Generative Adversarial Networks

Lecture 2

Introduction To Deep Learning (11-785 / 685 / 485) Spring 2022

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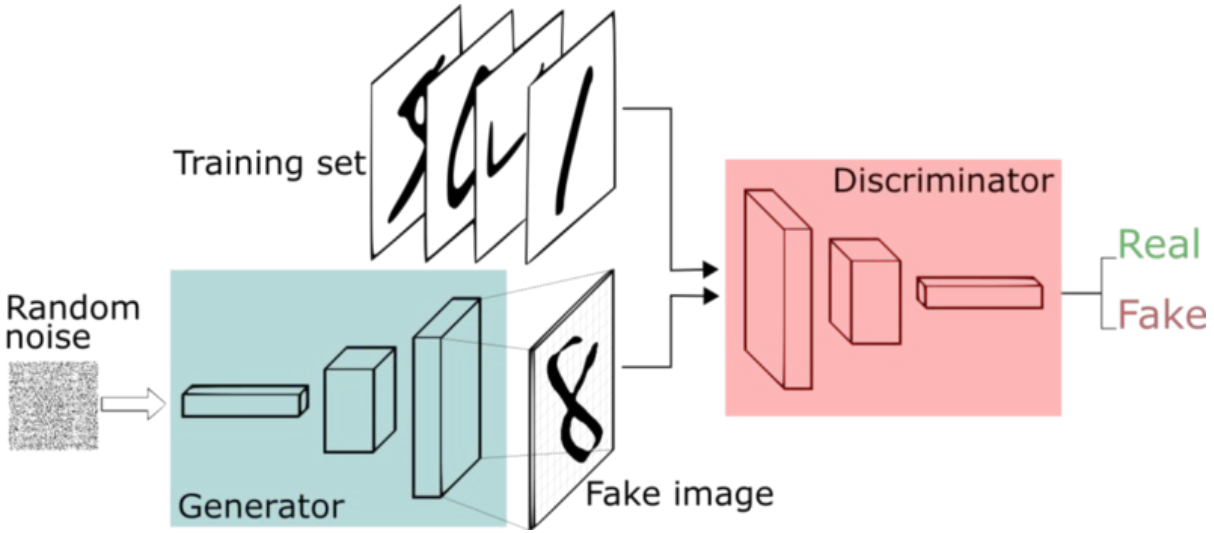
Overview

- GANs Quick Recap
- GAN Training
- Issues with GAN Training
- Remedies for GAN Training Issues
- GAN Architectures & Recent Progress

- Discriminative models learn the conditional probability $P(Y \mid X)$
- Generative models understand the joint probability $P(X, Y) = P(X \mid Y) \cdot P(Y)$
  - GANs model $P(X)$, the distribution of training data
  - Once sufficiently trained, GANs can generate new data from $P(X)$
- Generative networks are used to generate samples from an unlabeled distribution $P(X)$ given samples $X_1, \ldots, X_n$. For example:
  - Generate images given sample images
  - Generate music given sample music
  - Generate realistic text given sample corpus

GAN Framework



https://www.freecodecamp.org/news/an-intuitive-introduction-to-generative-adversarial-networks-gans-7a2264a81394

GAN Training

Description	Notation
Training samples	x
Latent noise vector	z
Discriminator	$D(x;\theta_d)$
Generator	$G(z;\theta_g)$
Probability distribution of real data	P_{data}
Probability distribution of fake data	P_g
Probability distribution of latent noise vector	P_z
Generator output - generated *fake* data	$G(z)$
Discriminator output for *fake* data	$D(G(z))$
Discriminator output for *real* data	$D(x)$

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Discriminator output for *real* data	$D(x)$

Discriminator objective:

- maximize output for *real* data $D(x)$
- minimize output for *fake* data $D(G(x))$

OR

- maximize output for *real* data $D(x)$
- maximize output for $1 - D(G(z))$

Generator objective:

- minimize output for *fake* data $1 - D(G(z))$

Since log is a monotonically increasing function:

Discriminator objective:

- maximize output for *real* data $\log(D(x))$
- maximize output for $\log(1 - D(G(z)))$

Generator objective:

- minimize output for *fake* data $\log(1 - D(G(z)))$

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Discriminator output for *real* data	$D(x)$

For one data point:

Discriminator objective:

- $\max_D(\log(D(x)) + \log(1 - D(G(z))))$

Generator objective:

- $\min_G(\log(1 - D(G(z))))$

For the entire distribution:

Discriminator objective:

- $\max_D E_{x\sim P_{data}(x)}[\log D(x)] + E_{z\sim P_z(z)}[1 - \log D(G(z))]$

Generator objective:

- $\min_G E_{z\sim P_z(z)}[1 - \log D(G(z))]$

Which stays same after adding a constant,

- $\min_G E_{x\sim P_{data}(x)}[\log D(x)] + E_{z\sim P_z(z)}[1 - \log D(G(z))]$

GAN Training

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Discriminator output for *real* data	$D(x)$

GAN objective:

- $\min_G \max_D E_{x\sim P_{data}(x)}[\log D(x)] + E_{z\sim P_z(z)}[1 - \log D(G(z))]$
- $\min_G \max_D P_{data}(x)(\log D(x)) + P_g(x)(1 - \log D(x))$

Jointly optimizing min-max is complicated, so we first find the current best D by taking the derivative of GAN objective:

- $-\frac{P_{data}(x)}{D(x)} + \frac{P_g(x)}{1-D(x)} = 0$
- $D(x) = \frac{P_{data}(x)}{P_{data}(x)+P_g(x)}$

Substituting this value in the GAN objective for Generator loss:

- $E_{x\sim P_{data}(x)}[\log \frac{P_{data}(x)}{\frac{1}{2}(P_{data}(x)+P_g(x))}] + E_{z\sim P_z(z)}[\log \frac{P_{data}(x)}{\frac{1}{2}(P_{data}(x)+P_g(x))}] - 2.\log 2$

Kullback-Leibler(KL) and Jensen-Shannon(JS) divergences are given by:

- $KL(P_1||P_2) = E_{x\sim P_1(x)}[\log \frac{P_1}{P_2}]$
- $JSD(P_1||P_2) = \frac{1}{2}KL(P_1||\frac{P_1+P_2}{2}) + \frac{1}{2}KL(P_2||\frac{P_1+P_2}{2})$

This makes the Generator loss:

- $2.JSD(P_{data}||P_g) - 2.log2$

GAN Training Algorithm (Ian Goodfellow et al.)

Algorithm 1 Minibatch stochastic gradient descent training of generative adversarial nets. The number of steps to apply to the discriminator, k, is a hyperparameter. We used $k = 1$, the least expensive option, in our experiments.

for number of training iterations **do**

  for k steps **do**

  - Sample minibatch of m noise samples $\{\boldsymbol{z}^{(1)}, \ldots, \boldsymbol{z}^{(m)}\}$ from noise prior $p_g(\boldsymbol{z})$.
  - Sample minibatch of m examples $\{\boldsymbol{x}^{(1)}, \ldots, \boldsymbol{x}^{(m)}\}$ from data generating distribution $p_{\text{data}}(\boldsymbol{x})$.
  - Update the discriminator by ascending its stochastic gradient:

$$\nabla_{\theta_d} \frac{1}{m} \sum_{i=1}^{m} \left[\log D\left(\boldsymbol{x}^{(i)}\right) + \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right) \right].$$

  end for

  - Sample minibatch of m noise samples $\{\boldsymbol{z}^{(1)}, \ldots, \boldsymbol{z}^{(m)}\}$ from noise prior $p_g(\boldsymbol{z})$.
  - Update the generator by descending its stochastic gradient:

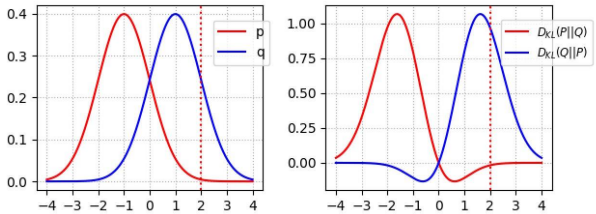
$$\nabla_{\theta_g} \frac{1}{m} \sum_{i=1}^{m} \log\left(1 - D\left(G\left(\boldsymbol{z}^{(i)}\right)\right)\right).$$

end for

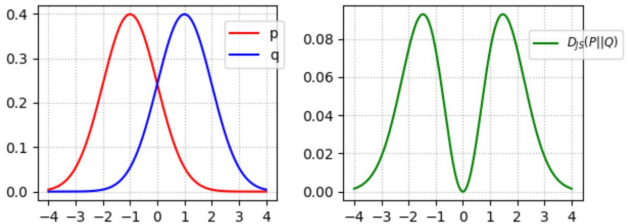
The gradient-based updates can use any standard gradient-based learning rule. We used momentum in our experiments.

Qualitative Effects of *JS-Divergence*

- The KL-divergence DL(p, q) penalizes the generator if it misses some modes of images: the penalty is high where p(x) > 0 but q(x) → 0. Nevertheless, it is acceptable that some images do not look real. The penalty is low when p(x) → 0 but q(x) > 0. (Poorer quality but more diverse samples)



- JS-divergence is symmetrical. Unlike KL-divergence, it will penalize poor images badly, but can allow less diversity



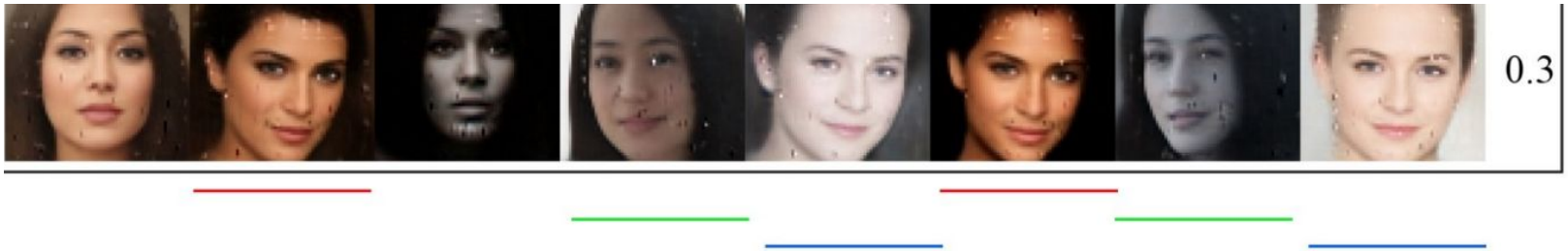
https://jonathan-hui.medium.com/gan-why-it-is-so-hard-to-train-generative-advisory-networks-819a86b3750b

GANs Training Issues

1. Mode Collapse
2. Vanishing Gradients
3. Convergence and Oscillation

GANs Training Issues - Mode Collapse

- If a generator produces an especially plausible output, the generator may learn to produce only that output
- The **Generator gets stuck** at a point where it only produces a limited variety of samples or one sample repeatedly during or after training
- Each iteration of Generator over-optimizes for a particular Discriminator, and the Discriminator never manages to learn its way out of the trap



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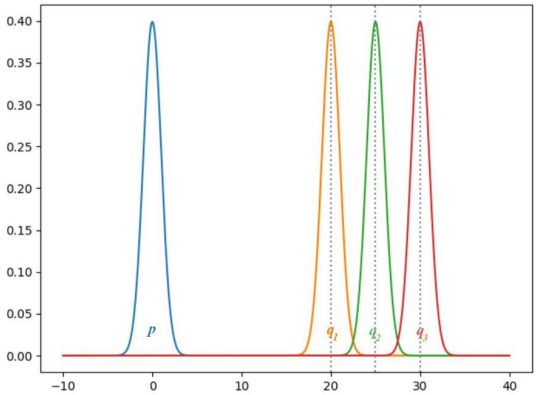
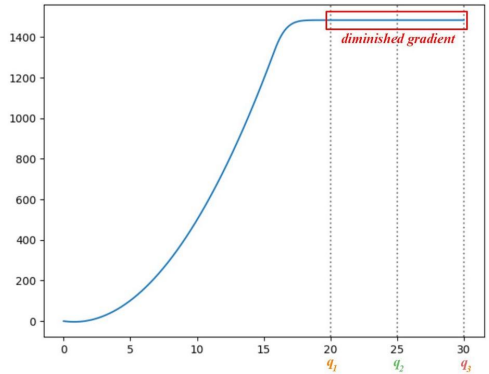
GANs Training Issues - Vanishing Gradients

- If the Discriminator is too good, then the Generator training can fail due to vanishing gradients. An optimal Discriminator doesn't provide enough information for the Generator to make progress
- Below image shows how gradients will vanish if the distribution of generated images (*p*) is too different than the distribution of real images (*q1, q2, q3*)





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GANs Training Issues - Convergence and Oscillation

- GAN training is based on a zero-sum, non-cooperative, minmax game. In short, if one wins the other loses
- In game theory, the GAN model converges when the discriminator and the generator reach a Nash equilibrium. This is the optimal point for the GAN objective
- Simplified example: Consider two player A and B which control the value of *x* and *y* respectively. Player A wants to maximize the value *xy* while B wants to minimize it

GANs Training Issues - Convergence and Oscillation

- We update the parameter *x* and *y* based on the gradient of the functions -

  $f(x) = xy$, $f(y) = -xy$

  $\partial f/\partial x = y$ and $\partial f/\partial y = -x$

  $x \rightarrow x - \alpha \cdot y$ and $y \rightarrow y + \alpha \cdot x$ (α is learning rate)

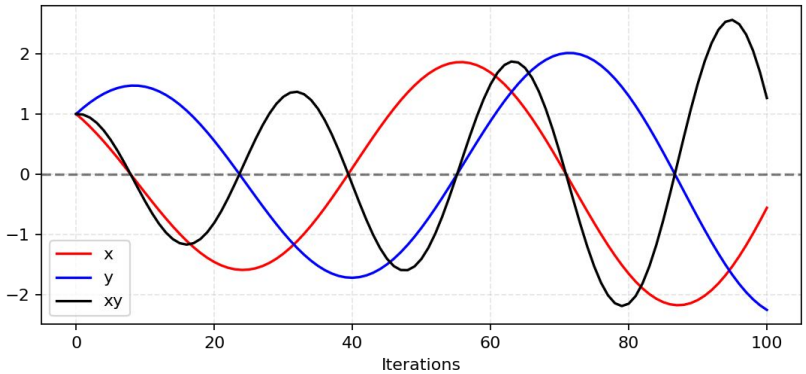
GANs Training Issues - Convergence and Oscillation

- We update the parameter *x* and *y* based on the gradient of the functions f(x) = xy, f(y) = -xy

  $\partial f/\partial x = y$ and $\partial f/\partial y = -x$

  $x \rightarrow x - \alpha \cdot y$ and $y \rightarrow y + \alpha \cdot x$ (α is learning rate)

- The Nash equilibrium is **x = y = 0**. This is the only state where the action of your opponent does not matter. It is the only state that any opponents' actions will not change the game outcome



https://medium.com/deep-math-machine-learning-ai/ch-14-general-adversarial-networks-gans-with-math-1318faf46b43

Feature Matching

Statistics of generated images should match statistics of real images.

Discriminator **produces multidimensional output, a “statistic” of data**.

Generator trained to minimize L2 between real and generated data.

Discriminator trained to maximize L2 between real and generated data.

Goal: matching features in real images

$$\|E_X D(X) - E_Z D(G(Z))\|_2^2$$

$$\|E_X f(X) - E_Z f(G(Z))\|_2^2$$

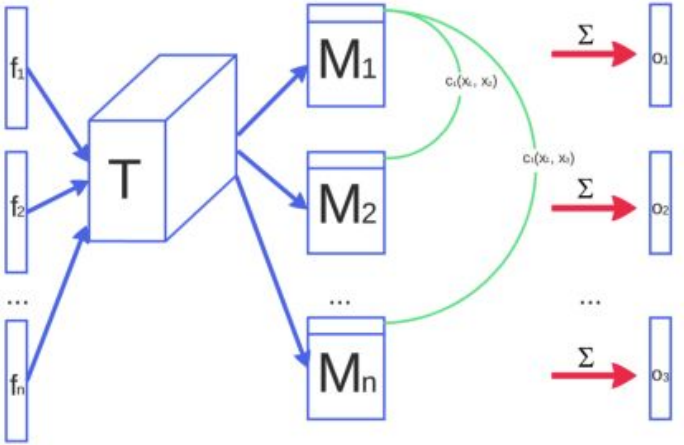
Tim Salimans, Ian J. Goodfellow, Wojciech Zaremba, Vicki Cheung, Alec Radford, and Xi Chen, Improved techniques f training gans, CoRR abs/1606.03498 (2016).

Minibatch Discrimination

Discriminator can look at multiple inputs at once and decide if those inputs come from the real or generated distribution.

- GANs frequently collapse to a single point
- Discriminator needs to differentiate between two distributions
- Easier task if looking at multiple samples

Append the **similarity between the image and other images in the same batch** in **one of the dense layers in the discriminator** to classify whether this image is real or generated.



Tim Salimans, Ian J. Goodfellow, Wojciech Zaremba, Vicki Cheung, Alec Radford, and Xi Chen, Improved techniques f training gans, CoRR abs/1606.03498 (2016).

Historical averaging

Dampen oscillations by encouraging updates to converge to a mean.

- GANs frequently create a cycle or experience oscillations
- **Add a term** to reduce oscillations that encourage the current parameters to be near a **moving average of the parameters**

$$\left\| \theta - \frac{1}{t} \sum_{i}^{t} \theta_i \right\|_2^2$$

Tim Salimans, Ian J. Goodfellow, Wojciech Zaremba, Vicki Cheung, Alec Radford, and Xi Chen, Improved techniques f training gans, CoRR abs/1606.03498 (2016).

One-sided Label Smoothing

Don’t over-penalize generated images

- Label smoothing is a common and easy technique that improves performance across many domains
  - Sigmoid tries hard to saturate to 0 or 1 but can never quite reach that goal
  - Provide targets that are epsilon or 1- epsilon so the sigmoid doesn’t saturate and overtrain
- Experimentally, **smooth the real targets** but **do not smooth the generated targets** when training the discriminator.

Tim Salimans, Ian J. Goodfellow, Wojciech Zaremba, Vicki Cheung, Alec Radford, and Xi Chen, Improved techniques f training gans, CoRR abs/1606.03498 (2016).

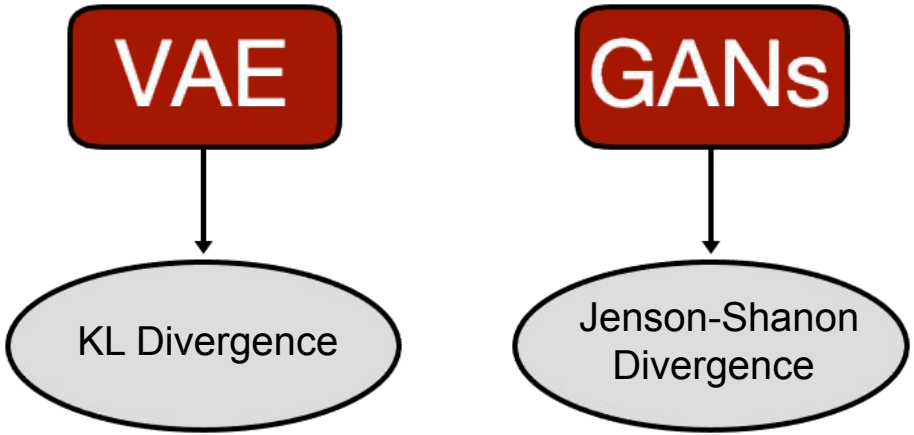
Virtual Batch Normalization

Use batch normalization to accelerate convergence

- Batch normalization accelerates convergence
- However, hard to apply in adversarial setting
- Collect statistics on **fixed batch** of real data and use to normalize other data.

Tim Salimans, Ian J. Goodfellow, Wojciech Zaremba, Vicki Cheung, Alec Radford, and Xi Chen, Improved techniques f training gans, CoRR abs/1606.03498 (2016).

Recap



Special Case:

KL-Divergence:

Let θ be the distance between the two peaks of the distribution.

If θ ≠ 0, $$KL(P\|Q) = 1log(\frac{1}{0}) = \infty$$

If θ = 0, $$KL(P\|Q) = 1log(\frac{1}{1}) = 0$$

Not differentiable w.r.t θ

Jenson-Shanon-Divergence:

$$m(X) = \frac{P_D + P_G}{2}$$

$$JS(P_D \| P_G) = \frac{1}{2} KL(P_D \| m) + \frac{1}{2} KL(P_G | m)$$

Let θ be the distance between the two peaks of the distribution.

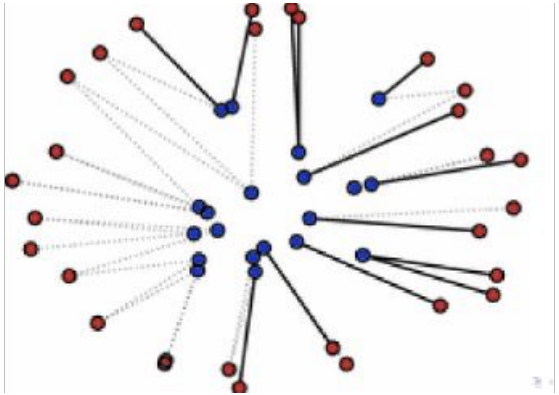
If θ ≠ 0, $JSD(P\|Q) = 0.5 * (1log(\frac{1}{0.5}) + 1log(\frac{1}{0.5})) = log2$

If θ = 0, $JSD(P\|Q) = 0.5 * (1log(\frac{1}{1}) + 1log(\frac{1}{1})) = 0$

Constant to θ

Wasserstein Distance: $W(\mathbb{P}_r, \mathbb{P}_g) = \inf_{\gamma \in \Pi(\mathbb{P}_r, \mathbb{P}_g)} \mathbb{E}_{(x,y)\sim\gamma}\left[\, \|x - y\| \,\right]$

- Minimum cost of turning one pile of dirt into another pile of dirt, when both distributions are treated as pile of dirt.
- The total Σ mass × mean distance required to transform one distribution to another



Red points, Blue points represent two different distributions.

Wasserstein Distance:

Let θ be the distance between the two peaks of the distribution.

$$W(P,Q) = | \theta |$$

Differentialble w.r.t θ

WGAN

$$W(\mathbb{P}_r, \mathbb{P}_g) = \inf_{\gamma \in \Pi(\mathbb{P}_r, \mathbb{P}_g)} \mathbb{E}_{(x,y)\sim\gamma} \left[\|x - y\| \right]$$

Kantorovich-Rubinstein duality

$$\min_G \max_{D \in \mathcal{D}} \mathbb{E}_{\boldsymbol{x}\sim\mathbb{P}_r} \left[D(\boldsymbol{x}) \right] - \mathbb{E}_{\tilde{\boldsymbol{x}}\sim\mathbb{P}_g} \left[D(\tilde{\boldsymbol{x}}) \right]$$

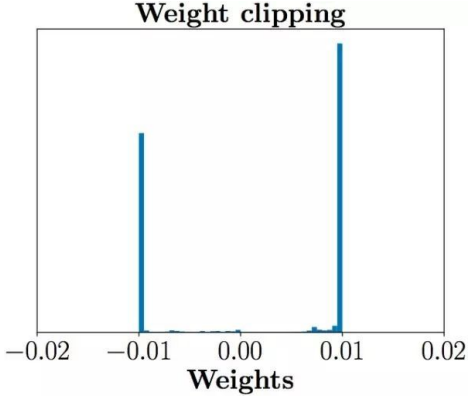
D should be a 1-Lipschitz function: $|f(x_1) - f(x_2)| \leq K|x_1 - x_2|$

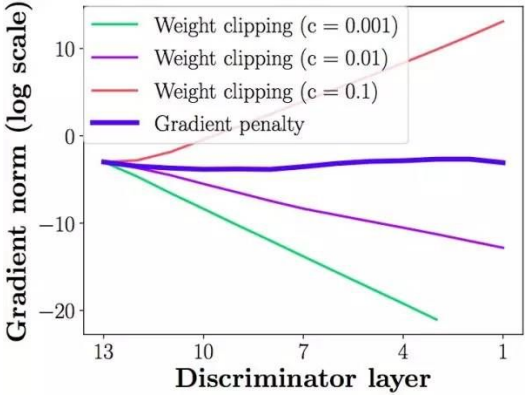
A function is K-Lipschitz if its gradients are at most K everywhere.

→ W(Pr, Pg) is continuous everywhere, and differentiable almost everywhere

Weight Clipping

Restrict weights between [-c, c]





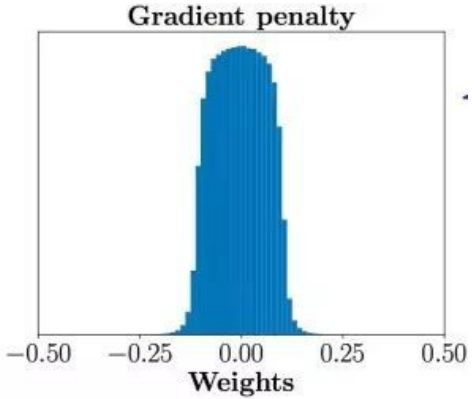
Arjovsky, Martin, Soumith Chintala, and Léon Bottou. "Wasserstein generative adversarial networks." International conference on machine learning. PMLR, 2017.

Gradient Penalty

$$L = \underbrace{\mathop{\mathbb{E}}_{\tilde{\boldsymbol{x}} \sim \mathbb{P}_g} [D(\tilde{\boldsymbol{x}})] - \mathop{\mathbb{E}}_{\boldsymbol{x} \sim \mathbb{P}_r} [D(\boldsymbol{x})]}_{\text{Original critic loss}} + \underbrace{\lambda \mathop{\mathbb{E}}_{\hat{\boldsymbol{x}} \sim \mathbb{P}_{\hat{\boldsymbol{x}}}} \left[(\|\nabla_{\hat{\boldsymbol{x}}} D(\hat{\boldsymbol{x}})\|_2 - 1)^2 \right]}_{\text{Our gradient penalty}}.$$

Gradient penalty introduces a softer constraint on gradients

- more stable training
- requires very little hyper-parameter tuning



Gulrajani, Ishaan, et al. “Improved training of wasserstein gans.” arXiv preprint arXiv:1704.00028 (2017).

Wasserstein

JSD

KL

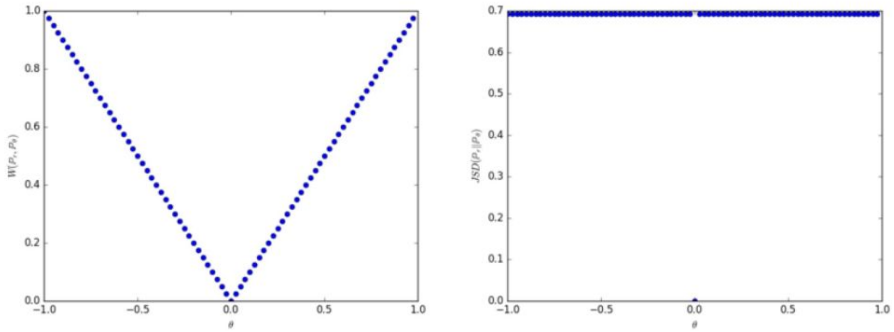


Figure 1: These plots show $\rho(\mathbb{P}_\theta, \mathbb{P}_0)$ as a function of θ when ρ is the EM distance (left plot) or the JS divergence (right plot). The EM plot is continuous and provides a usable gradient everywhere. The JS plot is not continuous and does not provide a usable gradient.

Differentiable

Meaningful Loss ($\theta \neq 0$)

Not Differentiable

Constant ($\theta \neq 0$)

Not Differentiable

Not symmetric

GAN Architectures

There are many variations of GANs for modeling different tasks. This is not meant to be exhaustive but a sample of the possibilities.

- GAN
- Conditional GAN
- LapGAN
- Recurrent Adversarial Network
- Categorical GAN
- InfoGAN
- AAE
- BiGAN
- CycleGAN

GAN

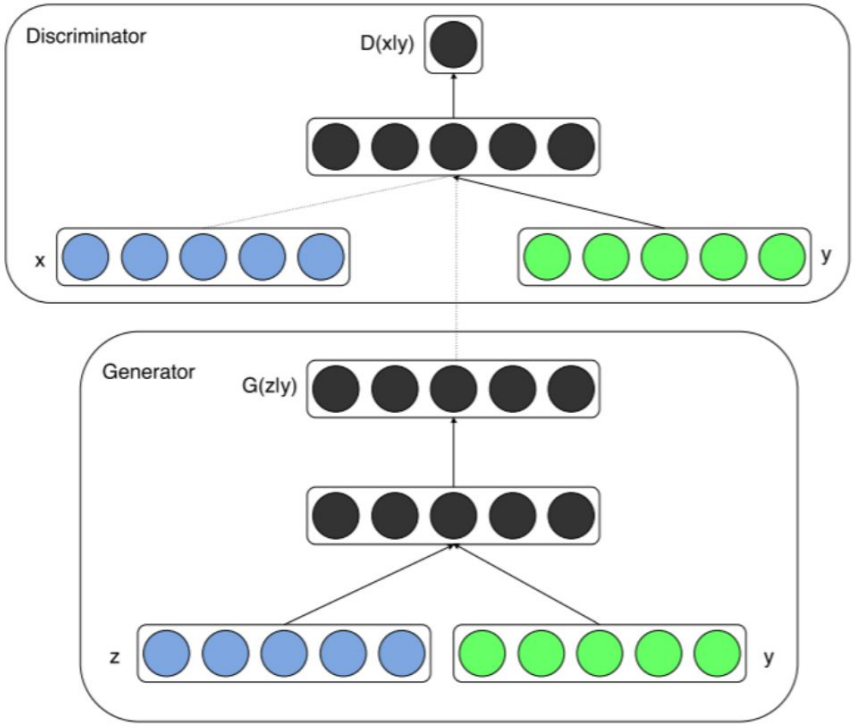
Unqualified, "GAN" typically refers to a simple model of $P(X)$ [GPM+14]. This is a vanilla GAN. Think unsupervised generation of unlabeled images, video, etc.

Conditional GANs

A conditional GAN models $P(X \mid Y)$. For example, generate samples of MNIST conditioned on the digit you are generating. [MO14]. The model is constructed by adding the labels Y as an input to both generator and discriminator.

$$\min_G \max_D V(D, G) = \mathbb{E}_X \log D(X, Y) + \mathbb{E}_Z \log D(G(Z, Y), Y)$$

Conditional GAN Architecture



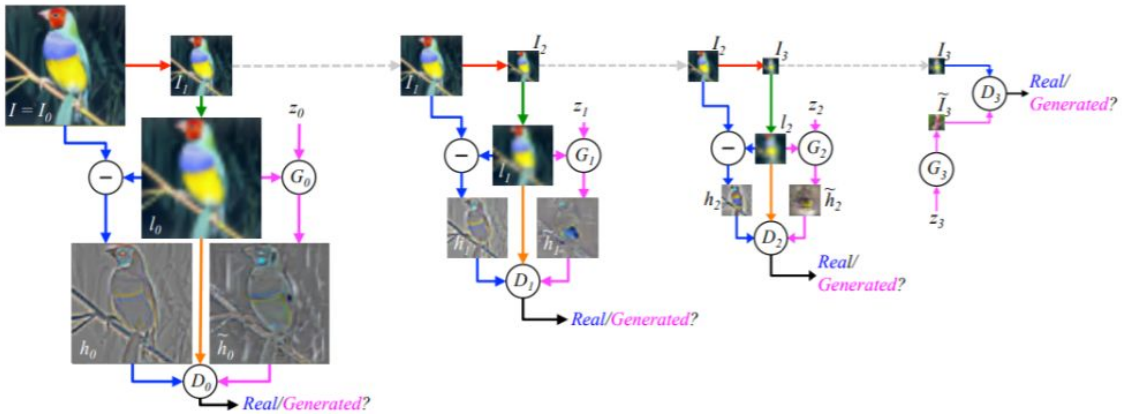
Conditional GAN Results

Figure 2: Generated MNIST digits, each row conditioned on one label

LapGAN

A Laplacian GAN is constructed of a chain of conditional GANs, to generate progressively larger images. A GAN generates small, blurry images. A conditional GAN generates larger images conditioned on the smaller image, repeated until you reach the desired size. [DCSF15]

LapGAN Architecture



BiGANs

A Bi-Directional Generative Adversarial Network trains an encoder/decoder pair in an elegant fashion. The discriminator tries to tell the difference between pairs of real data and encoded real data from data generated from prior samples and prior samples. [DKD16]

$$V(D, E, G) = \mathbb{E}_X \log D(X, E(X)) + \mathbb{E}_Z \log(1 - D(G(Z), Z))$$

This method simultaneously trains the pair and does not require any assumptions about the distance metric in either the hidden or output spaces.

BiGAN Architecture

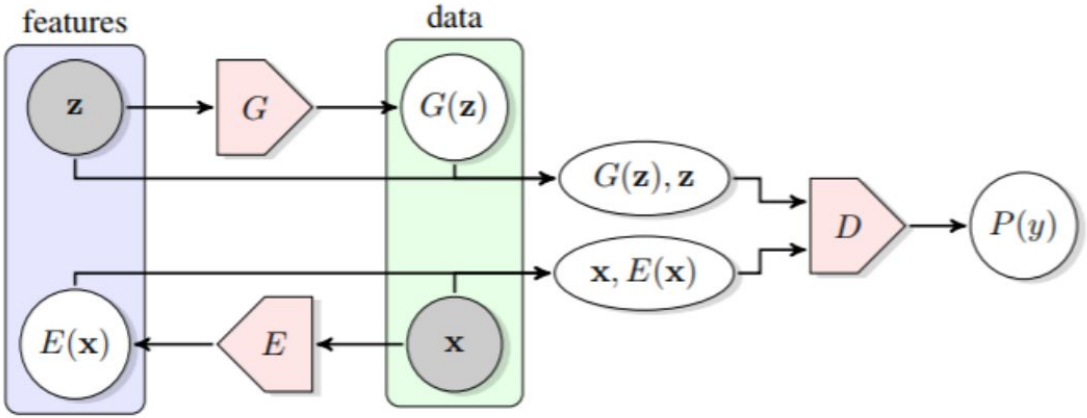
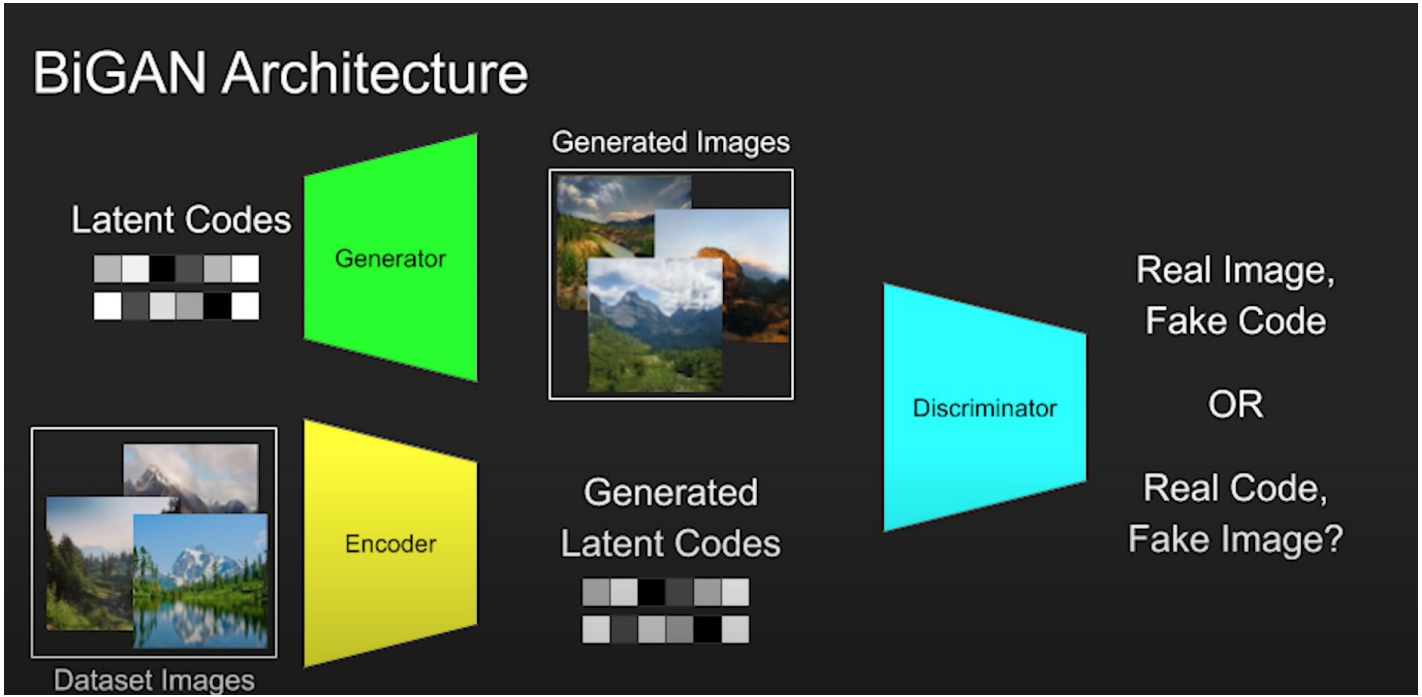


Figure 1: The structure of Bidirectional Generative Adversarial Networks (BiGAN).

BiGAN Architecture

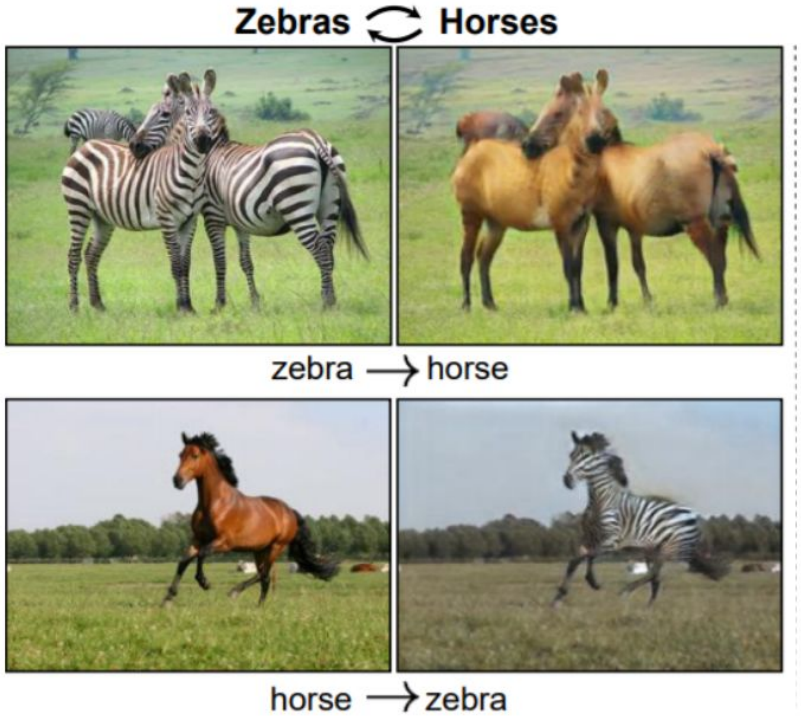


CycleGAN

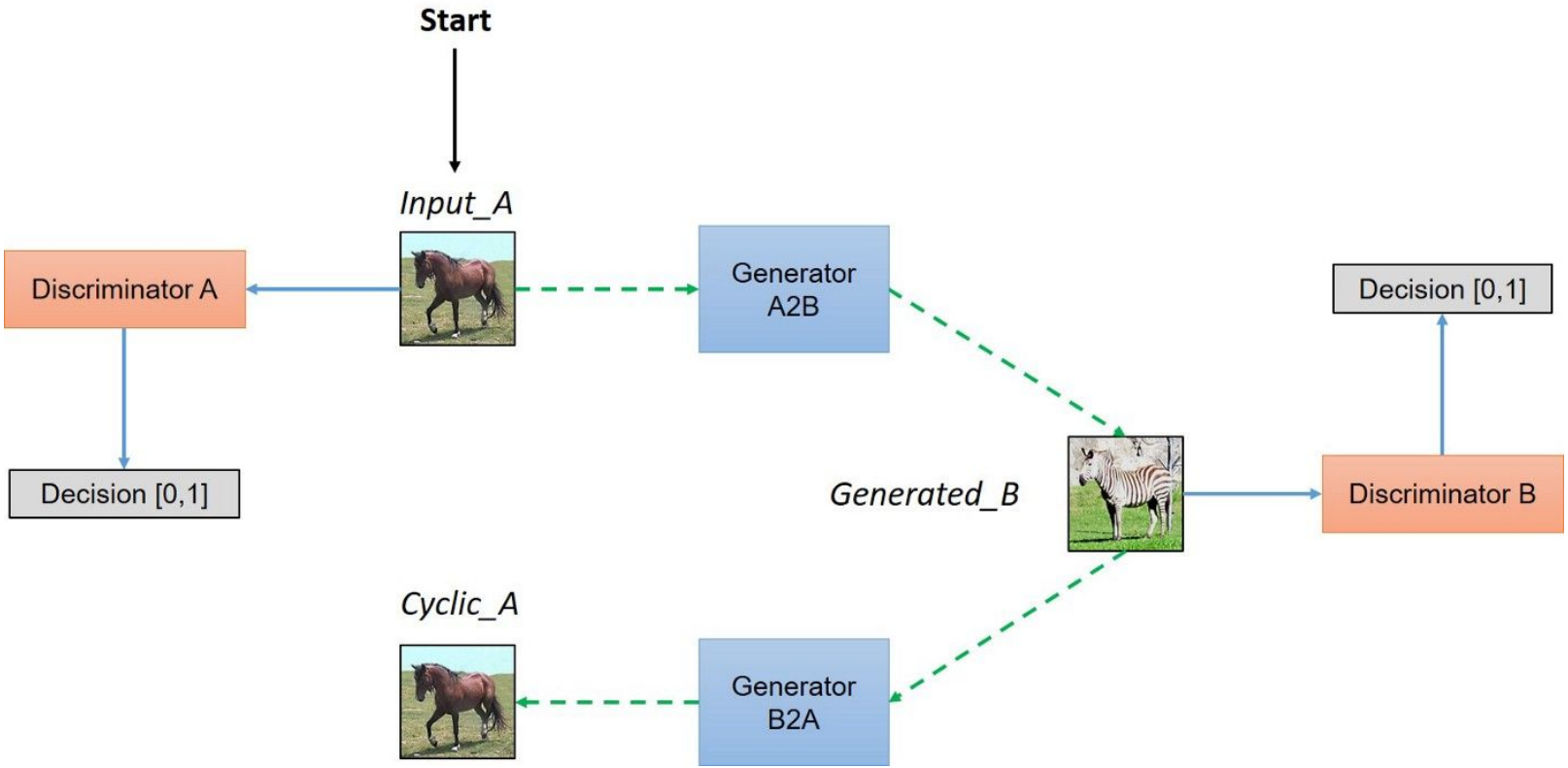
CycleGAN trains a pair of conditional GANs to perform image-to-image translation [ZPIE17].

- GAN A trained to convert from X to Y
- GAN B trained to convert from Y to X
- Additional "cycle-consistency" losses $\|Y - A(B(Y))\|_1$ and $\|X - B(A(X))\|$

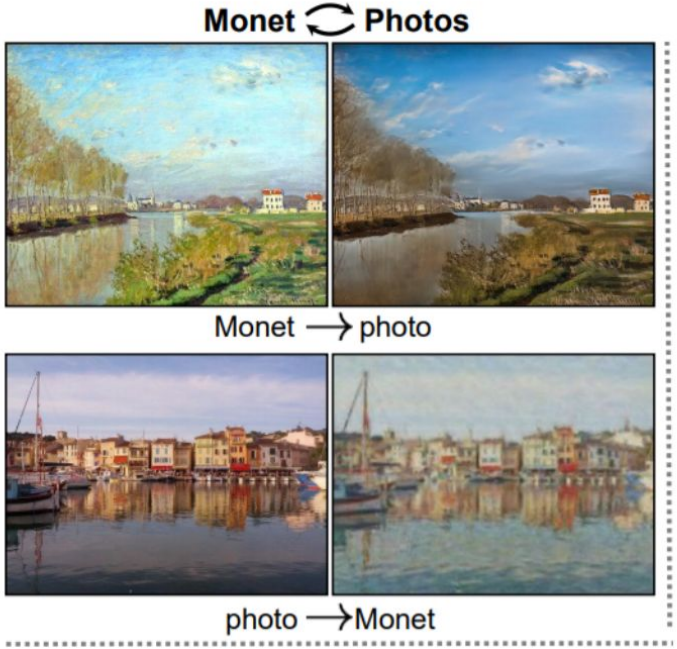
CycleGAN Results



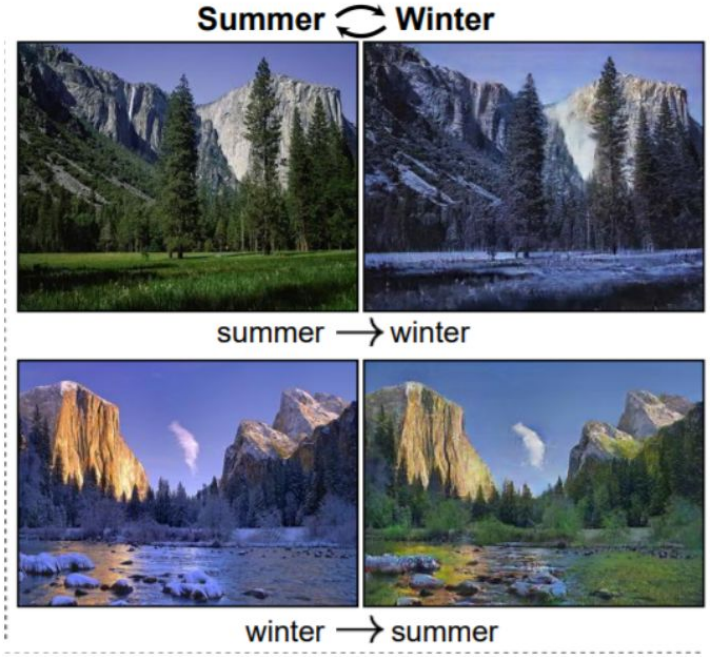
CycleGAN Architecture



CycleGAN Results



CycleGAN Results

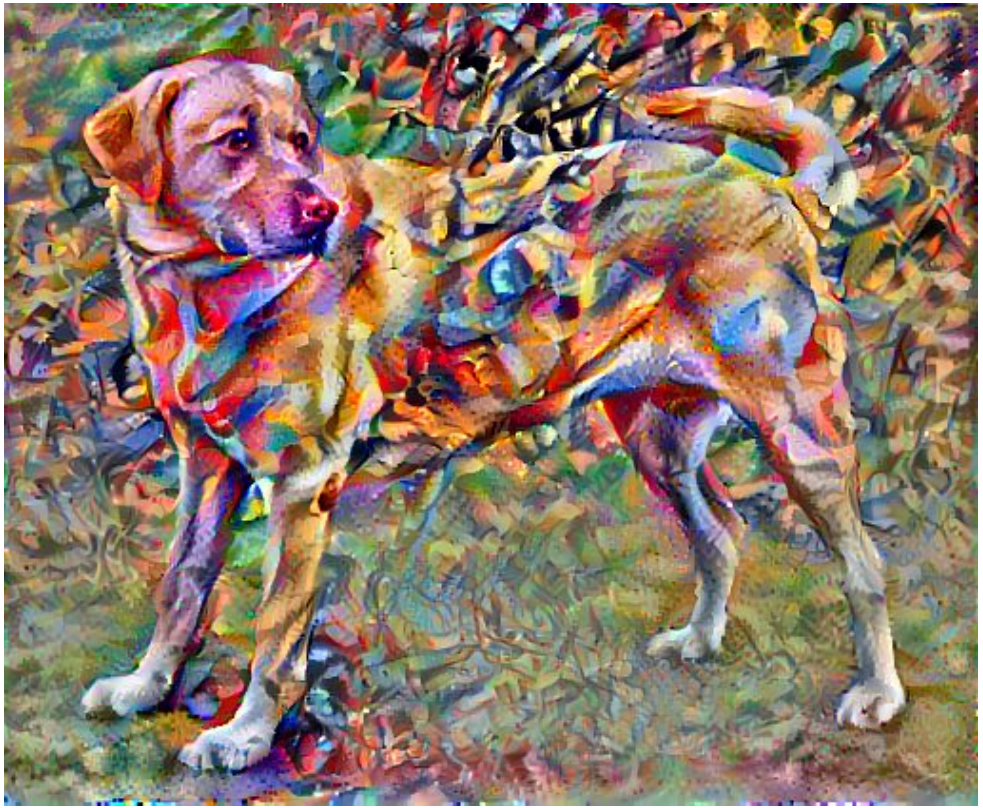


CycleGAN Lesson

- There is no paired dataset of zebras and horses
- So no easy discriminative method to train zebras from horses
- But using GANs, can train distributions to match

Neural Style Transfer Results

Neural Style Transfer Results



Style Transfer with CycleGAN

Neural Style Transfer vs CycleGAN

- Neural Style Transfer
  - Need a *content* and *style* image
  - Specific & small number of images
  - More control
  - https://reiinakano.com/arbitrary-image-stylization-tfjs/
- CycleGAN
  - Just need 2 domains of images. No need for specific *content* or *style* images
  - Many similar pictures
  - Specificity of images doesn’t really matter

[ADDITIONAL] Kaggle: CycleGAN for Monet Paintings

https://www.kaggle.com/code/dimitreoliveira/introduction-to-cyclegan-monet-paintings

I'm Something of a Painter Myself

Introduction to CycleGAN - Monet paintings


Just Think an Extra
couple of Seconds
before Assuming
Something is Real