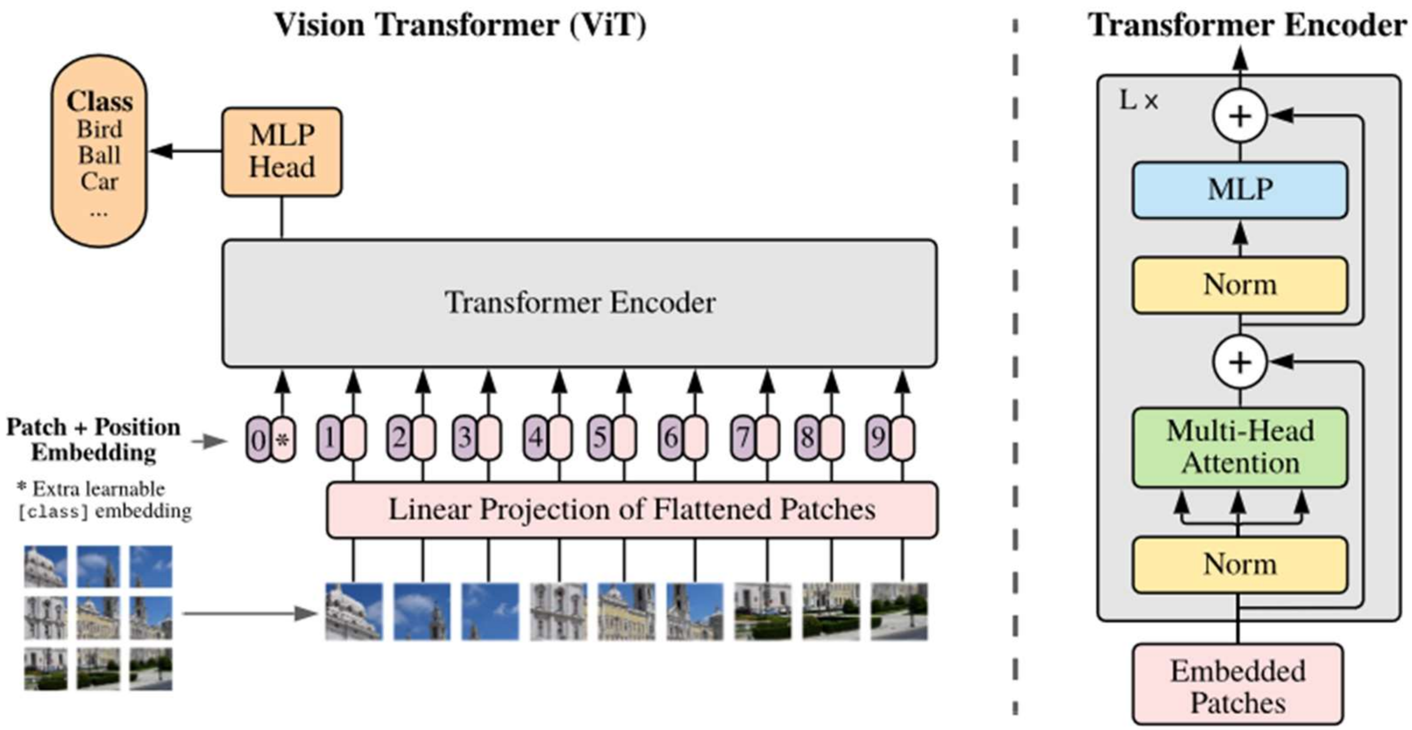
# Vision Transformers



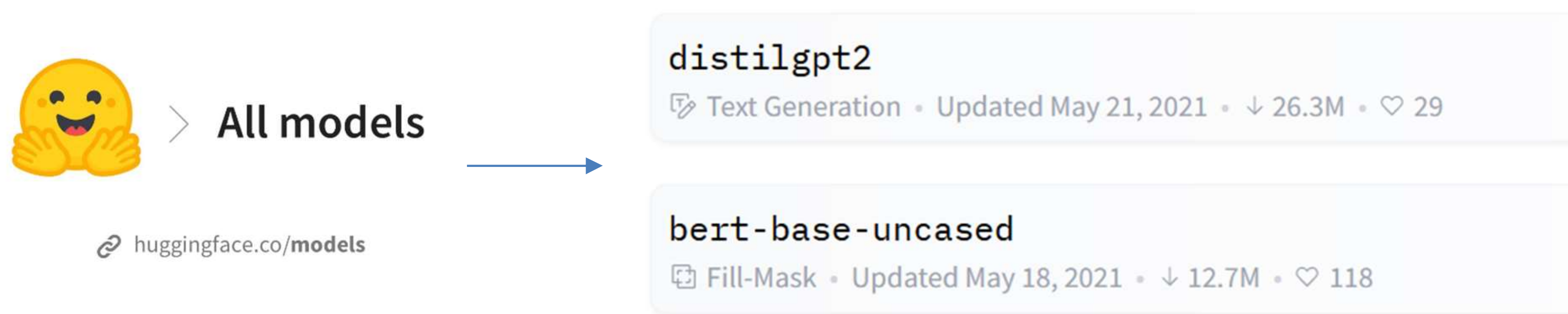

Dosovitskiy et al, *An Image is Worth 16x16 Words: Transformers for Image Recognition at Scale*, 2020

- Divide your image in patches with pos. encodings
- Apply Self-Attention!

→ Sequential and image problems are similar when using transformers

# Impact of Transformers

- Transformers have played a major role in the "uniformization" of DL-based tasks:
  - Find a pretrained "BERT-like" transformer (Text, Image, Speech)
  - Fine-tune on your task – or not! (Prompting…)
- This has helped democratize Deep Learning considerably




- **But…**

# Caveat 1

- Not all transformers are the same: Big/small, fast/slow, mono-/multilingual, contrastive/generative, regressive/autoencoding…
- Pick the right one!

# Caveat 2

- Transformers are not always the right choice.
- They often require more parameters than LSTMs at equal performance

→ Tricky on small hardware (phones, IoT, etc)