

**Deep Learning**

**Sequence to Sequence models:  
Connectionist Temporal  
Classification**

**Spring 2025**  
**Attendance: @787**

# Sequence-to-sequence modelling

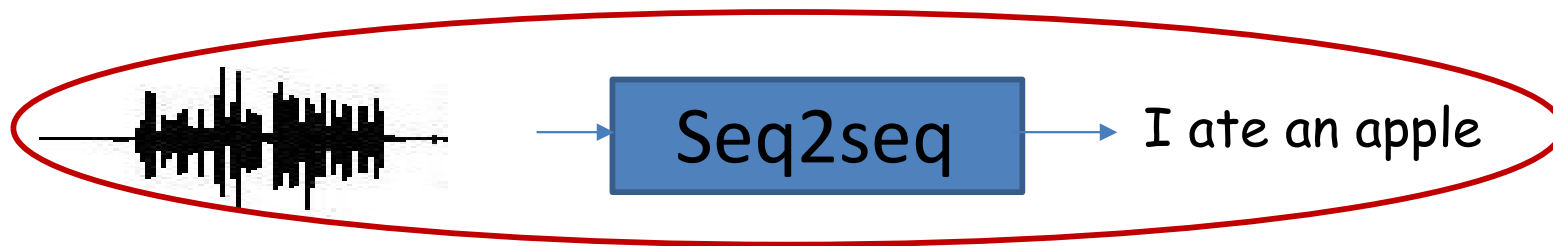
- Problem:
  - A sequence  $X_1 \dots X_N$  goes in
  - A different sequence  $Y_1 \dots Y_M$  comes out
- E.g.
  - Speech recognition: Speech goes in, a word sequence comes out
    - Alternately output may be phoneme or character sequence
  - Machine translation: Word sequence goes in, word sequence comes out
  - Dialog : User statement goes in, system response comes out
  - Question answering : Question comes in, answer goes out
- In general  $N \neq M$ 
  - No synchrony between  $X$  and  $Y$ .

# Sequence to sequence



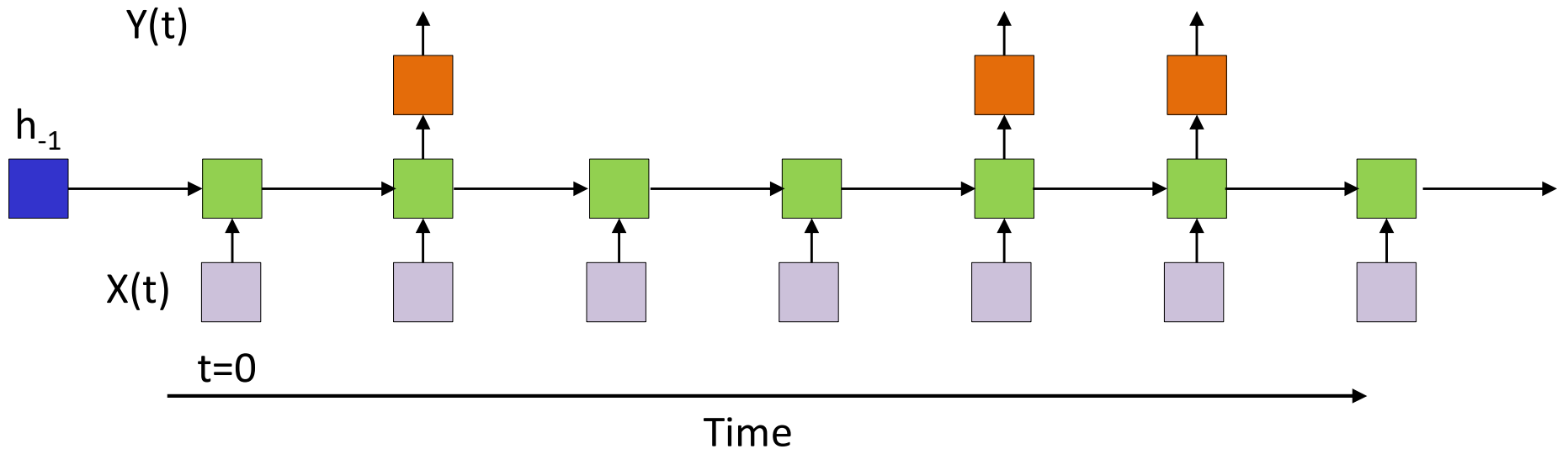
- Sequence goes in, sequence comes out
- No notion of “time synchrony” between input and output
  - May even not even maintain order of symbols
    - E.g. “I ate an apple” → “Ich habe einen apfel gegessen”
  - Or even seem related to the input
    - E.g. “My screen is blank” → “Please check if your computer is plugged in.”

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# Case 1: Order-aligned but not time synchronous



- The input and output sequences happen in the same order
  - Although they may not be *time synchronous*, they can be “aligned” against one another
  - E.g. Speech recognition
    - The input speech can be aligned to the phoneme sequence output

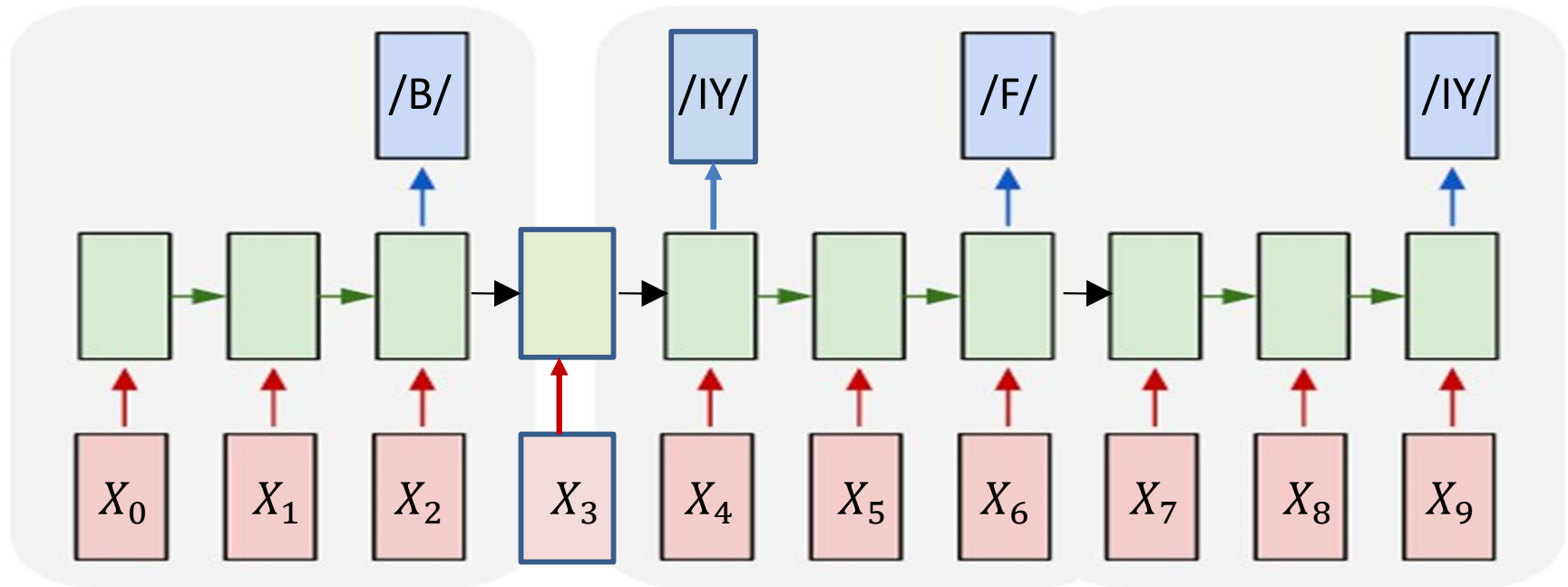
# Problems

- How do we perform *inference* on such a model
  - How to output time-asynchronous sequences
- How do we *train* such models

# Problems

- How do we perform *inference* on such a model *Partially addressed*
  - How to output time-asynchronous sequences
- How do we *train* such models

# The inference problem



- Objective: Given a sequence of inputs, asynchronously output a sequence of symbols
  - “Decoding”
- Find most likely symbol sequence given inputs

$$S_0 \dots S_{K-1} = \operatorname{argmax}_{S'_0 \dots S'_{K-1}} \operatorname{prob}(S'_0 \dots S'_{K-1} | X_0 \dots X_{N-1})$$

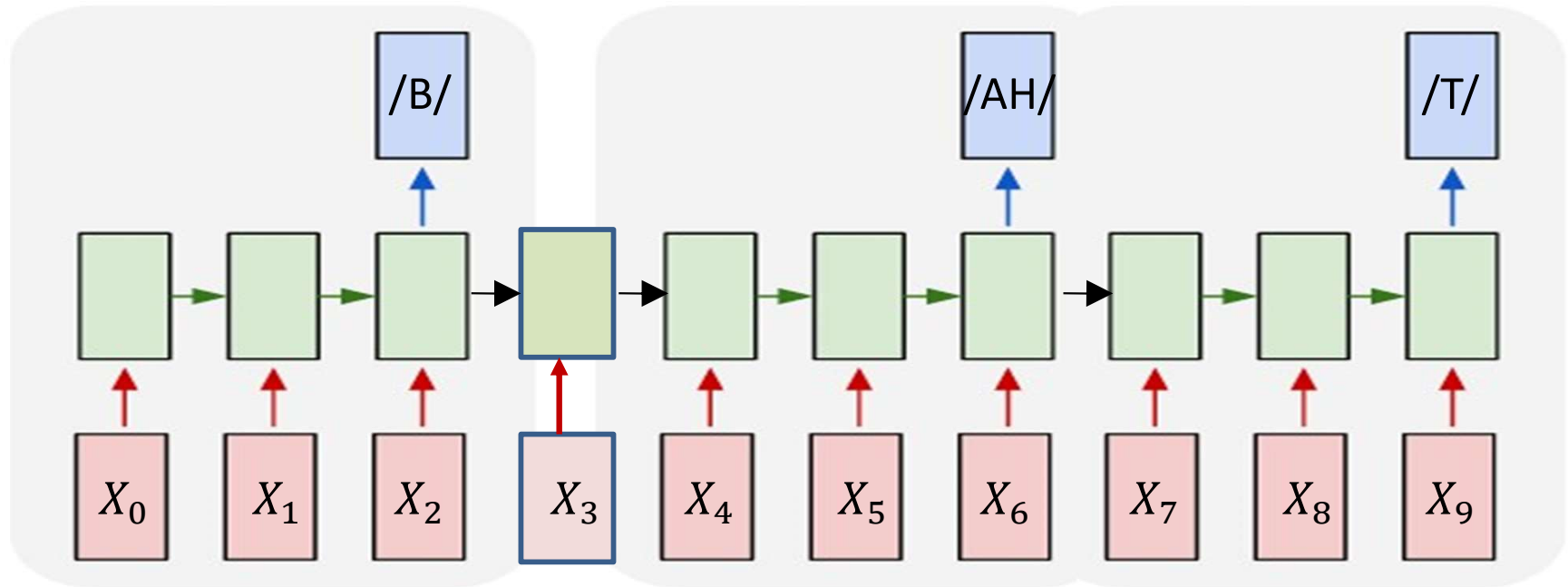


# Problems

- How do we perform *inference* on such a model
  - How to output time-asynchronous sequences

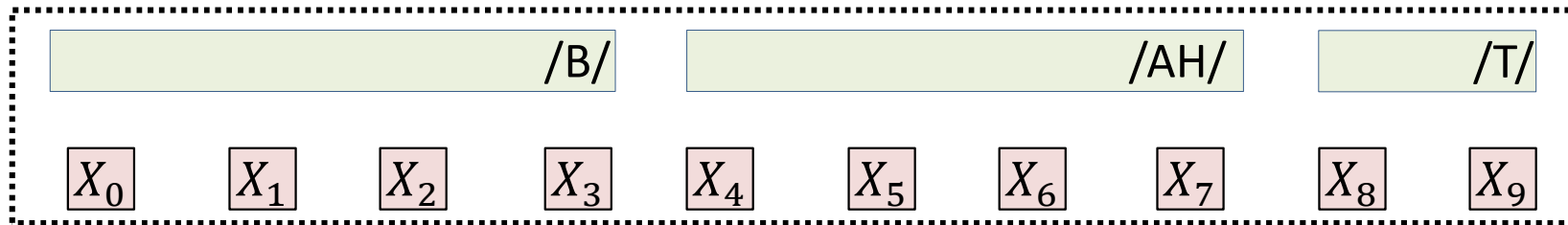
- How do we *train* such models

# Recap: Training with alignment



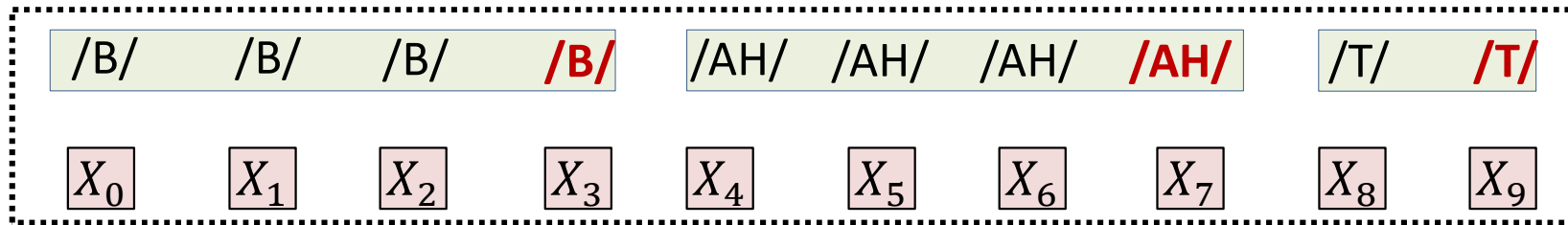
- Training data: input sequence + output sequence
  - Output sequence length  $\leq$  input sequence length
- Given the *alignment* of the output to the input
  - The phoneme /B/ ends at  $X_2$ , /AH/ at  $X_6$ , /T/ at  $X_9$

# Recap: Characterizing an alignment



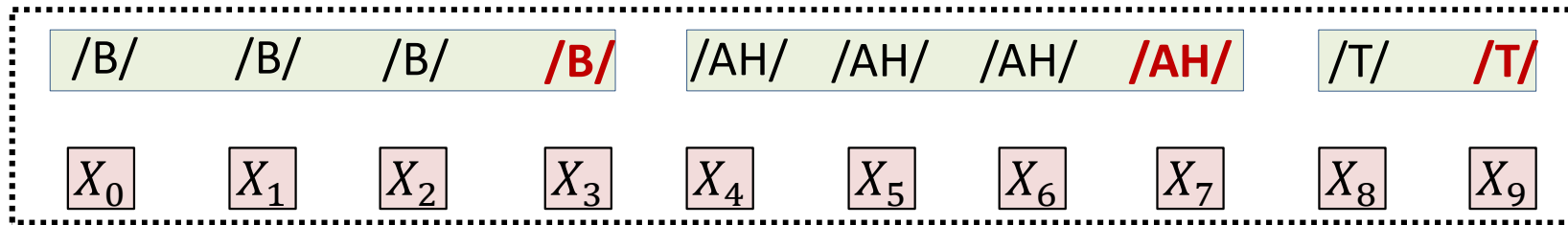
- Given only the order-synchronous sequence and its time stamps
  - $S_0(T_0), S_1(T_1), \dots, S_{K-1}(T_{K-1})$
  - E.g.  $S_0 = /B/ (3)$ ,  $S_1 = /B/ (7)$ ,  $S_2 = /T/ (9)$ ,

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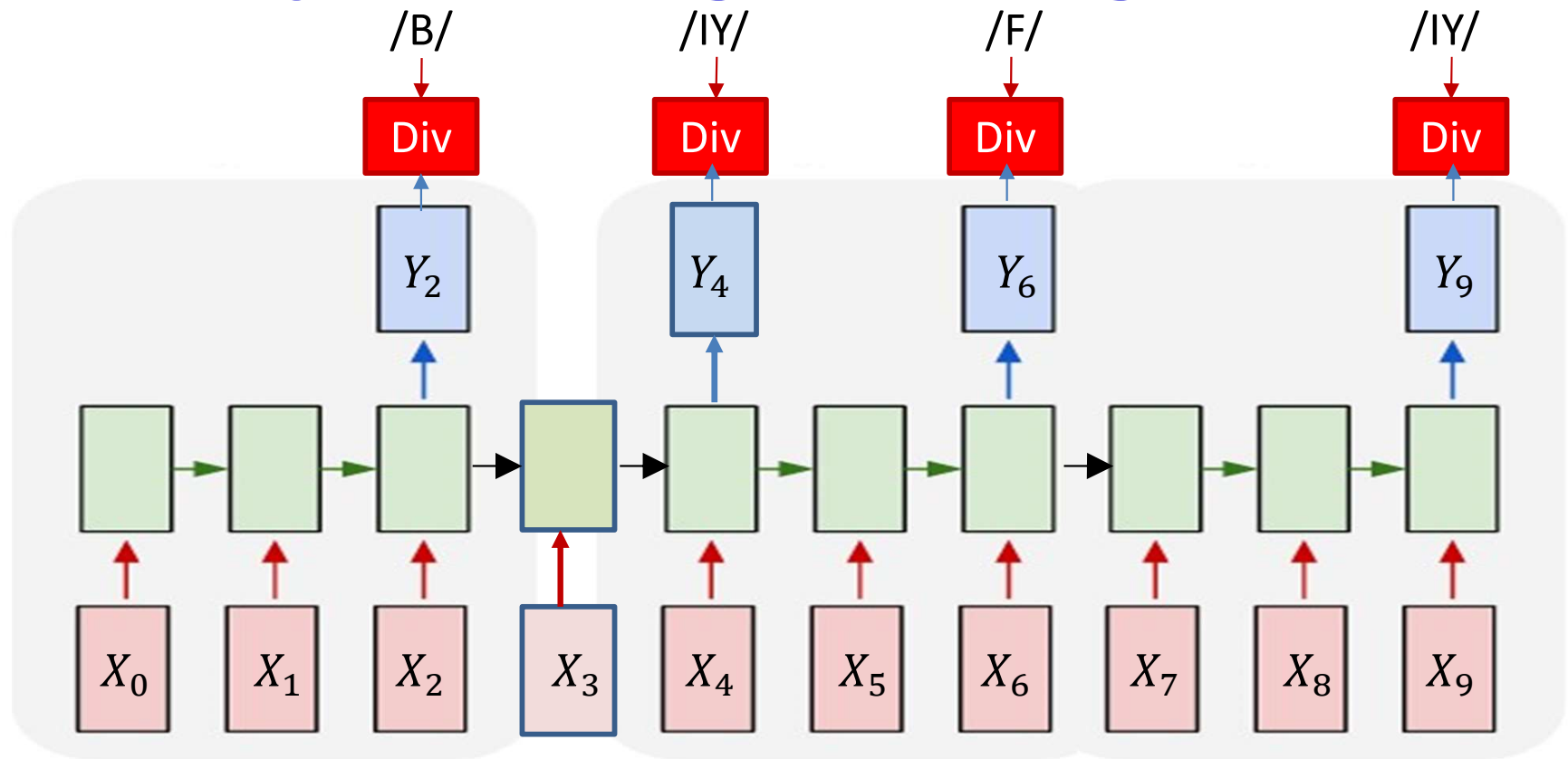
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  - $S_0(T_0), S_1(T_1), \dots, S_{K-1}(T_{K-1})$
  - E.g.  $S_0 = /B/ (3), S_1 = /B/ (7), S_2 = /T/ (9),$
- Repeat symbols to convert it to a time-synchronous sequence
  - $s_0, s_1, \dots, s_{N-1} = S_0, S_0, \dots, (T_0 \text{ times}), S_1, S_1, \dots, (T_1 \text{ times}), \dots, S_{K-1}$
  - E.g.  $s_0, s_1, \dots, s_9 = /B//B//B//B//AH//AH//AH//AH//AH//T//T/$

# Recap: Characterizing an alignment

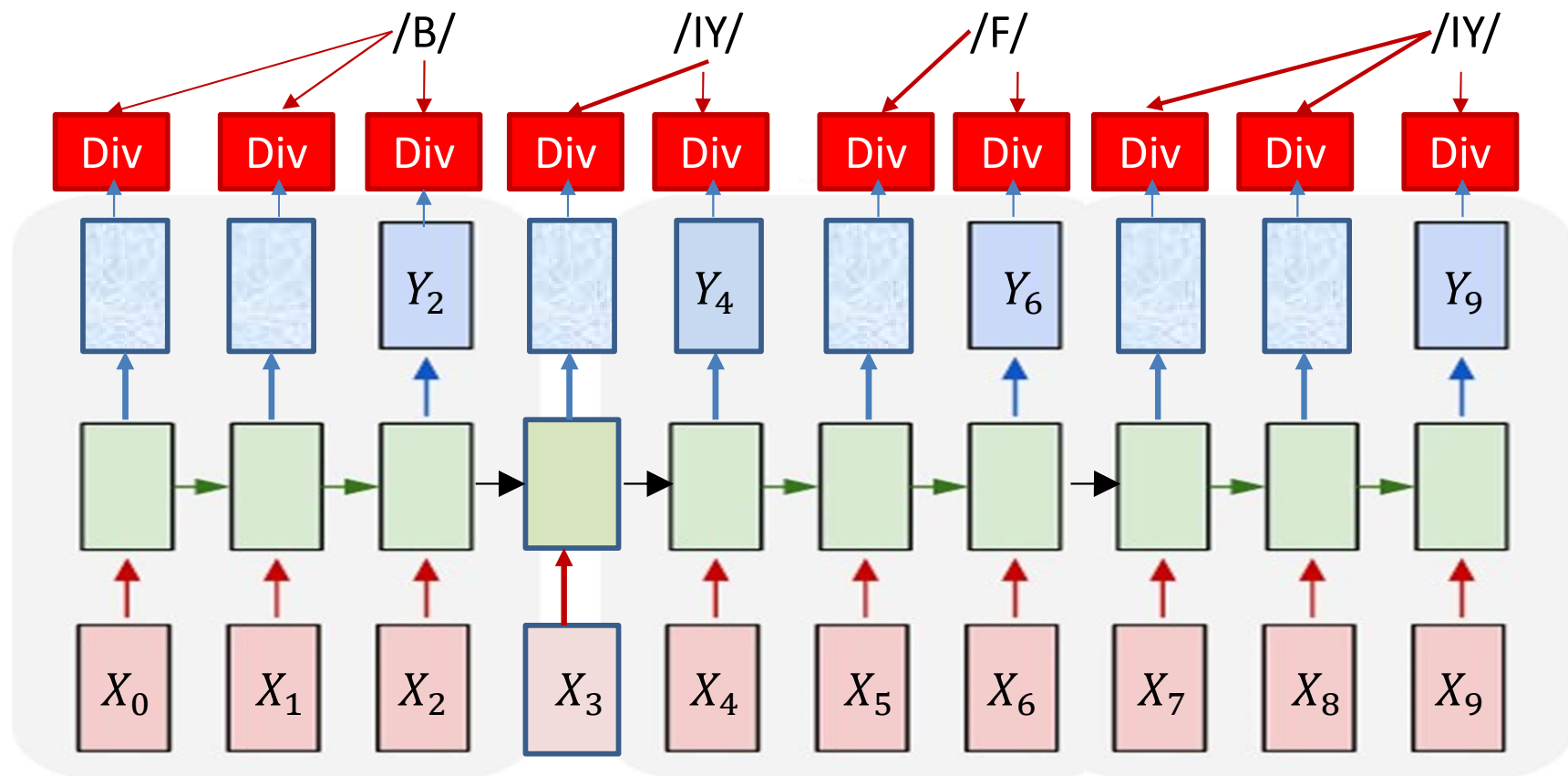


- Given only the order-synchronous sequence and its time stamps
  - $S_0(T_0), S_1(T_1), \dots, S_{K-1}(T_{K-1})$
  - E.g.  $S_0 = /B/ (3), S_1 = /B/ (7), S_2 = /T/ (9),$
- Repeat symbols to convert it to a time-synchronous sequence
  - $s_0 = S_0, s_1 = S_0, \dots, s_{T_0} = S_0, s_{T_0+1} = S_1, \dots, s_{T_1} = S_1, s_{T_1+1} = S_2, \dots, s_{N-1} = S_{K-1}$
  - E.g.  $s_0, s_1, \dots, s_9 = /B//B//B//B//AH//AH//AH//AH//AH//T//T/$
- For our purpose an alignment of  $S_0 \dots S_{K-1}$  to an input of length  $N$  has the form
  - $s_0, s_1, \dots, s_{N-1} = S_0, S_0, \dots, S_0, S_1, S_1, \dots, S_1, S_2, \dots, S_{K-1}$  (of length  $N$ )
- Any sequence of this kind of length  $N$  that contracts (by eliminating repetitions) to  $S_0 \dots S_{K-1}$  is a candidate alignment of  $S_0 \dots S_{K-1}$

# Recap: Training with alignment

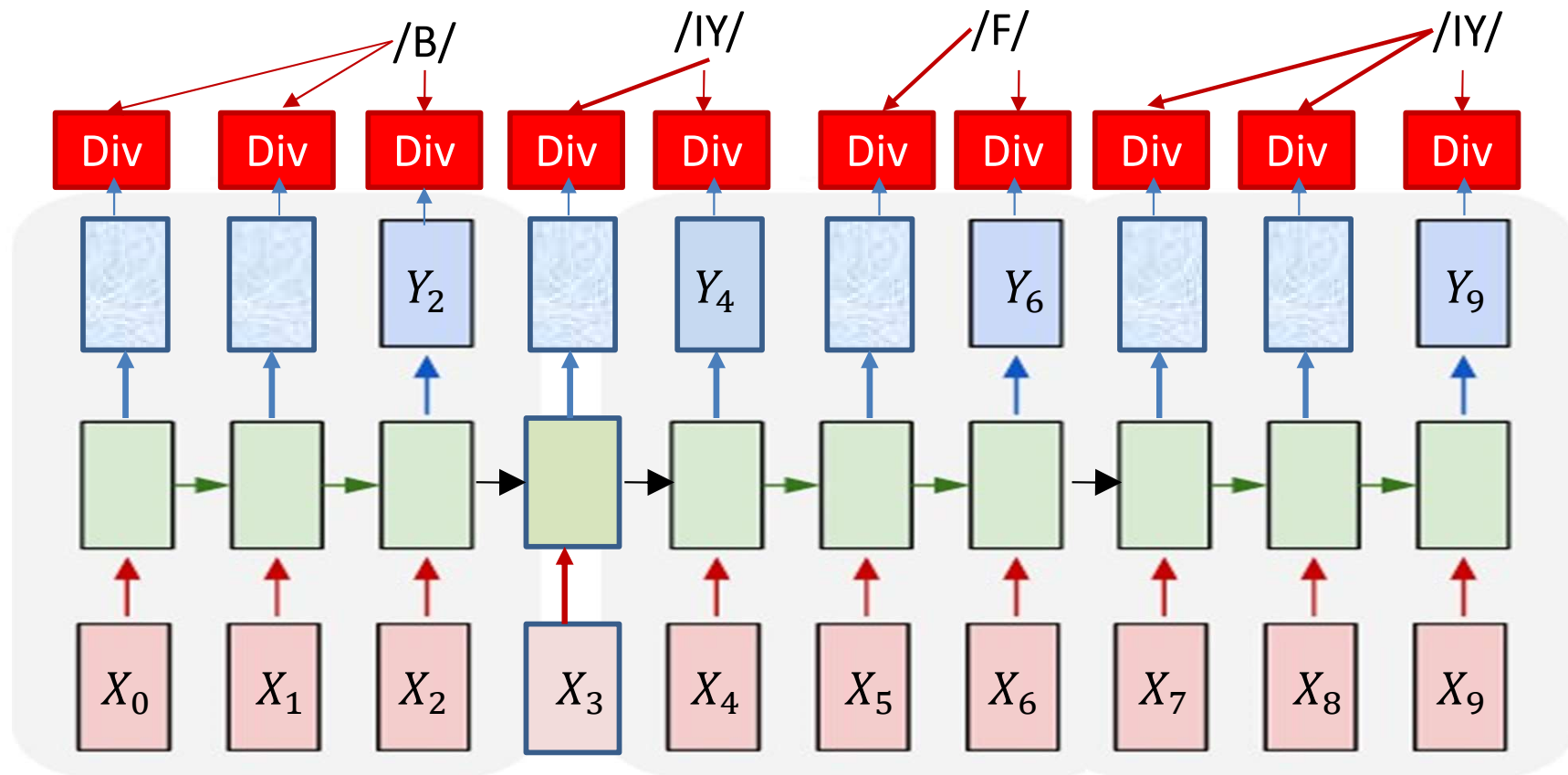


- Given the order-aligned output sequence with timing



- Given the order aligned output sequence with timing
  - Convert it to a time-synchronous alignment by repeating symbols
- Compute the divergence from the time-aligned sequence

$$DIV = \sum_t KL(Y_t, symbol_t) = - \sum_t \log Y(t, symbol_t)$$



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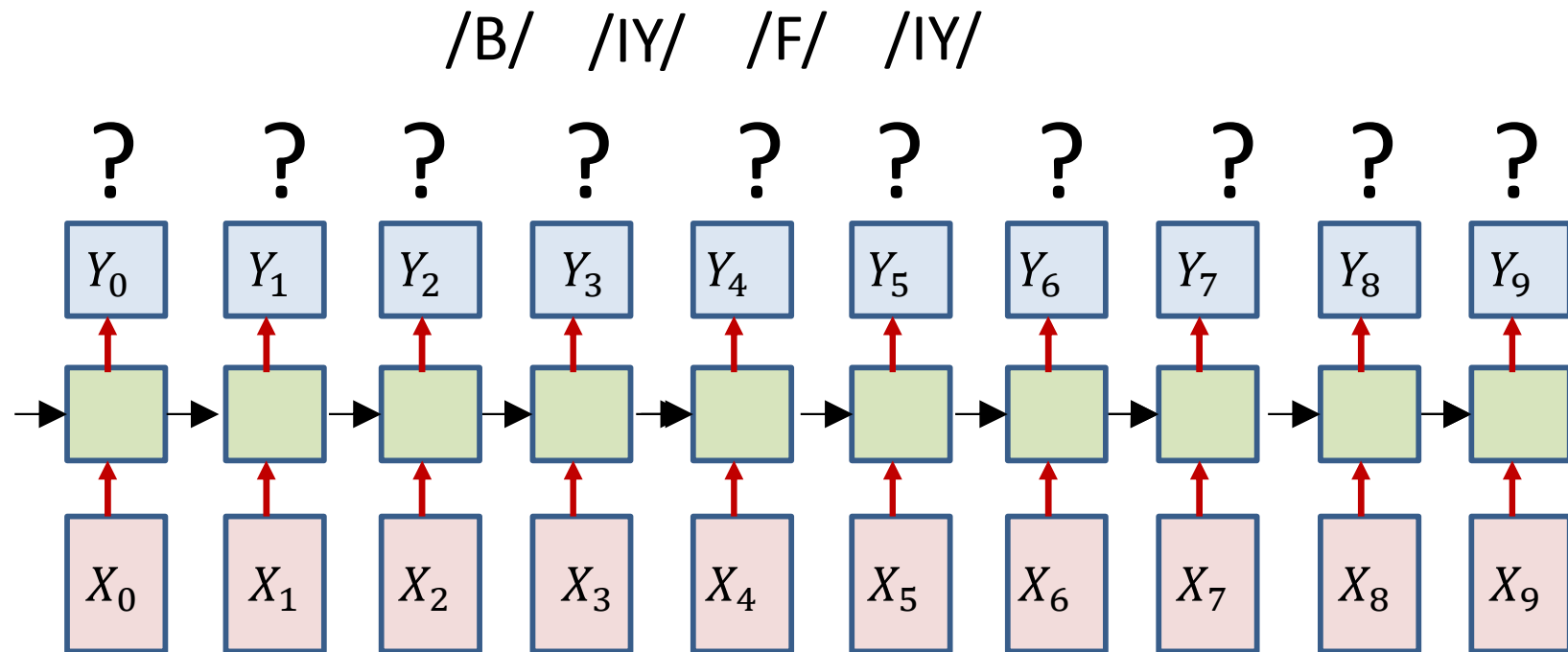
- The gradient w.r.t the  $t$ -th output vector  $Y_t$

$$\nabla_{Y_t} DIV = \begin{bmatrix} 0 & 0 & \dots & \frac{-1}{Y(t, symbol_t)} & 0 & \dots & 0 \end{bmatrix}$$

- Zeros except at the component corresponding to the target aligned to that time

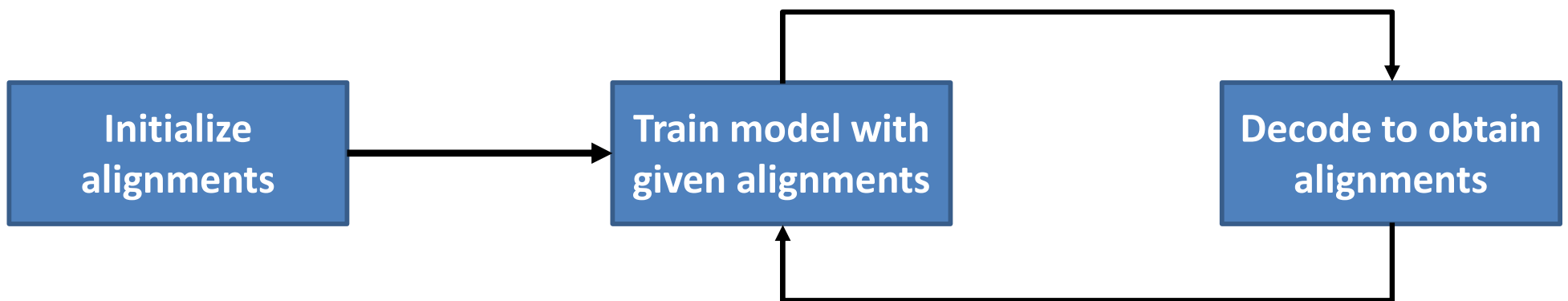
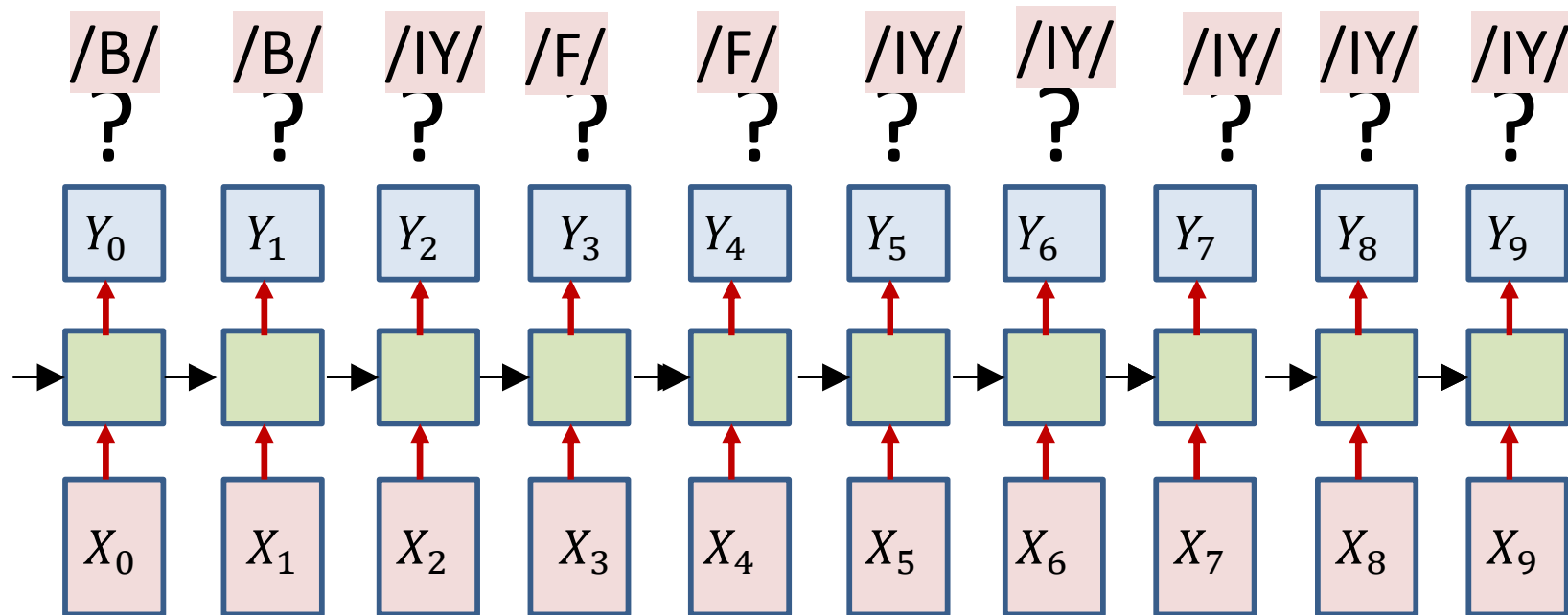


# Problem: Alignment not provided



- Only the sequence of output symbols is provided for the training data
  - But no timing information

# Solution 1: *Guess the alignment*



# Poll 1 (@788, @789)

Viterbi training explicitly estimates the alignment of each training instance and computes the divergence for the estimated alignment (T/F)

- True
- False

Viterbi training requires reestimation of alignments in every iteration (T/F)

- True
- False

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# Iterative update: Problem

- Approach heavily dependent on initial alignment
- Prone to poor local optima
- Alternate solution: Do not commit to an alignment during any pass..

# Recap: Training *without* alignment

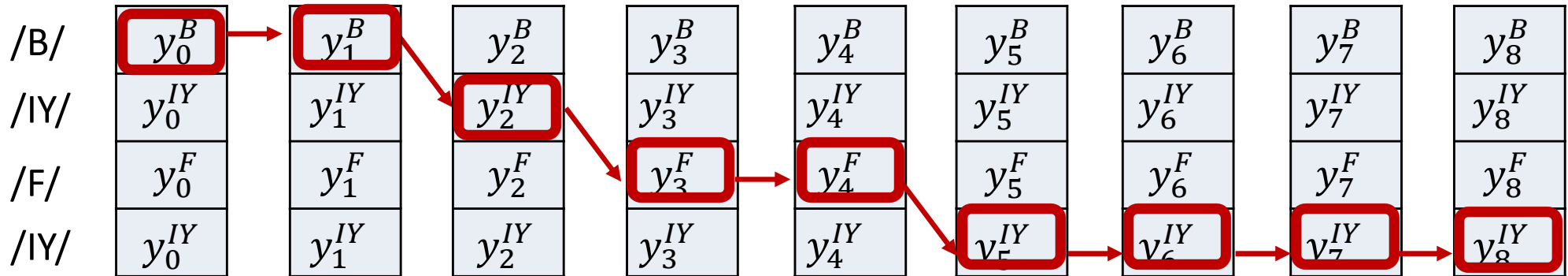
- We know how to train if the alignment is provided
- Problem: Alignment is *not* provided
- Solution:
  1. *Guess* the alignment
  2. Consider *all possible* alignments

# Recap: The “aligned” table

/B/	$y_0^B$	$y_1^B$	$y_2^B$	$y_3^B$	$y_4^B$	$y_5^B$	$y_6^B$	$y_7^B$	$y_8^B$
/IY/	$y_0^{IY}$	$y_1^{IY}$	$y_2^{IY}$	$y_3^{IY}$	$y_4^{IY}$	$y_5^{IY}$	$y_6^{IY}$	$y_7^{IY}$	$y_8^{IY}$
/F/	$y_0^F$	$y_1^F$	$y_2^F$	$y_3^F$	$y_4^F$	$y_5^F$	$y_6^F$	$y_7^F$	$y_8^F$
/IY/	$y_0^{IY}$	$y_1^{IY}$	$y_2^{IY}$	$y_3^{IY}$	$y_4^{IY}$	$y_5^{IY}$	$y_6^{IY}$	$y_7^{IY}$	$y_8^{IY}$
/AH/	$y_0^{AH}$	$y_1^{AH}$	$y_2^{AH}$	$y_3^{AH}$	$y_4^{AH}$	$y_5^{AH}$	$y_6^{AH}$	$y_7^{AH}$	$y_8^{AH}$
/B/	$y_0^B$	$y_1^B$	$y_2^B$	$y_3^B$	$y_4^B$	$y_5^B$	$y_6^B$	$y_7^B$	$y_8^B$
/D/	$y_0^D$	$y_1^D$	$y_2^D$	$y_3^D$	$y_4^D$	$y_5^D$	$y_6^D$	$y_7^D$	$y_8^D$
/EH/	$y_0^{EH}$	$y_1^{EH}$	$y_2^{EH}$	$y_3^{EH}$	$y_4^{EH}$	$y_5^{EH}$	$y_6^{EH}$	$y_7^{EH}$	$y_8^{EH}$
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/G/	$y_0^G$	$y_1^G$	$y_2^G$	$y_3^G$	$y_4^G$	$y_5^G$	$y_6^G$	$y_7^G$	$y_8^G$

Arrange the constructed table so that from top to bottom it has the exact sequence of symbols required

# The reason for suboptimality



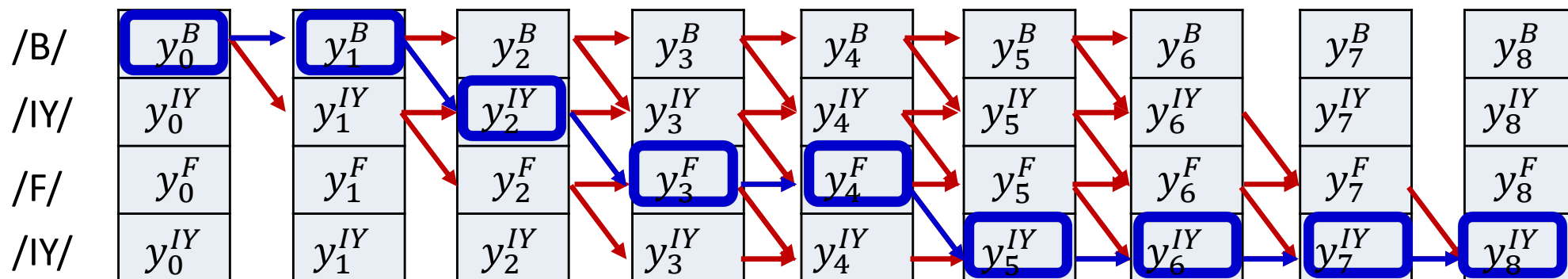
- We *commit* to the single “best” estimated alignment
  - The *most likely* alignment

$$DIV = - \sum_t \log Y(t, symbol_t^{bestpath})$$

- This can be way off, particularly in early iterations, or if the model is poorly initialized



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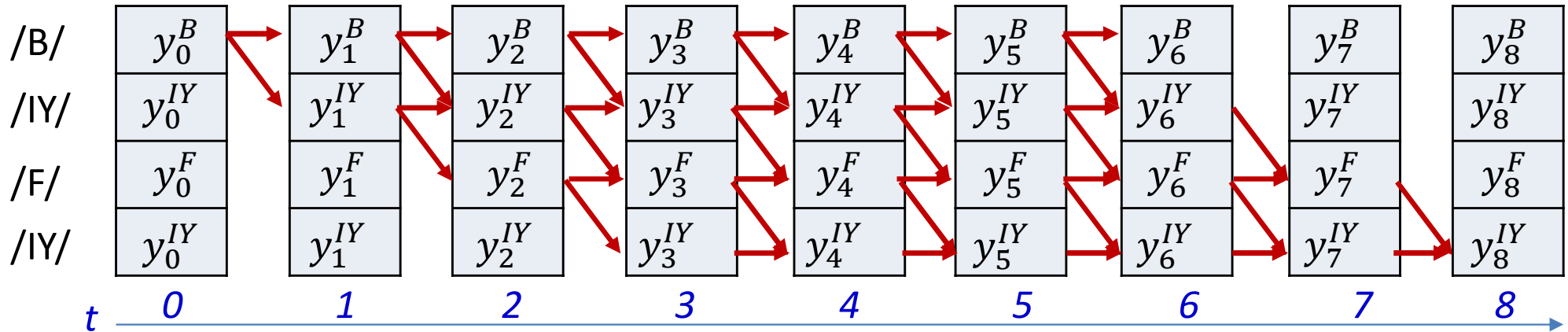


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$$DIV = - \sum_t \log Y(t, symbol_t^{bestpath})$$

- This can be way off, particularly in early iterations, or if the model is poorly initialized
- **Alternate view:** there is a probability distribution over alignments of the target Symbol sequence (to the input)
  - *Selecting a single alignment is the same as drawing a single sample from it*
  - Selecting the most likely alignment is the same as deterministically always drawing the most probable value from the distribution

# Averaging over *all* alignments



- Instead of only selecting the most likely alignment, use the statistical expectation over *all* possible alignments

$$DIV = E \left[ - \sum_t \log Y(t, s_t) \right]$$

- Use the *entire distribution of alignments*
- This will mitigate the issue of suboptimal selection of alignment

# Poll 2 (@790, @791)

The “training-without-alignment” procedure computes the average divergence over *all possible* alignments of the label sequence to the input (T/F)

- True
- False

The “training-without-alignment” requires explicit estimation of the alignment of the label sequence to the input

- True
- False

# Poll 2

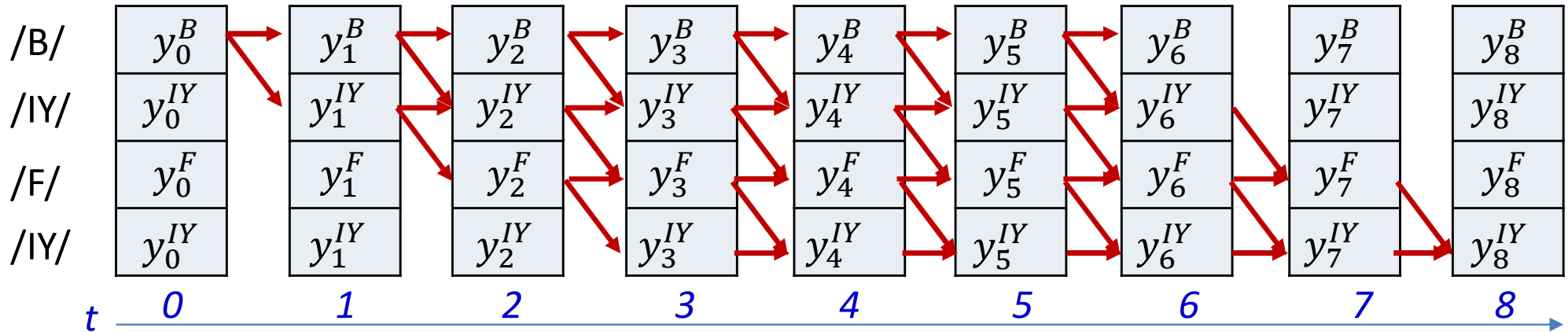
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# The expectation over *all* alignments



$$DIV = E \left[ - \sum_t \log Y(t, s_t) \right]$$

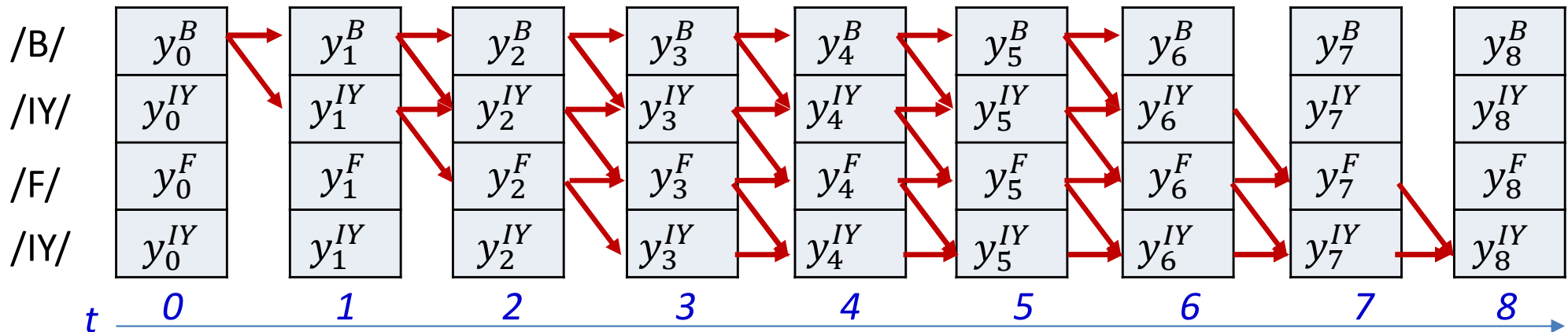
- Using the linearity of expectation

$$DIV = - \sum_t E[\log Y(t, s_t)]$$

- This reduces to finding the expected divergence *at each input*

$$DIV = - \sum_t \sum_{S \in S_1 \dots S_K} P(s_t = S | \mathbf{S}, \mathbf{X}) \log Y(t, s_t = S)$$

# The expectation over *all* alignments



The probability of aligning the specific symbol  $s$  at time  $t$ , given that unaligned sequence  $\mathbf{S} = S_0 \dots S_{K-1}$  and given the input sequence  $\mathbf{X} = X_0 \dots X_{N-1}$

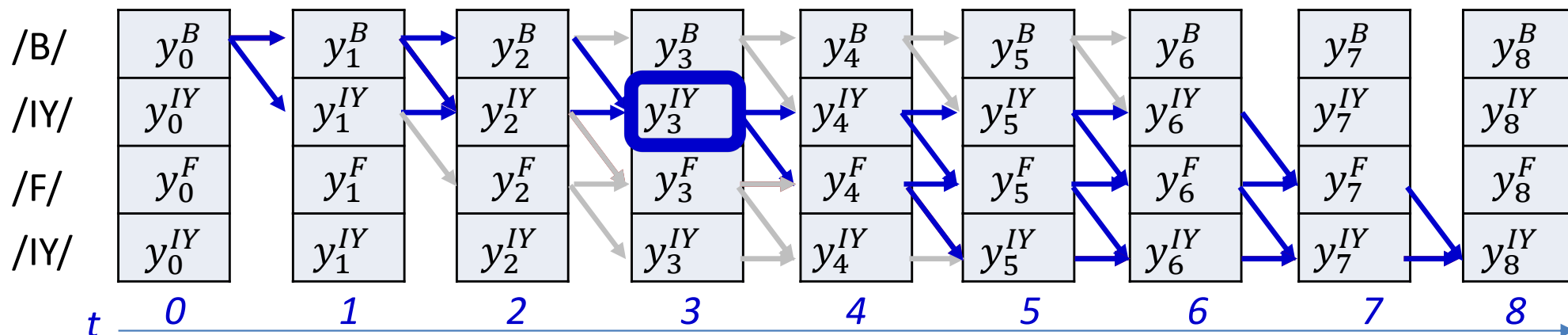
- We need to be able to compute this

$$DIV = - \sum_t E[\log Y(t, s_t)]$$

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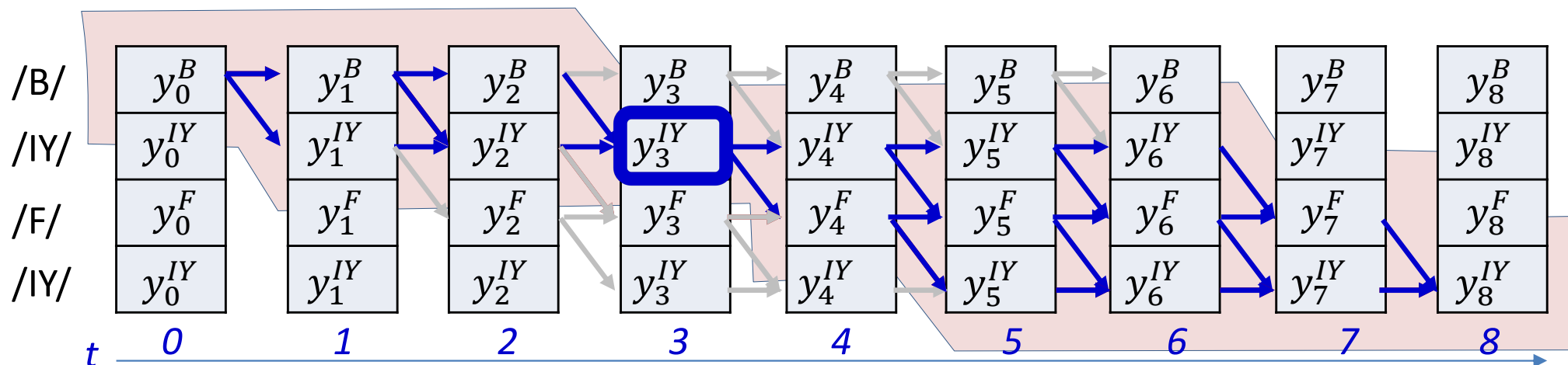
# A posteriori probabilities of symbols



$$P(s_t = S_r | \mathbf{S}, \mathbf{X}) \propto P(s_t = S_r, \mathbf{S} | \mathbf{X})$$

- $P(s_t = S_r, \mathbf{S} | \mathbf{X})$  is the total probability of all valid paths *in the graph for target sequence  $\mathbf{S}$*  that go through the symbol  $S_r$  (the  $r^{\text{th}}$  symbol in the sequence  $S_0 \dots S_{K-1}$ ) at time  $t$
- We will compute this using the “forward-backward” algorithm

# A posteriori probabilities of symbols

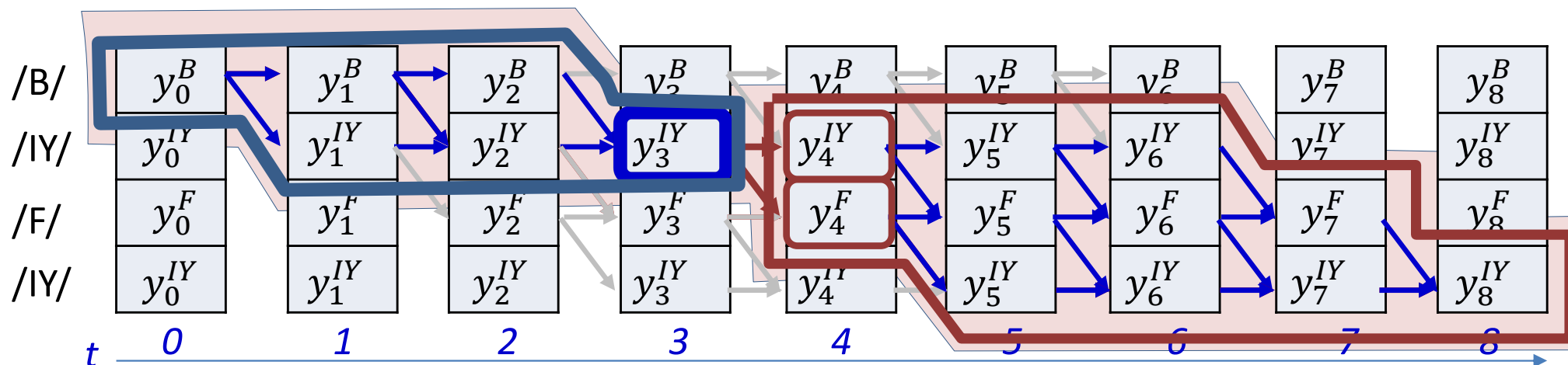


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# A posteriori probabilities of symbols



- $P(s_t = S_r, \mathbf{S}|\mathbf{X})$  can be decomposed as

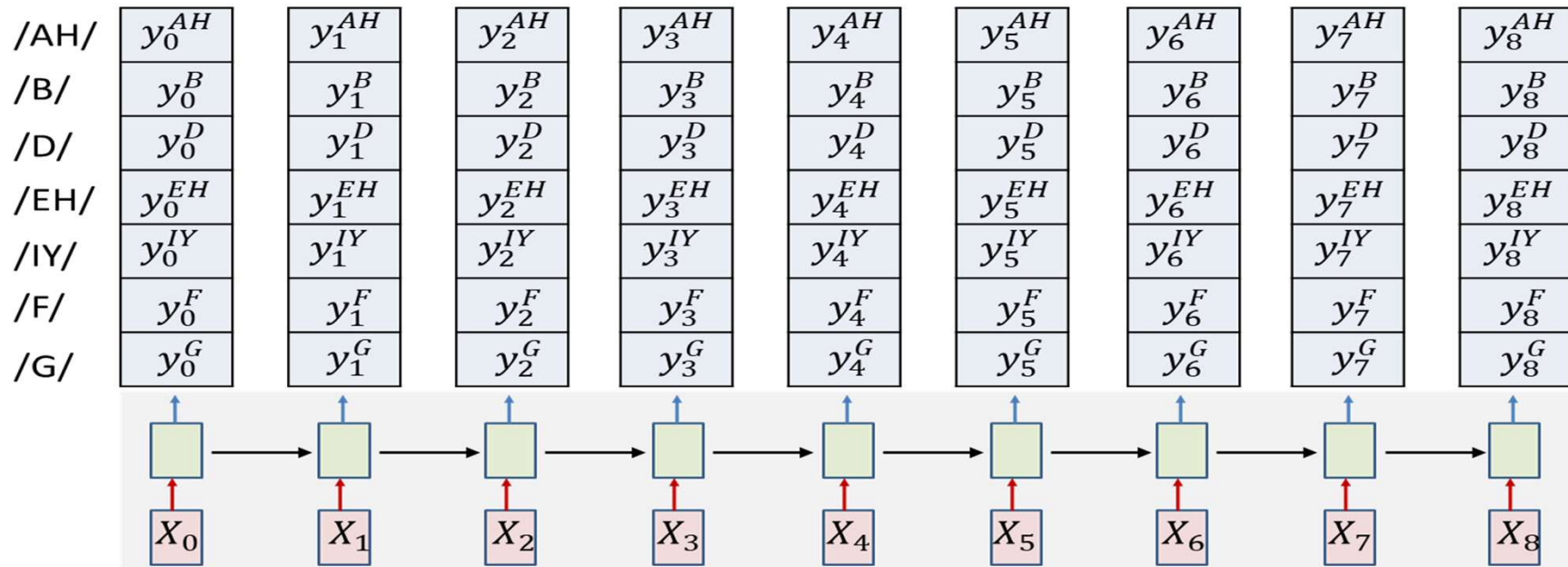
$$\begin{aligned}
 P(s_t = S_r, \mathbf{S}|\mathbf{X}) &= P(S_0, \dots, S_r, \dots, S_{K-1}, s_t = S_r | \mathbf{X}) \\
 &= P(S_0 \dots S_r, s_t = S_r, s_{t+1} \in \text{succ}(S_r), \text{succ}(S_r), \dots, S_{K-1} | \mathbf{X})
 \end{aligned}$$

- Using Bayes Rule

$$= P(S_0 \dots S_r, s_t = S_r | \mathbf{X}) P(s_{t+1} \in \text{succ}(S_r), \text{succ}(S_r), \dots, S_{K-1} | S_0 \dots S_r, s_t = S_r \mathbf{X})$$

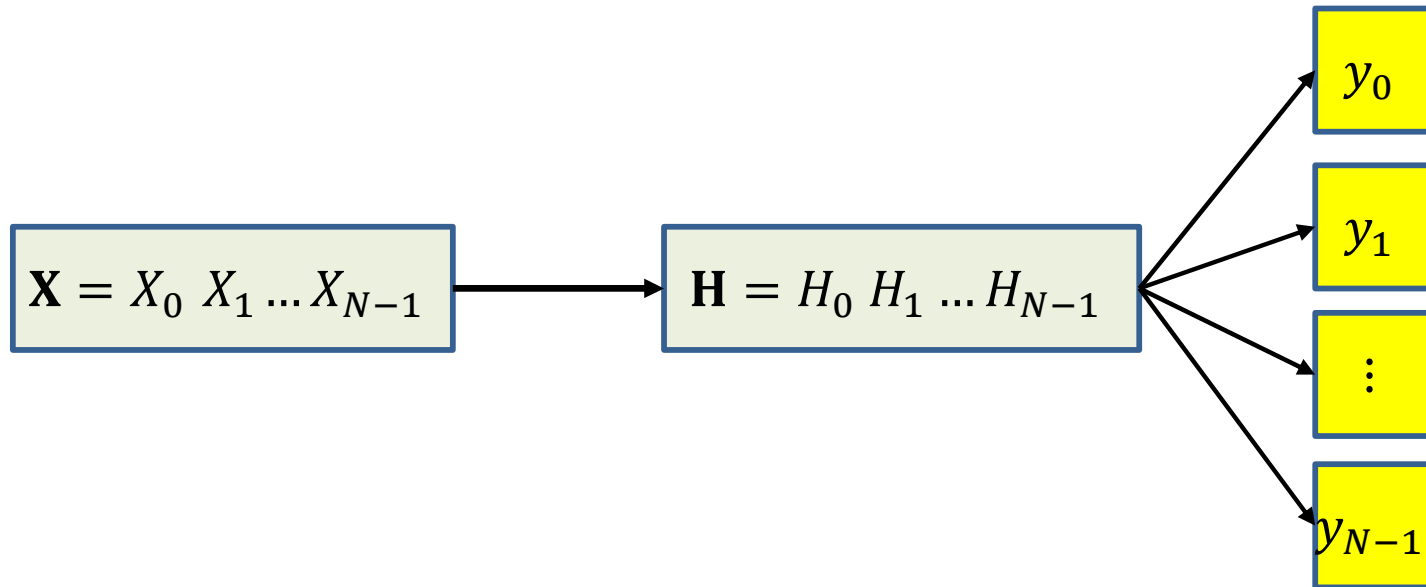
- The probability of the subgraph in the blue outline, times the conditional probability of the red-encircled subgraph, given the blue subgraph

# Conditional independence



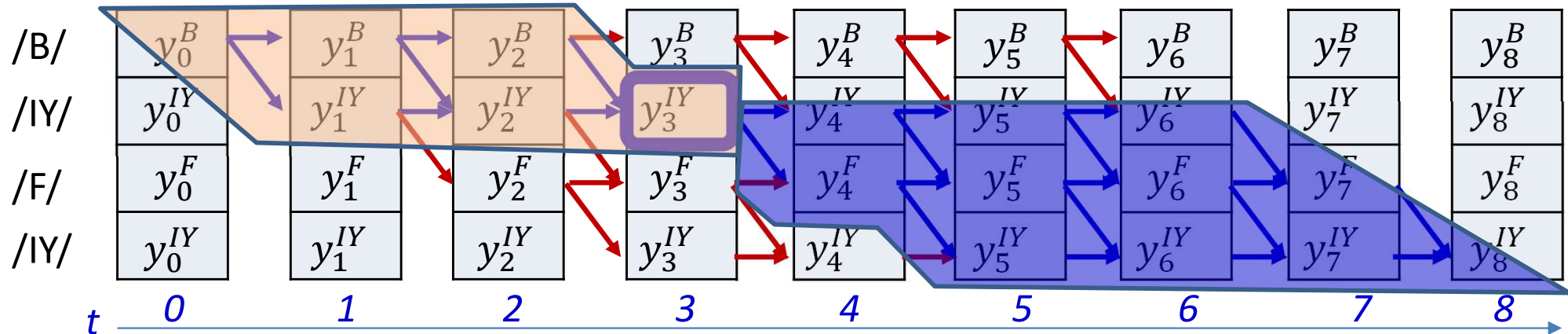
- The vector of probabilities at any time depends *only* on the hidden state, and not the probability vector at other times
  - I.e. the probabilities of the symbols at time  $t$  do not directly influence the probabilities at  $t + 1$  or any other time
  - If we were to *draw* a sample symbol from the probability distribution at time  $t$ , exactly which symbol we drew at  $t$  has no influence on the symbol probabilities at other times
- Given the input the symbol probabilities at time  $t$  are independent of symbol probabilities at  $t' \neq t$ 
  - The two are conditionally independent given the input

# Conditional independence



- **Dependency graph:** Input sequence  $\mathbf{X} = X_0 X_1 \dots X_{N-1}$  governs hidden variables  $\mathbf{H} = H_0 H_1 \dots H_{N-1}$
- Hidden variables govern output predictions  $y_0, y_1, \dots, y_{N-1}$  individually
- $y_0, y_1, \dots, y_{N-1}$  are conditionally independent given  $\mathbf{H}$
- Since  $\mathbf{H}$  is deterministically derived from  $\mathbf{X}$ ,  $y_0, y_1, \dots, y_{N-1}$  are also conditionally independent given  $\mathbf{X}$ 
  - This wouldn't be true if past  $y$ s affected future  $y$ s, i.e. if the  $y$ s at any time went back into the net as inputs

# A posteriori symbol probability

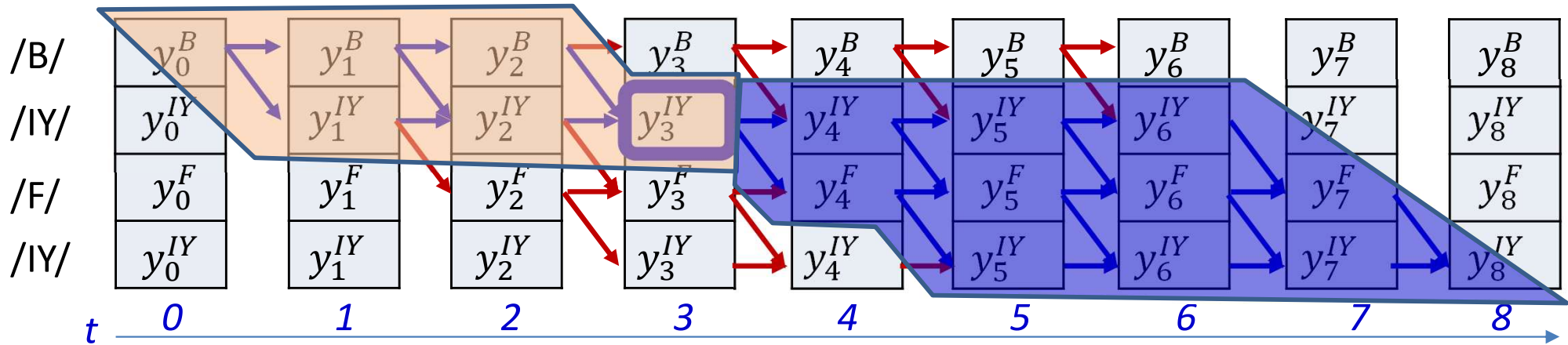


$$P(s_t = S_r, \mathbf{S} | \mathbf{X})$$

$$= \underbrace{P(S_0 \dots S_r, s_t = S_r | \mathbf{X})}_{\text{forward probability}} \underbrace{P(s_{t+1} \in \text{succ}(S_r), \text{succ}(S_r), \dots, S_{K-1} | \mathbf{X})}_{\text{backward probability}}$$

- We will call the first term the *forward probability*  $\alpha(t, r)$
- We will call the second term the *backward probability*  $\beta(t, r)$

# A posteriori symbol probability

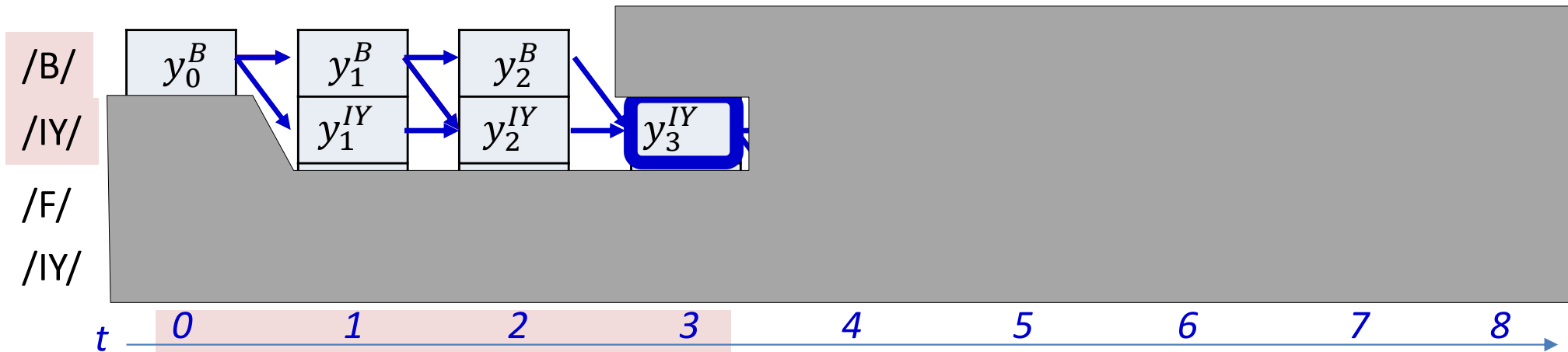


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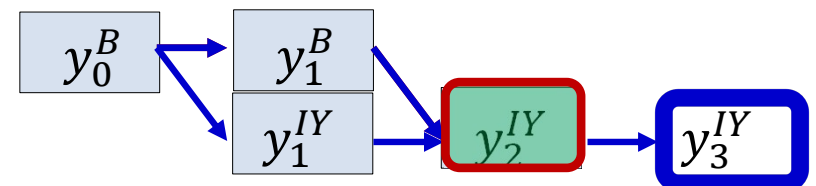
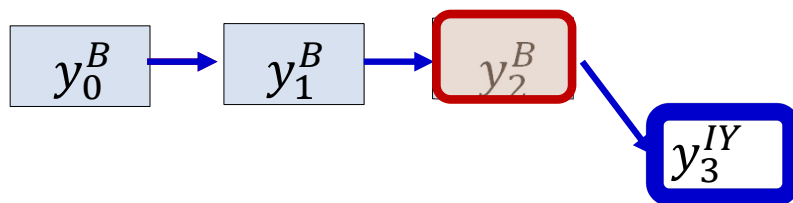
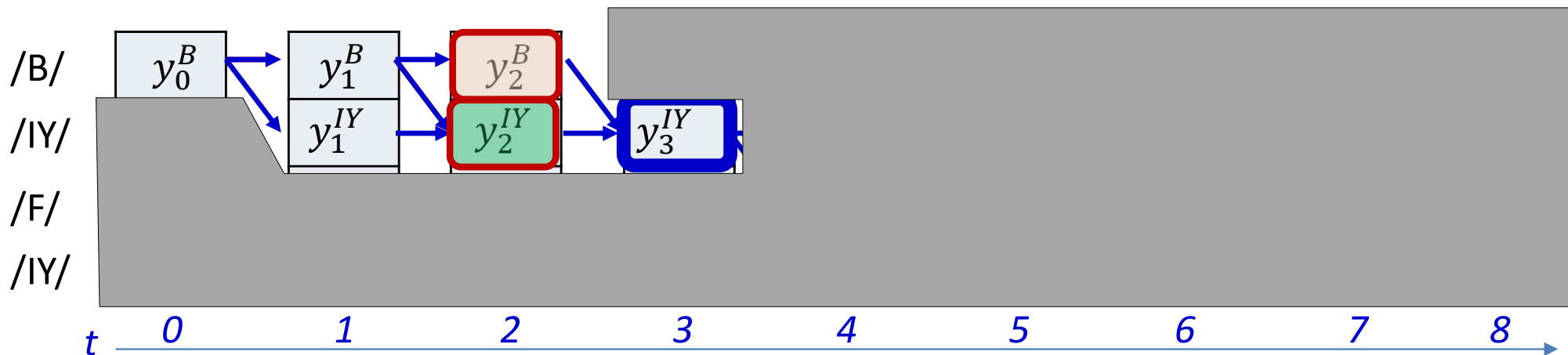
# Computing $\alpha(t, r)$ : Forward algorithm



$$\alpha(t, r) = P(S_0 \dots S_r, s_t = S_r | \mathbf{X})$$

- The  $\alpha(t, r)$  is the total probability of the subgraph shown
  - The total probability of all paths leading to the alignment of  $S_r$  to time  $t$

# Computing $\alpha(t, r)$ : Forward algorithm



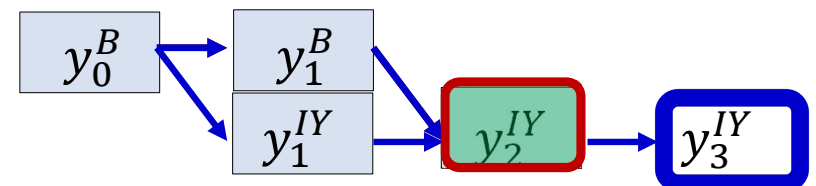
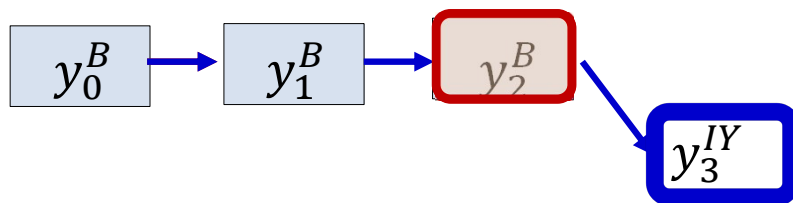
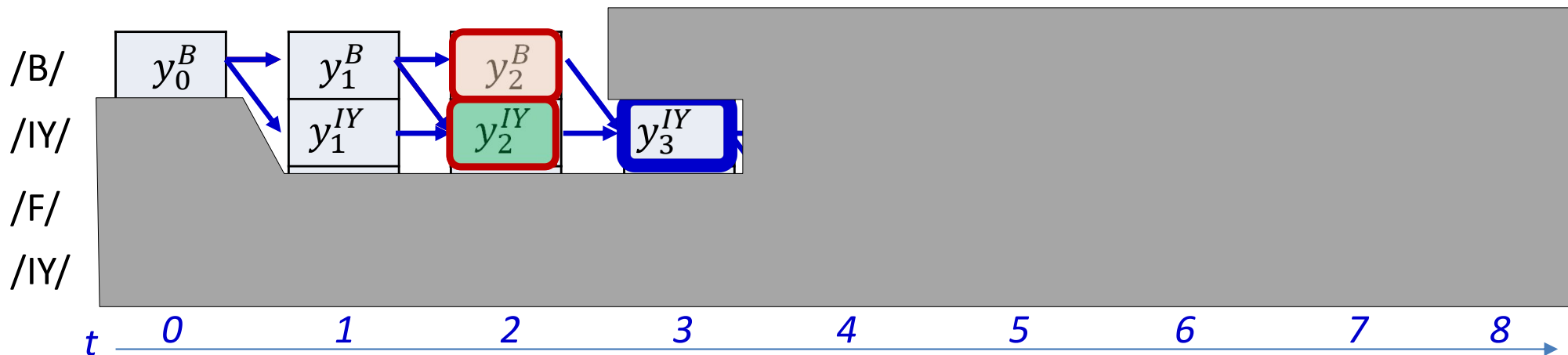
$$\alpha(3, IY) = P(S_0 \dots S_r, s_t = S_r | \mathbf{X})$$

$$\alpha(3, IY) = P(\text{subgraph ending at } (2, B))y_3^{IY} + P(\text{subgraph ending at } (2, IY))y_3^{IY}$$

$$\alpha(t, r) = \sum_{q: S_q \in \text{pred}(S_r)} P(\text{subgraph ending at } (t-1, q))Y_t^{S(r)}$$

- Where  $\text{pred}(S_r)$  is any symbol that is permitted to come before an  $S_r$  and may include  $S_r$
- $q$  is its row index, and can take values  $r$  and  $r - 1$  in this example

# Computing $\alpha(t, r)$ : Forward algorithm



$$\alpha(t, r) = P(S_0 \dots S_r, s_t = S_r | \mathbf{X})$$

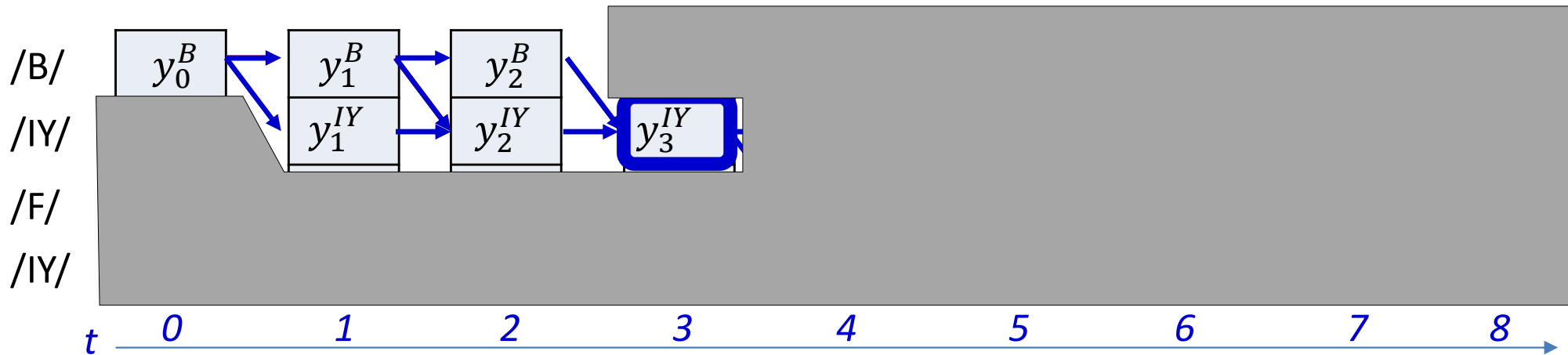
$$\alpha(3, IY) = \alpha(2, B)y_3^{IY} + \alpha(2, IY)y_3^{IY}$$

$$\alpha(t, r) = \sum_{q: S_q \in \text{pred}(S_r)} \alpha(t-1, q) Y_t^{S(r)}$$

- Where  $\text{pred}(S_r)$  is any symbol that is permitted to come before an  $S_r$  and may include  $S_r$
- $q$  is its row index, and can take values  $r$  and  $r - 1$  in this example



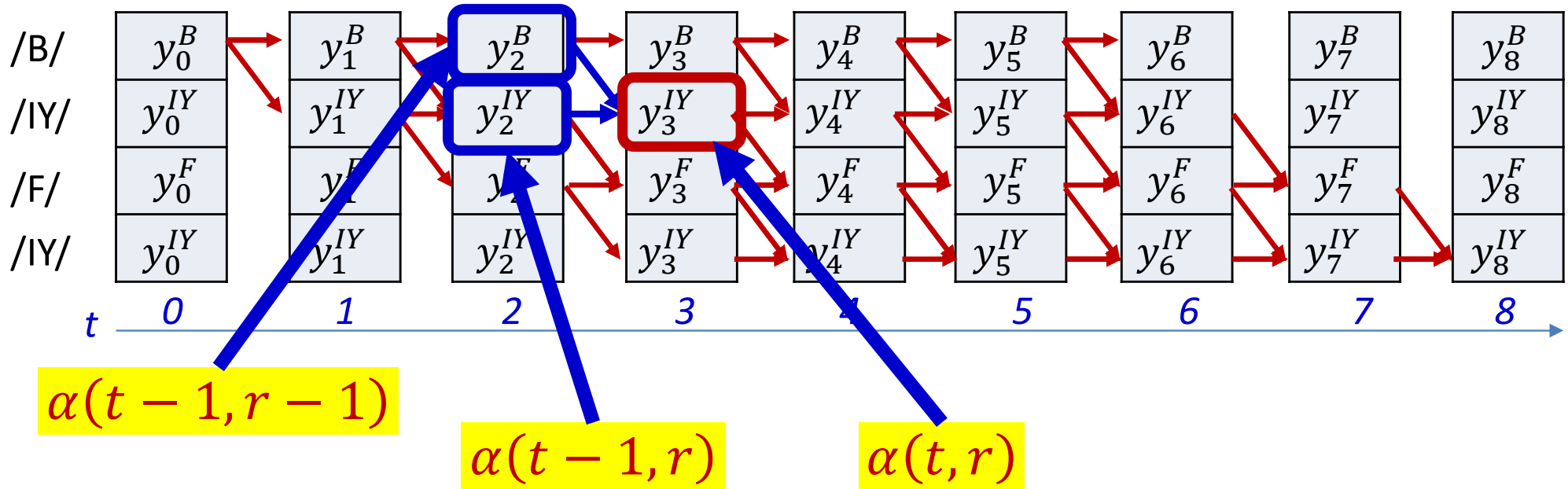
# Forward algorithm



$$\alpha(t, r) = \sum_{q: S_q \in \text{pred}(S_r)} \alpha(t-1, q) y_t^{S_r}$$

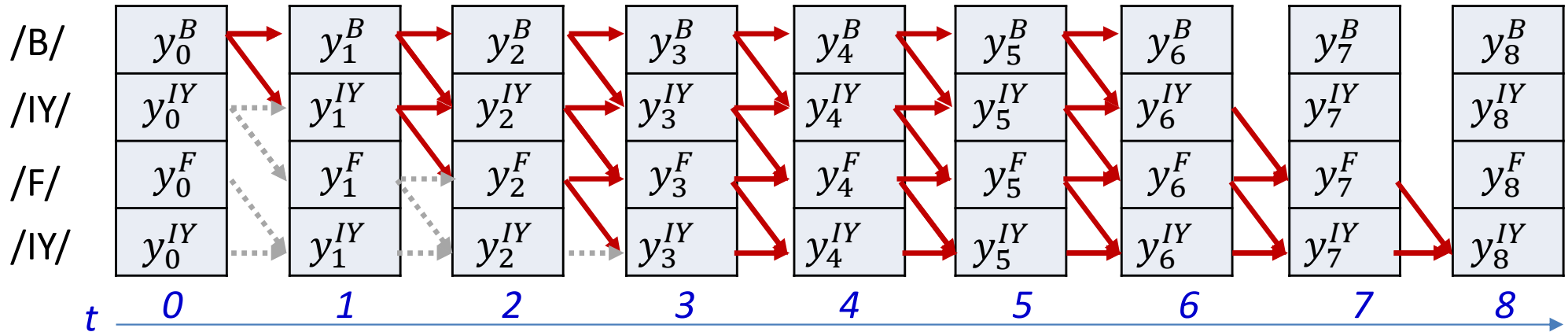
- The  $\alpha(t, r)$  is the total probability of the subgraph shown

# Forward algorithm



$$\alpha(t, r) = (\alpha(t-1, r) + \alpha(t-1, r-1))y_t^{S(r)}$$

# Forward algorithm



- Initialization:

$$\alpha(0,0) = y_0^{S(0)}, \quad \alpha(0,r) = 0, \quad r > 0$$

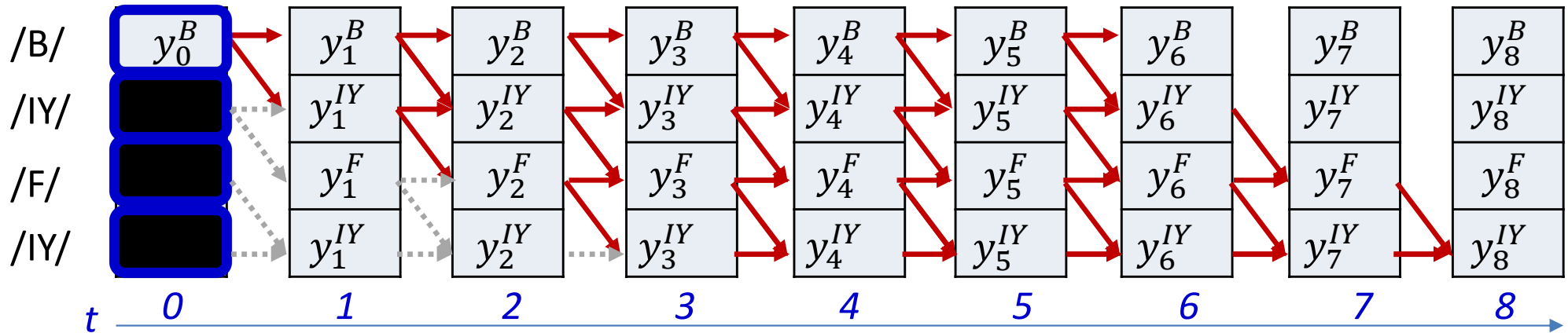
- for  $t = 1 \dots T - 1$

$$\alpha(t,0) = \alpha(t-1,0)y_t^{S(0)}$$

for  $l = 1 \dots K - 1$

$$\alpha(t,l) = (\alpha(t-1,l) + \alpha(t-1,l-1))y_t^{S(l)}$$

# Forward algorithm



- Initialization:

$$\alpha(0,0) = y_0^{S(0)}, \quad \alpha(0,r) = 0, \quad r > 0 \quad \leftarrow$$

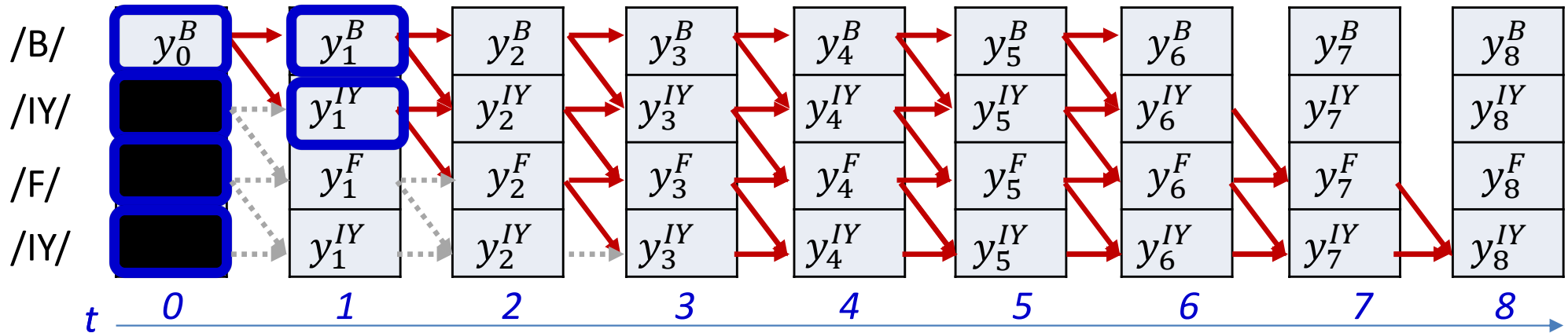
- for  $t = 1 \dots T - 1$

$$\alpha(t,0) = \alpha(t-1,0)y_t^{S(0)}$$

for  $l = 1 \dots K - 1$

- $\alpha(t,l) = (\alpha(t-1,l) + \alpha(t-1,l-1))y_t^{S(l)}$

# Forward algorithm



- Initialization:

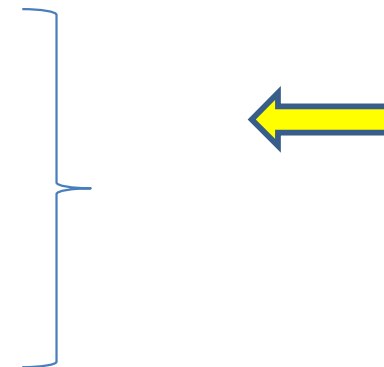
$$\alpha(0,0) = y_0^{S(0)}, \quad \alpha(0,r) = 0, \quad r > 0$$

- for  $t = 1 \dots T - 1$

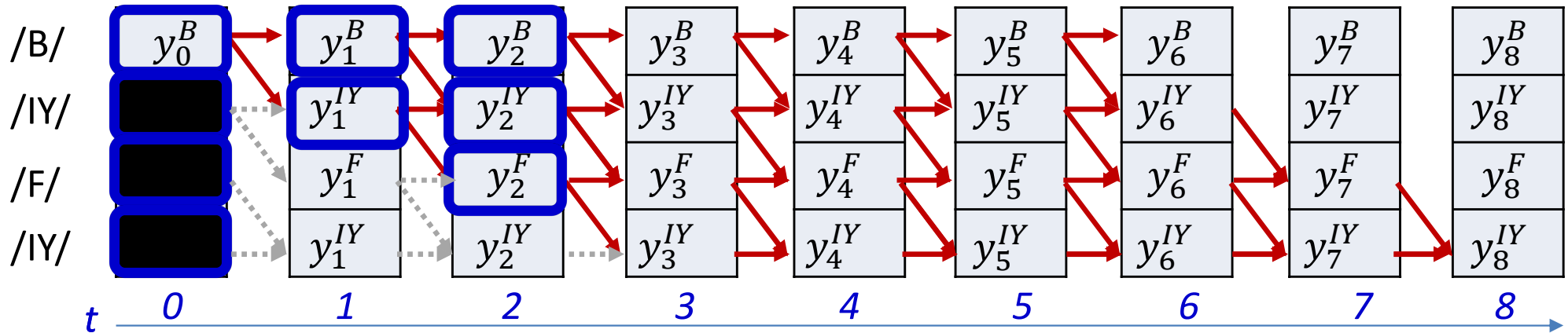
$$\alpha(t,0) = \alpha(t-1,0)y_t^{S(0)}$$

for  $l = 1 \dots K - 1$

- $\alpha(t,l) = (\alpha(t-1,l) + \alpha(t-1,l-1))y_t^{S(l)}$



# Forward algorithm



- Initialization:

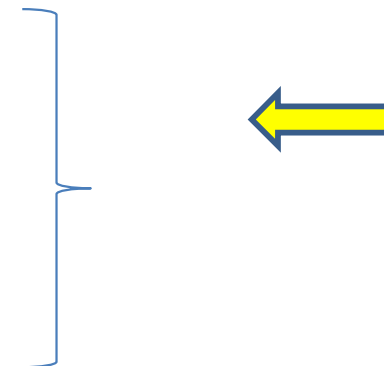
$$\alpha(0,0) = y_0^{S(0)}, \quad \alpha(0,r) = 0, \quad r > 0$$

- for  $t = 1 \dots T - 1$

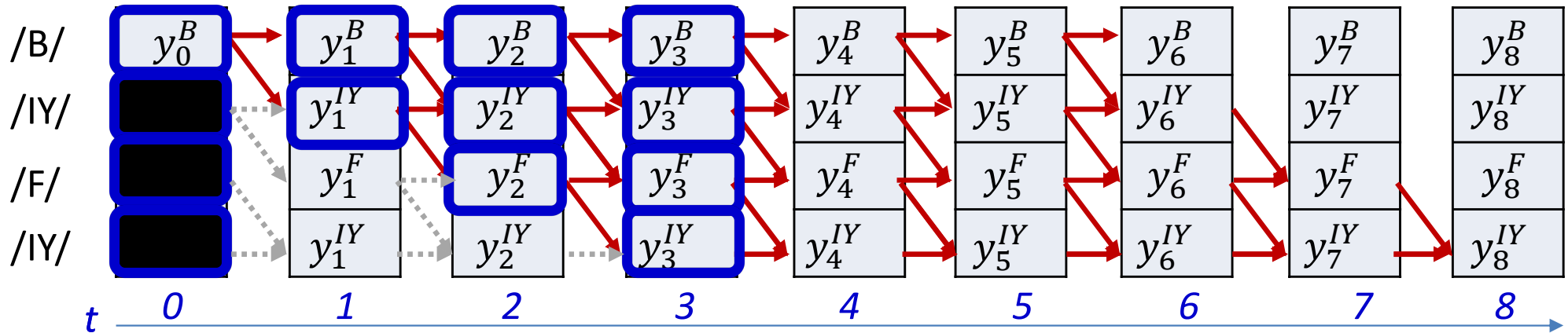
$$\alpha(t,0) = \alpha(t-1,0)y_t^{S(0)}$$

for  $l = 1 \dots K - 1$

- $\alpha(t,l) = (\alpha(t-1,l) + \alpha(t-1,l-1))y_t^{S(l)}$



# Forward algorithm



- Initialization:

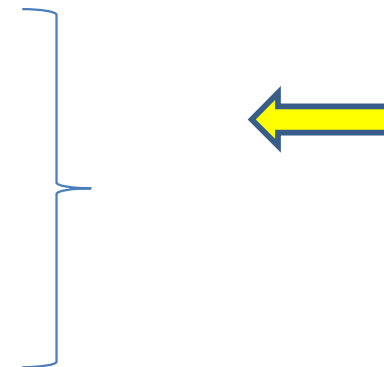
$$\alpha(0,0) = y_0^{S(0)}, \quad \alpha(0,r) = 0, \quad r > 0$$

- for  $t = 1 \dots T - 1$

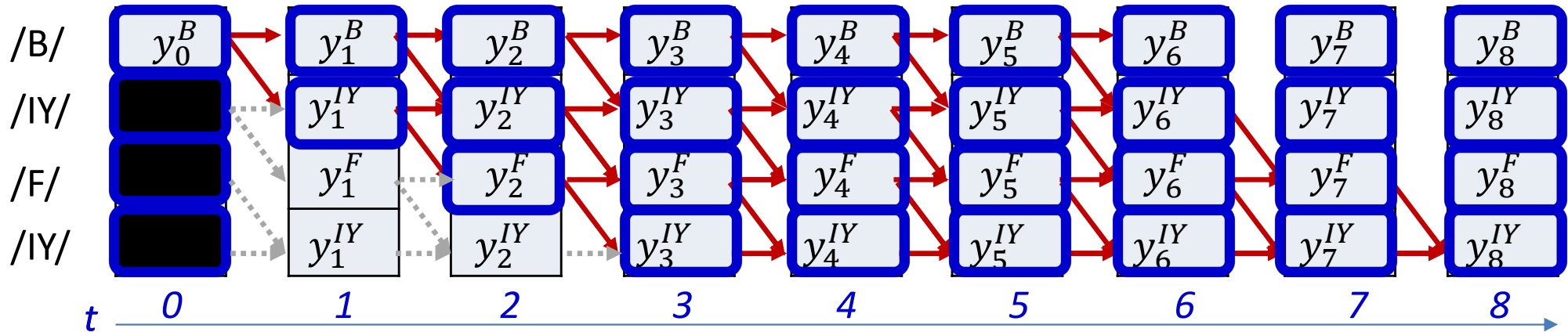
$$\alpha(t,0) = \alpha(t-1,0)y_t^{S(0)}$$

for  $l = 1 \dots K - 1$

- $\alpha(t,l) = (\alpha(t-1,l) + \alpha(t-1,l-1))y_t^{S(l)}$



# Forward algorithm



- Initialization:

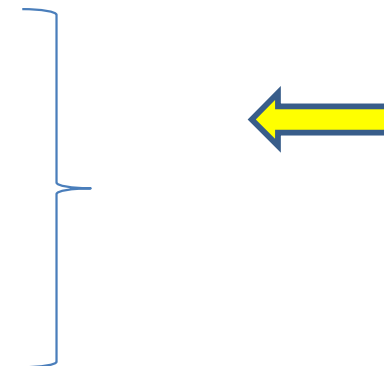
$$\alpha(0,0) = y_0^{S(0)}, \quad \alpha(0,r) = 0, \quad r > 0$$

- for  $t = 1 \dots T - 1$

$$\alpha(t,0) = \alpha(t-1,0)y_t^{S(0)}$$

for  $l = 1 \dots K - 1$

- $\alpha(t,l) = (\alpha(t-1,l) + \alpha(t-1,l-1))y_t^{S(l)}$





# In practice..

- The recursion

$$\alpha(t, l) = (\alpha(t - 1, l) + \alpha(t - 1, l - 1))y_t^{S(l)}$$

will generally underflow

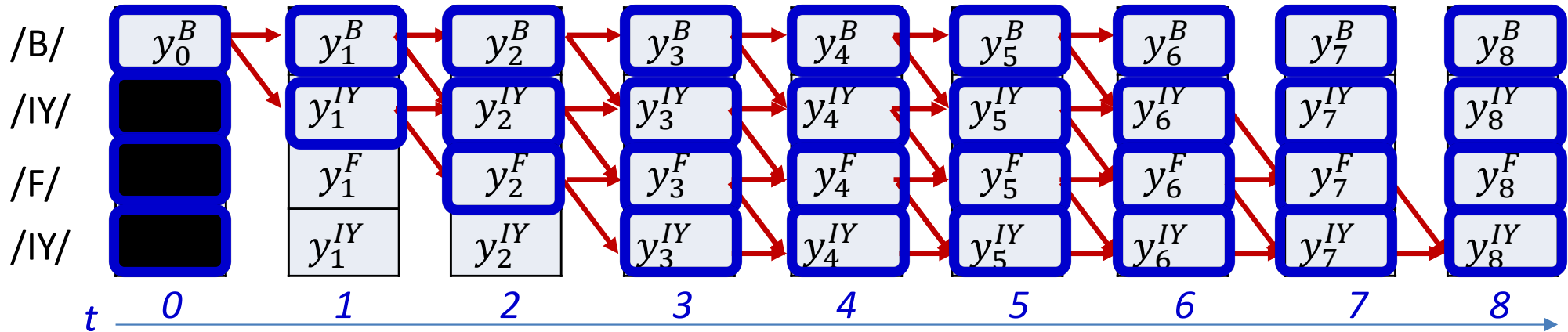
- Instead we can do it in the *log* domain

$$\log \alpha(t, l)$$

$$= \log(e^{\log \alpha(t-1, l)} + e^{\log \alpha(t-1, l-1)}) + \log y_t^{S(l)}$$

– This can be computed entirely without underflow

# Forward algorithm: Alternate statement



- The algorithm can also be stated as follows which separates the graph probability from the observation probability. This is needed to compute derivatives
- Initialization:

$$\hat{\alpha}(0,0) = 1, \quad \hat{\alpha}(0,r) = 0, \quad r > 0$$

$$\alpha(0,r) = \hat{\alpha}(0,r)y_0^{S(r)}, \quad 0 \leq r \leq K - 1$$

- for  $t = 1 \dots T - 1$

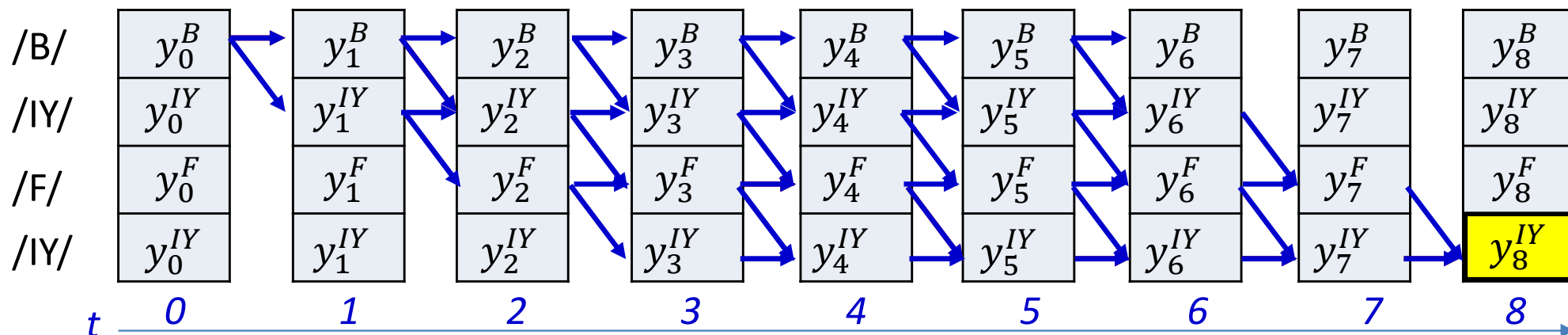
$$\hat{\alpha}(t,0) = \alpha(t-1,0)$$

for  $l = 1 \dots K - 1$

- $\hat{\alpha}(t,l) = \alpha(t-1,l) + \alpha(t-1,l-1)$

$$\alpha(t,r) = \hat{\alpha}(t,r)y_t^{S(r)}, \quad 0 \leq r \leq K - 1$$

# The final forward probability $\alpha(t, r)$



$$\alpha(T - 1, K - 1) = P(S_0 \dots S_{K-1} | \mathbf{X})$$

- The probability of the entire symbol sequence is the alpha at the bottom right node

# SIMPLE FORWARD ALGORITHM

#N is the number of symbols in the target output

#S(i) is the ith symbol in target output

#y(t,i) is the output of the network for the ith symbol at time t

#T = length of input

**#First create output table**

```
For i = 1:N
    s(1:T,i) = y(1:T, S(i))
```

**#The forward recursion**

# First, at t = 1

```
alpha(1,1) = s(1,1)
```

```
alpha(1,2:N) = 0
```

```
for t = 2:T
```

```
    alpha(t,1) = alpha(t-1,1)*s(t,1)
```

```
    for i = 2:N
```

```
        alpha(t,i) = alpha(t-1,i-1) + alpha(t-1,i)
```

```
        alpha(t,i) *= s(t,i)
```

Can actually be done without explicitly composing the output table

Using 1..N and 1..T indexing, instead of 0..N-1, 0..T-1, for convenience of notation

# SIMPLE FORWARD ALGORITHM

#N is the number of symbols in the target output

#S(i) is the ith symbol in target output

#y(t,i) is the network output for the ith symbol at time t

#T = length of input

## #The forward recursion

# First, at  $t = 1$

$\alpha(1,1) = y(1,S(1))$

$\alpha(1,2:N) = 0$

for  $t = 2:T$

$\alpha(t,1) = \alpha(t-1,1) * y(t,S(1))$

    for  $i = 2:N$

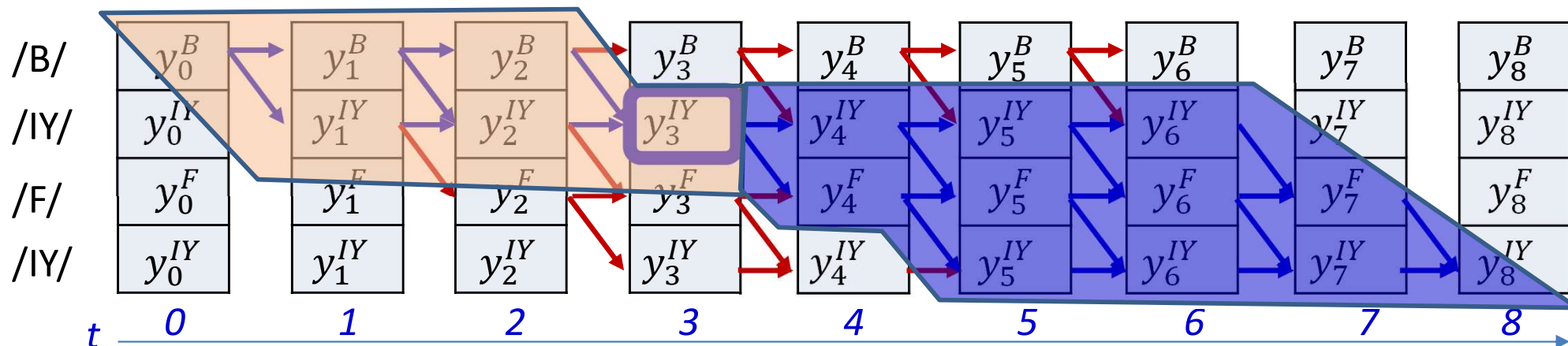
$\alpha(t,i) = \alpha(t-1,i-1) + \alpha(t-1,i)$

$\alpha(t,i) *= y(t,S(i))$

Without explicitly composing the output table

Using 1..N and 1..T indexing, instead of 0..N-1, 0..T-1, for convenience of notation

# A posteriori symbol probability



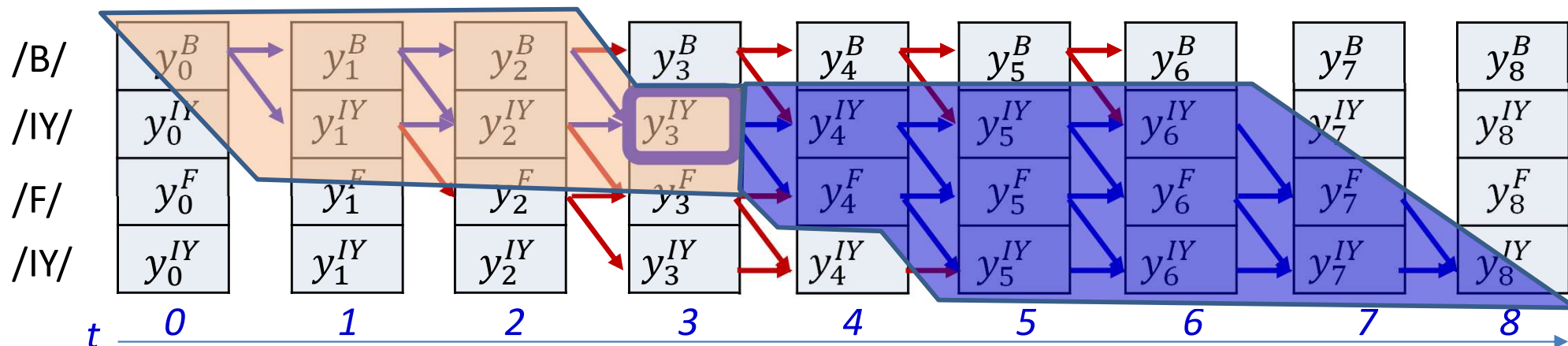
$$P(s_t = S_r, \mathbf{S} | \mathbf{X})$$

$$= \underbrace{P(S_0 \dots S_r, s_t = S_r | \mathbf{X})}_{\text{forward probability}} \underbrace{P(s_{t+1} \in \text{succ}(S_r), \text{succ}(S_r), \dots, S_{K-1} | \mathbf{X})}_{\text{backward probability}}$$

- We will call the first term the *forward probability*  $\alpha(t, r)$
- We will call the second term the *backward probability*  $\beta(t, r)$

We have seen how to compute this

# A posteriori symbol probability

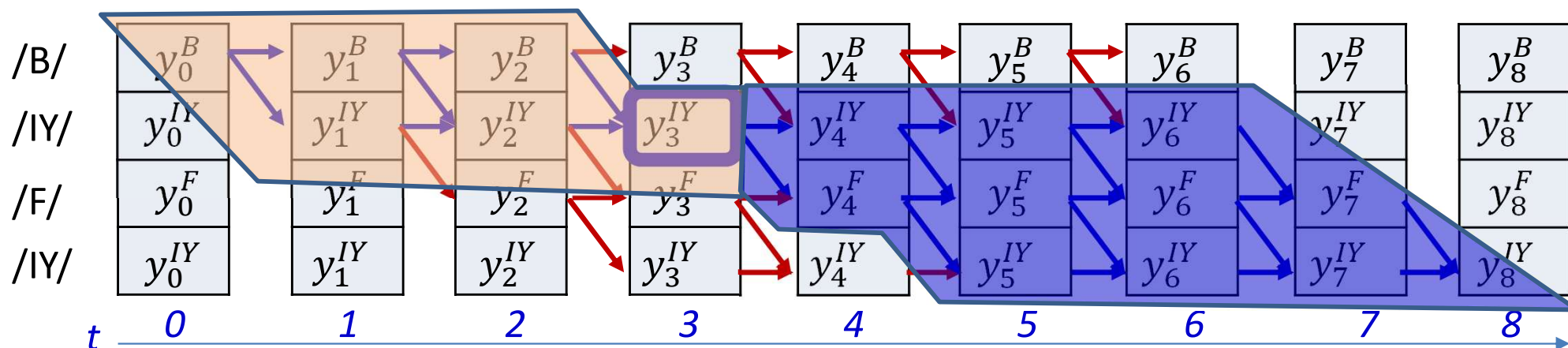


$$P(s_t = S_r, \mathbf{S} | \mathbf{X}) = \alpha(t, r) P(s_{t+1} \in \text{succ}(S_r), \text{succ}(S_r), \dots, S_{K-1} | \mathbf{X})$$

- We will call the first term the *forward probability*  $\alpha(t, r)$
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We have seen how to compute this

# A posteriori symbol probability



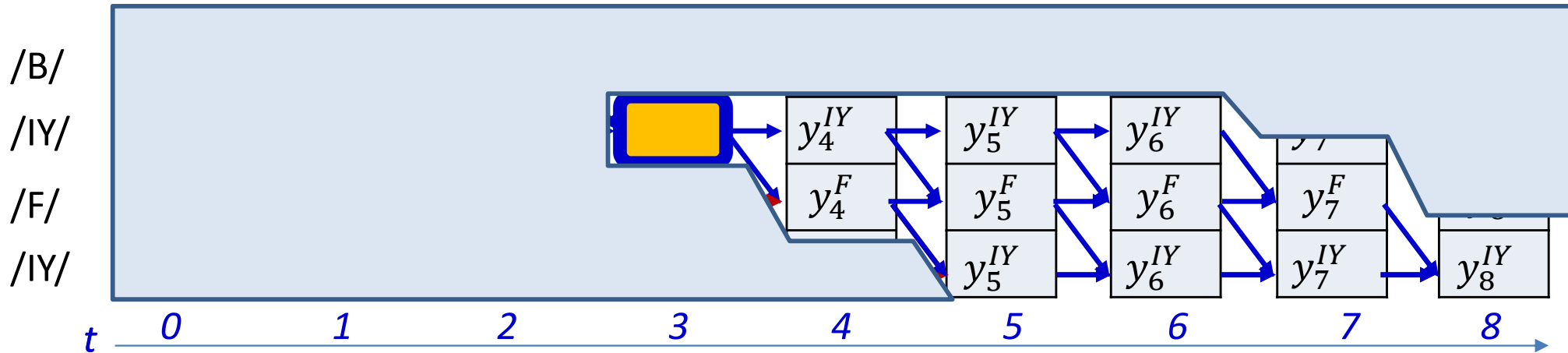
$$P(s_t = S_r, \mathbf{S} | \mathbf{X}) = \alpha(t, r) P(s_{t+1} \in \text{succ}(S_r), \text{succ}(S_r), \dots, S_{K-1} | \mathbf{X})$$

- We will call the first term the *forward probability*  $\alpha(t, r)$
- We will call the second term the *backward probability*  $\beta(t, r)$

Lets look at this

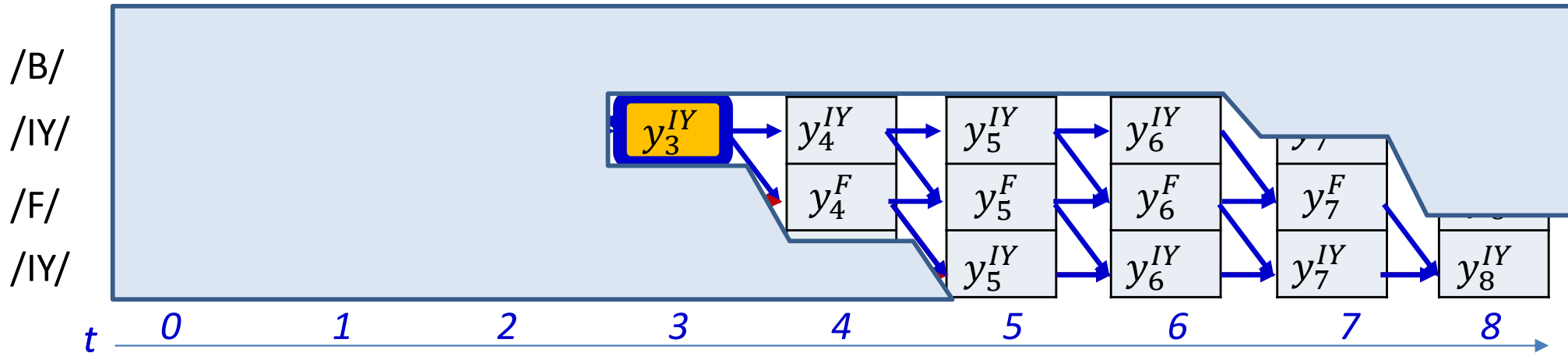


# Backward probability



- $\beta(t, r)$  is the probability of the exposed subgraph, not including the orange shaded box

# Backward probability



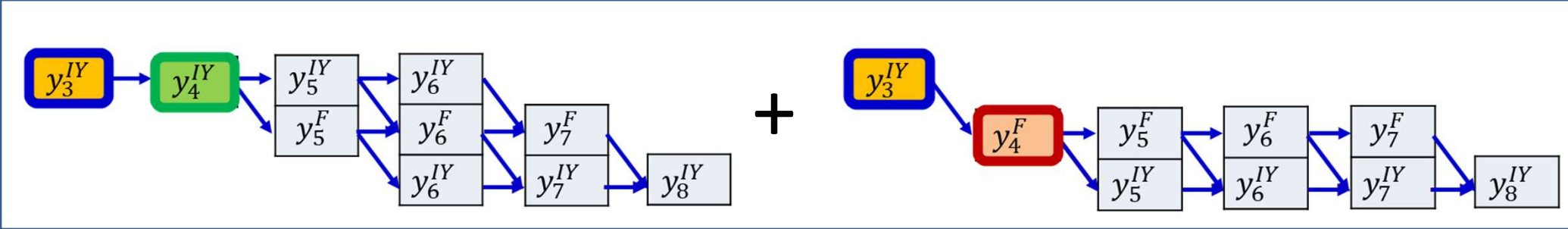
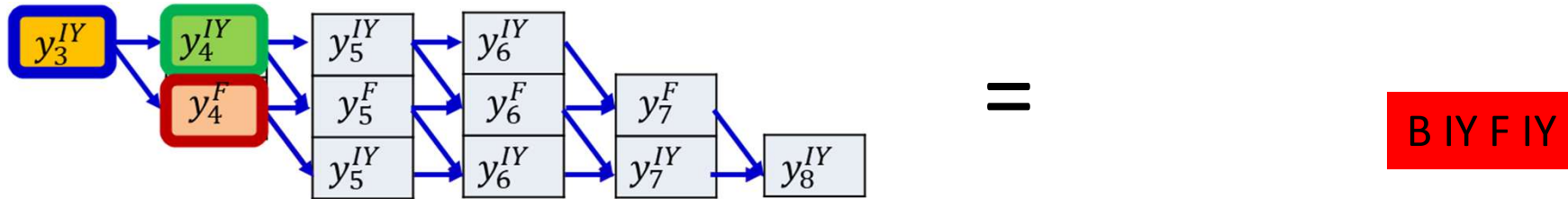
- $\beta(t, r)$  is the probability of the exposed subgraph, not including the orange shaded box
- For convenience, let us include the box in the graph, and factor it out later

$\hat{\beta}(t, r)$  = probability of graph including node at  $(t, r)$

$$\beta(t, r) = \frac{1}{y_t^{S_r}} \hat{\beta}(t, r)$$

- We will develop an algorithm to compute  $\hat{\beta}(t, r)$  and compute  $\beta(t, r)$  from it by dividing out  $y_t^{S_r}$  later

# Backward probability

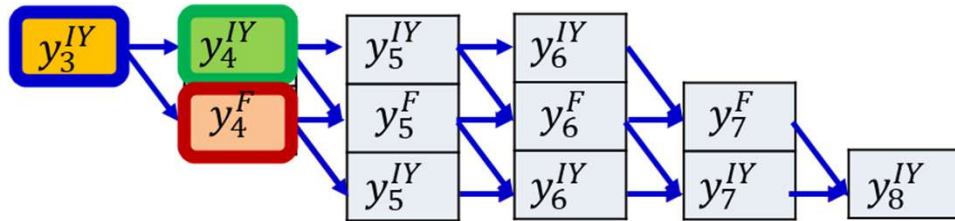


- Using the same logic as in the forward algorithm:

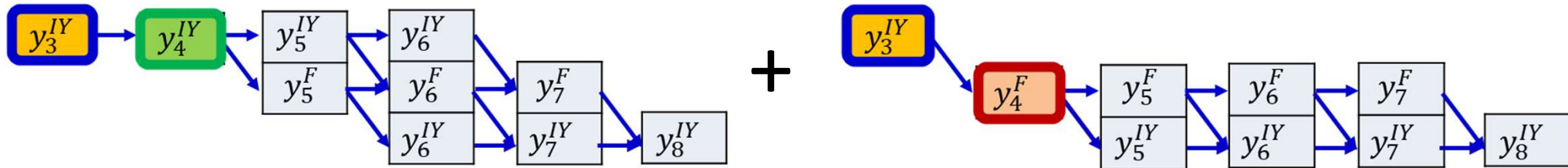
$$\hat{\beta}(3, IY)$$

$$= y_3^{IY} P(\text{subgraph starting at } (4, IY)) + y_3^{IY} P(\text{subgraph starting at } (4, F))$$

# Backward probability



=



- Using the same logic as in the forward algorithm:

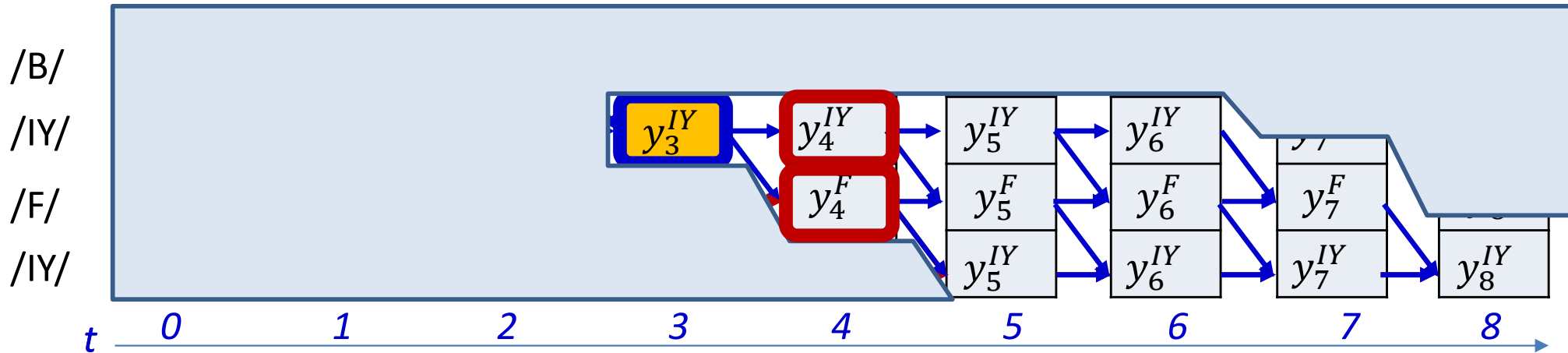
$$\hat{\beta}(3, IY)$$

$$= y_3^{IY} P(\text{subgraph starting at } (4, IY)) + y_3^{IY} P(\text{subgraph starting at } (4, F))$$

- We recognize these terms:

$$\hat{\beta}(3, IY) = y_3^{IY} \left( \hat{\beta}(4, IY) + \hat{\beta}(4, F) \right)$$

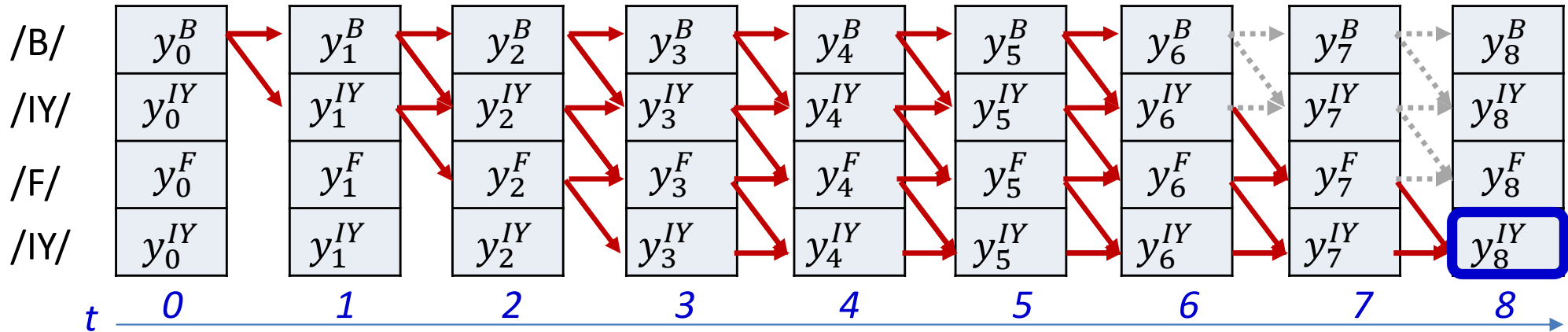
# Backward algorithm



$$\hat{\beta}(t, r) = y_t^{S_r} \sum_{q \in \text{succ}(r)} \hat{\beta}(t+1, q)$$

- The  $\hat{\beta}(t, r)$  is the total probability of the subgraph shown
  - Including the node at (t, r)
- The  $\hat{\beta}(t, r)$  terms at any time  $t$  are defined recursively in terms of the  $\hat{\beta}(t+1, q)$  terms at the next time

# Backward algorithm



- Entire backward algorithm:
  - Note : some nodes (bottom row) have more successors than others

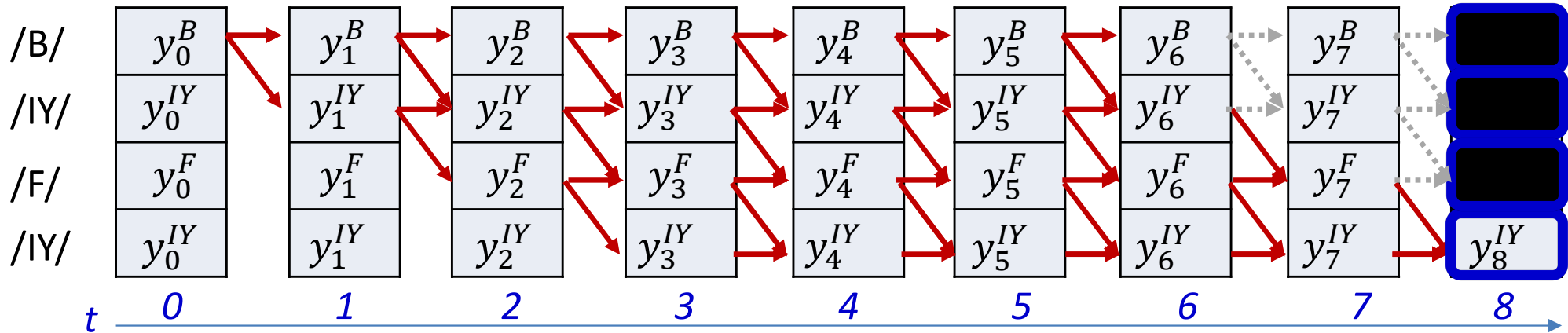
- Initialization:

$$\hat{\beta}(T-1, K-1) = y_{T-1}^{S(K-1)}, \quad \hat{\beta}(T-1, r) = 0, \quad r < K-1$$

- for  $t = T-2$  downto 0  
for  $r = K-1 \dots 0$

$$\hat{\beta}(t, r) = y_t^{S(r)} \sum_{q \in \text{succ}(r)} \hat{\beta}(t+1, q)$$

# Backward algorithm



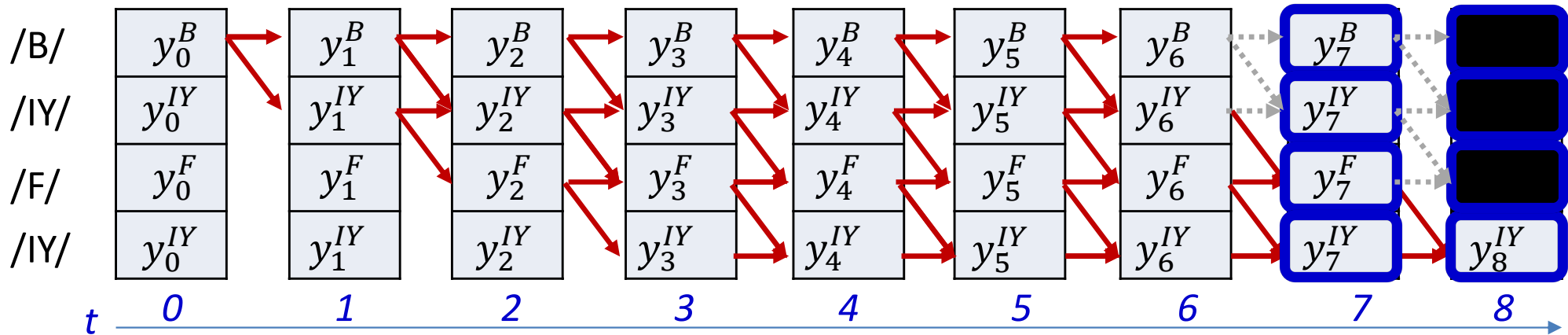
- Initialization:

$$\hat{\beta}(T-1, K-1) = y_{T-1}^{S(K-1)}, \quad \hat{\beta}(T-1, r) = 0, \quad r < K-1$$

- for  $t = T-2$  downto 0  
for  $r = K-1 \dots 0$

$$\hat{\beta}(t, r) = y_t^{S(r)} \sum_{q \in \text{succ}(r)} \hat{\beta}(t+1, q)$$

# Backward algorithm



- Initialization:

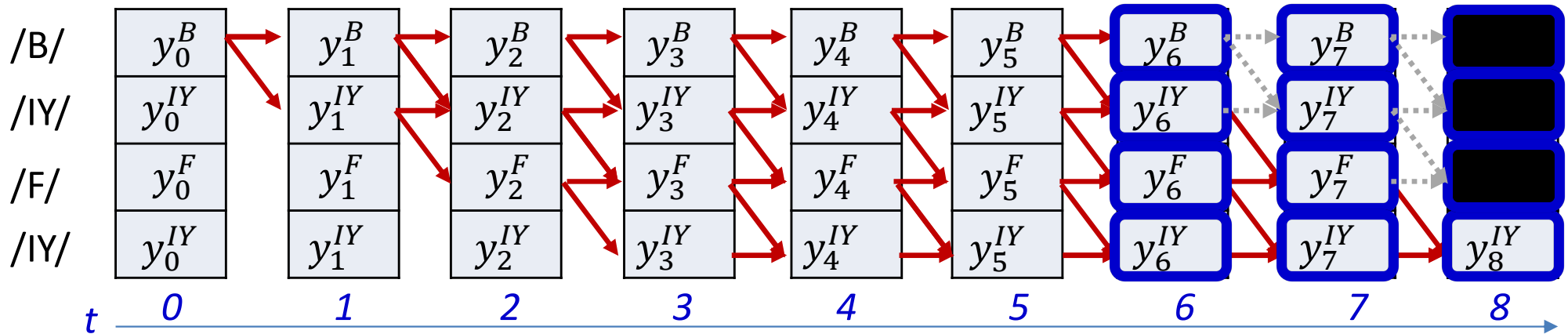
$$\hat{\beta}(T-1, K-1) = y_{t+1}^{S(K-1)}, \hat{\beta}(T-1, r) = 0, r < K-1$$

- for  $t = T-2$  downto 0  
for  $r = K-1 \dots 0$

$$\hat{\beta}(t, r) = y_t^{S(r)} \sum_{q \in \text{succ}(r)} \hat{\beta}(t+1, q)$$



# Backward algorithm



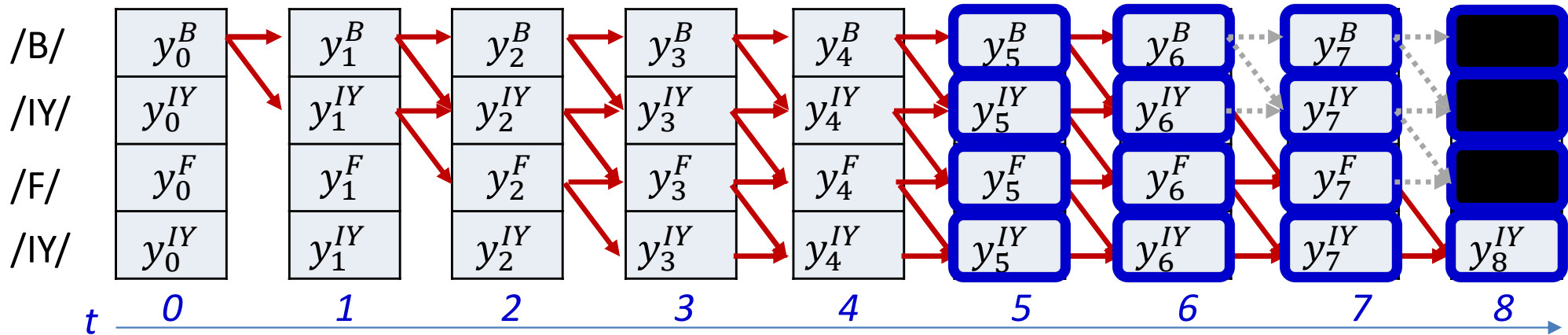
- Initialization:

$$\hat{\beta}(T-1, K-1) = y_{t+1}^{S(K-1)}, \hat{\beta}(T-1, r) = 0, r < K-1$$

- for  $t = T-2$  downto 0  
for  $r = K-1 \dots 0$

$$\hat{\beta}(t, r) = y_t^{S(r)} \sum_{q \in \text{succ}(r)} \hat{\beta}(t+1, q)$$

# Backward algorithm



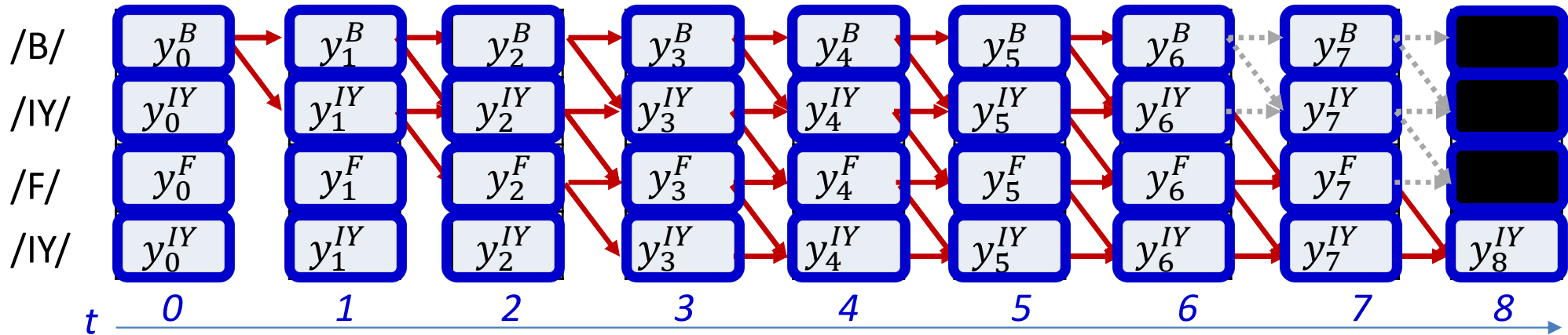
- Initialization:

$$\hat{\beta}(T-1, K-1) = y_{t+1}^{S(K-1)}, \hat{\beta}(T-1, r) = 0, r < K-1$$

- for  $t = T-2$  downto 0  
for  $r = K-1 \dots 0$

$$\hat{\beta}(t, r) = y_t^{S(r)} \sum_{q \in \text{succ}(r)} \hat{\beta}(t+1, q)$$

# Backward algorithm



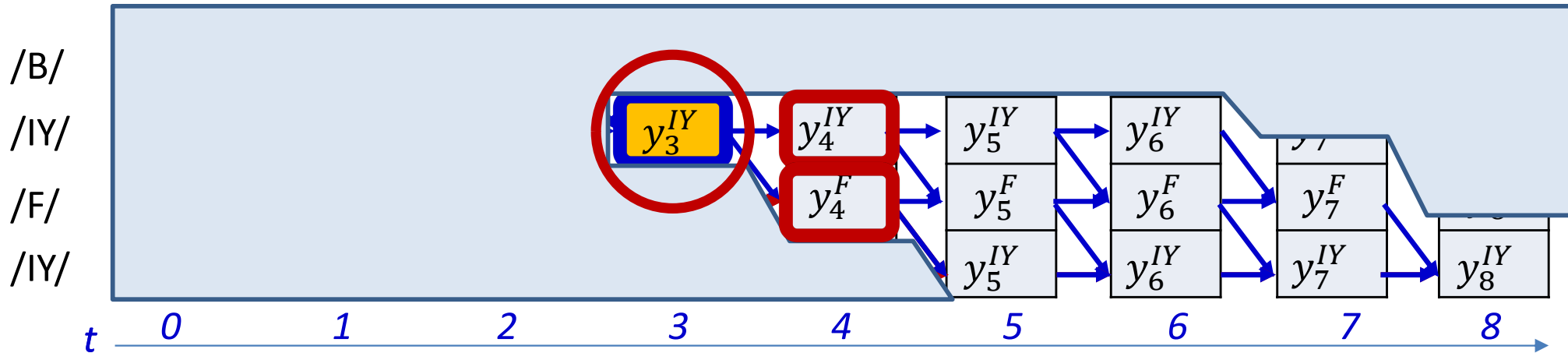
- Initialization:

$$\hat{\beta}(T-1, K-1) = y_{t+1}^{S(K-1)}, \hat{\beta}(T-1, r) = 0, r < K-1$$

- for  $t = T-2$  downto 0  
for  $r = K-1 \dots 0$

$$\hat{\beta}(t, r) = y_t^{S(r)} \sum_{q \in \text{succ}(r)} \hat{\beta}(t+1, q)$$

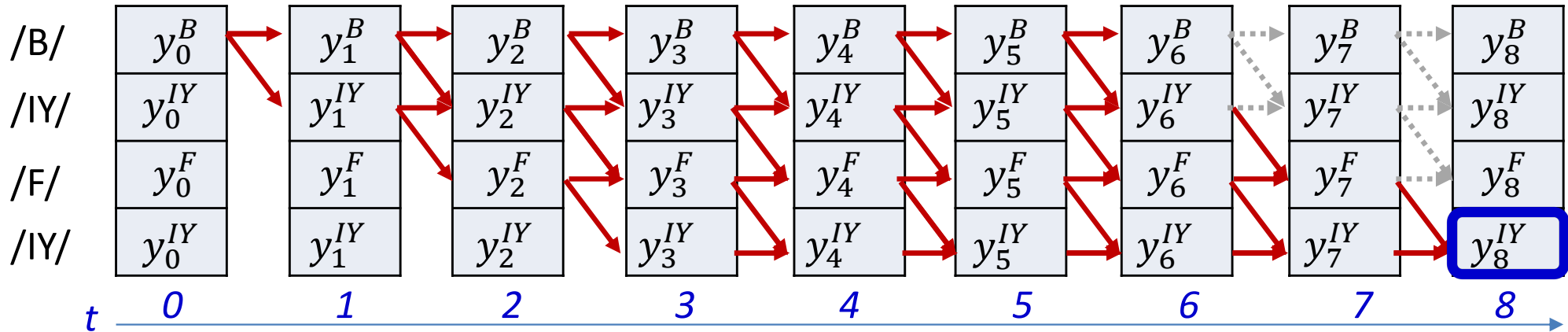
# Backward algorithm



- This recursion gives us  $\hat{\beta}(t, r)$  which includes the node at  $(t, r)$
- The actual backward probability is obtained as

$$\beta(t, r) = \frac{1}{y_t^{s_r}} \hat{\beta}(t, r)$$

# Backward algorithm



- Initialization:

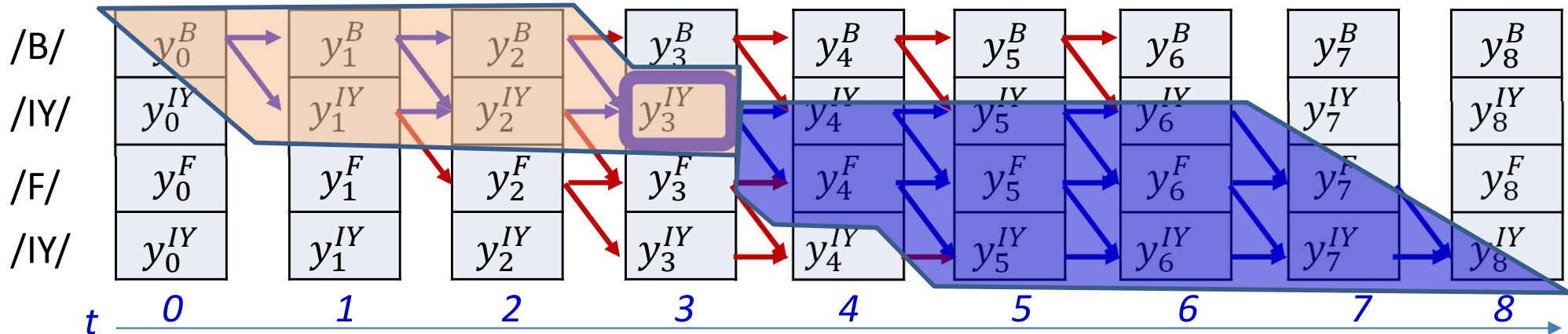
$$\hat{\beta}(T-1, K-1) = y_{T-1}^{S(K-1)}, \quad \hat{\beta}(T-1, r) = 0, \quad r < K-1$$

- for  $t = T-2$  downto 0  
for  $r = K-1 \dots 0$

$$\hat{\beta}(t, r) = y_t^{S(r)} \sum_{q \in \text{succ}(r)} \hat{\beta}(t+1, q)$$

$$\beta(t, r) = \frac{1}{y_t^{S(r)}} \hat{\beta}(t, r)$$

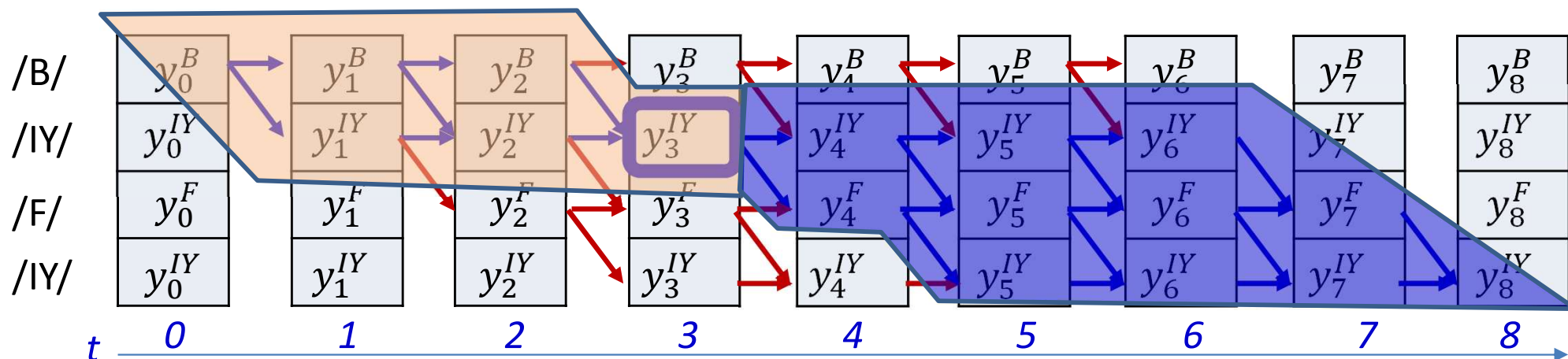
# A posteriori symbol probability



$$P(s_t = S_r, \mathbf{S} | \mathbf{X}) = \alpha(t, r) P(\text{blue graph})$$

- We will call the first term the *forward probability*  $\alpha(t, r)$
- We will call the second term the *backward probability*  $\beta(t, r)$

# The joint probability



$$P(s_t = S_r, \mathbf{S} | \mathbf{X}) = \alpha(t, r) \beta(t, r)$$

- We will call the first term the *forward probability*  $\alpha(t, r)$
- We will call the second term the *backward probability*  $\beta(t, r)$

Forward algo

Backward algo

# SIMPLE BACKWARD ALGORITHM

#N is the number of symbols in the target output

#S(i) is the ith symbol in target output

#y(t,i) is the output of the network for the ith symbol at time t

#T = length of input

**#First create output table**

```
For i = 1:N
    s(1:T,i) = y(1:T, S(i))
```

**#The backward recursion to compute betahat**

# First, at t = T

```
betahat(T,N) = s(T,N)
```

```
betahat(T,1:N-1) = 0
```

```
for t = T-1 downto 1
```

```
    betahat(t,N) = s(t,N)*betahat(t+1,N)
```

```
    for i = N-1 downto 1
```

```
        betahat(t,i) = s(t,i)*(betahat(t+1,i) + betahat(t+1,i+1))
```

**#Compute beta from betahat**

```
for t = T downto 1
```

```
    for i = N downto 1
```

```
        beta(t,i) = betahat(t,i)/s(t,i)
```

Can actually be done without explicitly composing the output table

Using 1..N and 1..T indexing, instead of 0..N-1, 0..T-1, for convenience of notation



# BACKWARD ALGORITHM

#N is the number of symbols in the target output

#S(i) is the ith symbol in target output

#y(t,i) is the output of the network for the ith symbol at time t

#T = length of input

**#The backward recursion for betahat**

# First, at  $t = T$

$\text{betahat}(T, N) = y(T, S(N))$

$\text{betahat}(T, 1:N-1) = 0$

for  $t = T-1$  downto 1

$\text{betahat}(t, N) = y(t, S(N)) * \text{betahat}(t+1, N)$

    for  $i = N-1$  downto 1

$\text{betahat}(t, i) = y(t, S(i)) * (\text{betahat}(t+1, i) + \text{betahat}(t+1, i+1))$

**#Compute beta from betahat**

for  $t = T$  downto 1

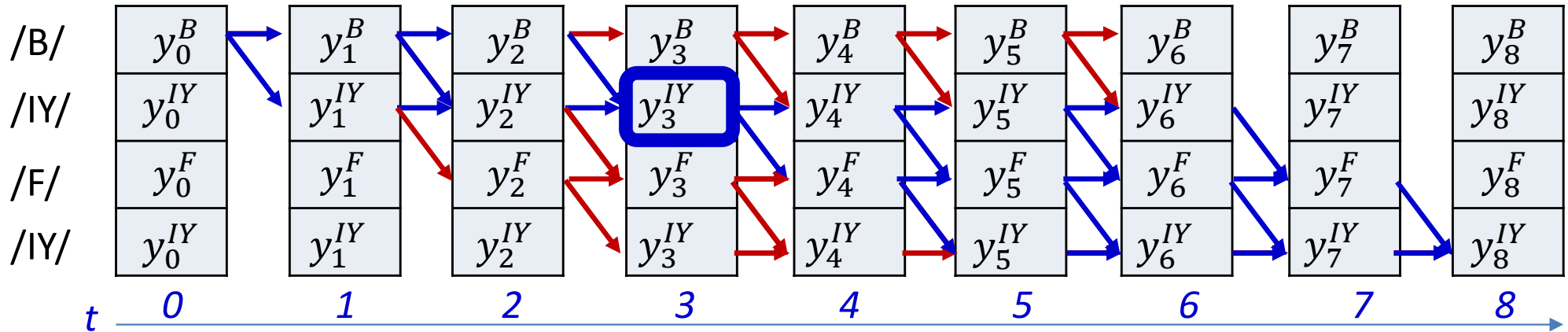
    for  $i = N$  downto 1

$\text{beta}(t, i) = \text{betahat}(t, i) / y(t, S(i))$

Without explicitly composing the output table

Using 1..N and 1..T indexing, instead of 0..N-1, 0..T-1, for convenience of notation

# The posterior probability

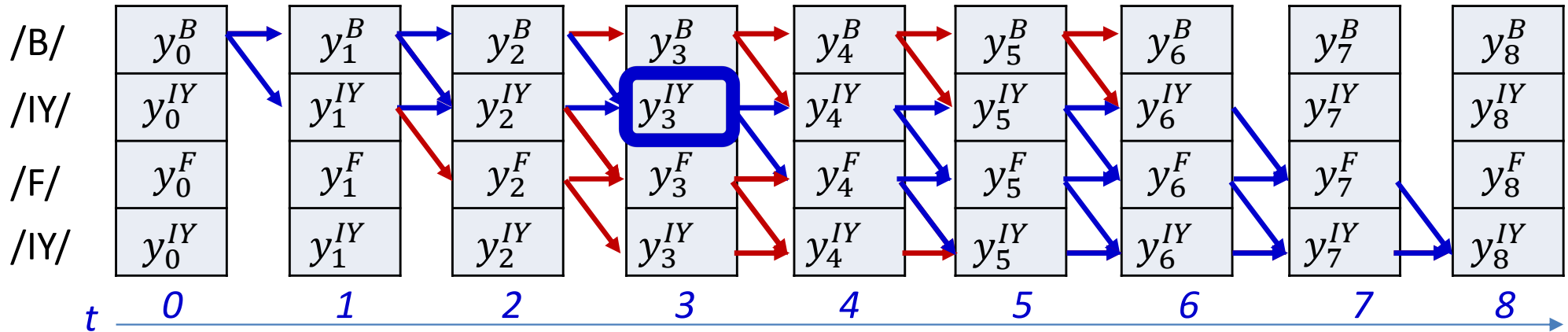


$$P(s_t = S_r, \mathbf{S} | \mathbf{X}) = \alpha(t, r) \beta(t, r)$$

- The *posterior* is given by

$$P(s_t = S_r | \mathbf{S}, \mathbf{X}) = \frac{P(s_t = S_r, \mathbf{S} | \mathbf{X})}{\sum_{S'_r} P(s_t = S'_r, \mathbf{S} | \mathbf{X})} = \frac{\alpha(t, r) \beta(t, r)}{\sum_{r'} \alpha(t, r') \beta(t, r')}$$

# The posterior probability



- Let the posterior  $P(s_t = S_r | \mathbf{S}, \mathbf{X})$  be represented by  $\gamma(t, r)$

$$\gamma(t, r) = \frac{\alpha(t, r)\beta(t, r)}{\sum_{r'} \alpha(t, r')\beta(t, r')}$$

# COMPUTING POSTERIORIORS

#N is the number of symbols in the target output  
#S(i) is the ith symbol in target output  
#y(t,i) is the output of the network for the ith symbol at time t  
#T = length of input

#Assuming the forward are completed first

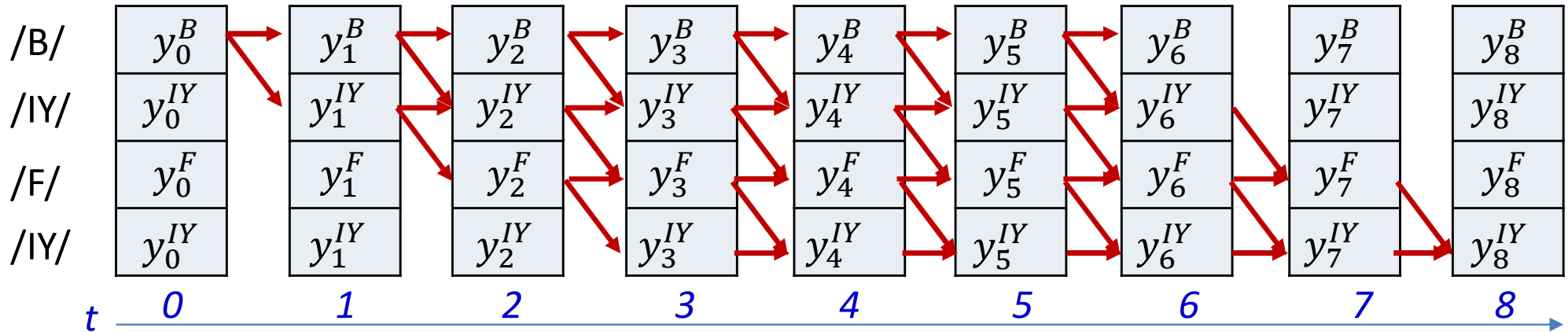
```
alpha = forward(y, S)    # forward probabilities computed  
beta  = backward(y, S)   # backward probabilities computed
```

#Now compute the posteriors

```
for t = 1:T  
    sumgamma(t) = 0  
    for i = 1:N  
        gamma(t,i) = alpha(t,i) * beta(t,i)  
        sumgamma(t) += gamma(t,i)  
    end  
    for i=1:N  
        gamma(t,i) = gamma(t,i) / sumgamma(t)
```

Using 1..N and 1..T indexing, instead of 0..N-1, 0..T-1, for convenience of notation

# The expected divergence



$$DIV = - \sum_t \sum_{s \in S_0 \dots S_{K-1}} P(s_t = s | \mathbf{S}, \mathbf{X}) \log Y(t, s_t = s)$$

$$DIV = - \sum_t \sum_r \gamma(t, r) \log y_t^{S(r)}$$

# Poll 3 (@792)

Select all that are true

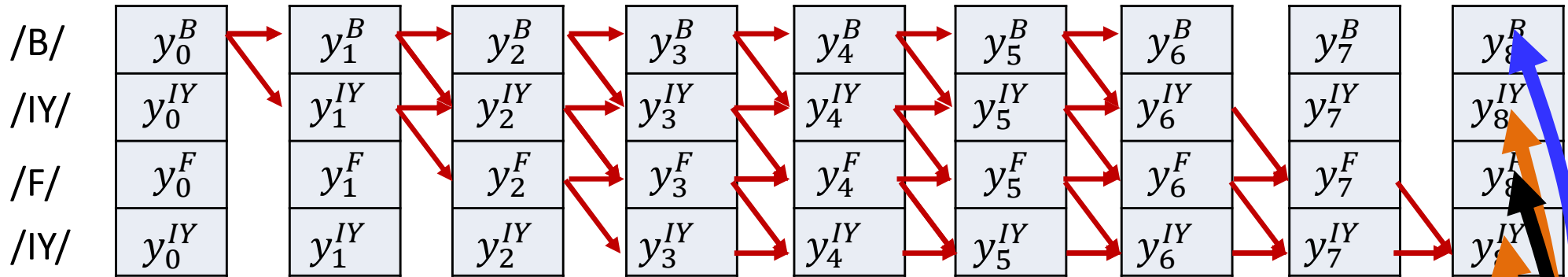
- The forward-backward algorithm is used to compute the a posteriori probability of aligning each symbol in the compressed sequence to each input
- These probabilities are required to compute the expected divergence across all alignments of the compressed symbol sequence to the input

# Poll 3

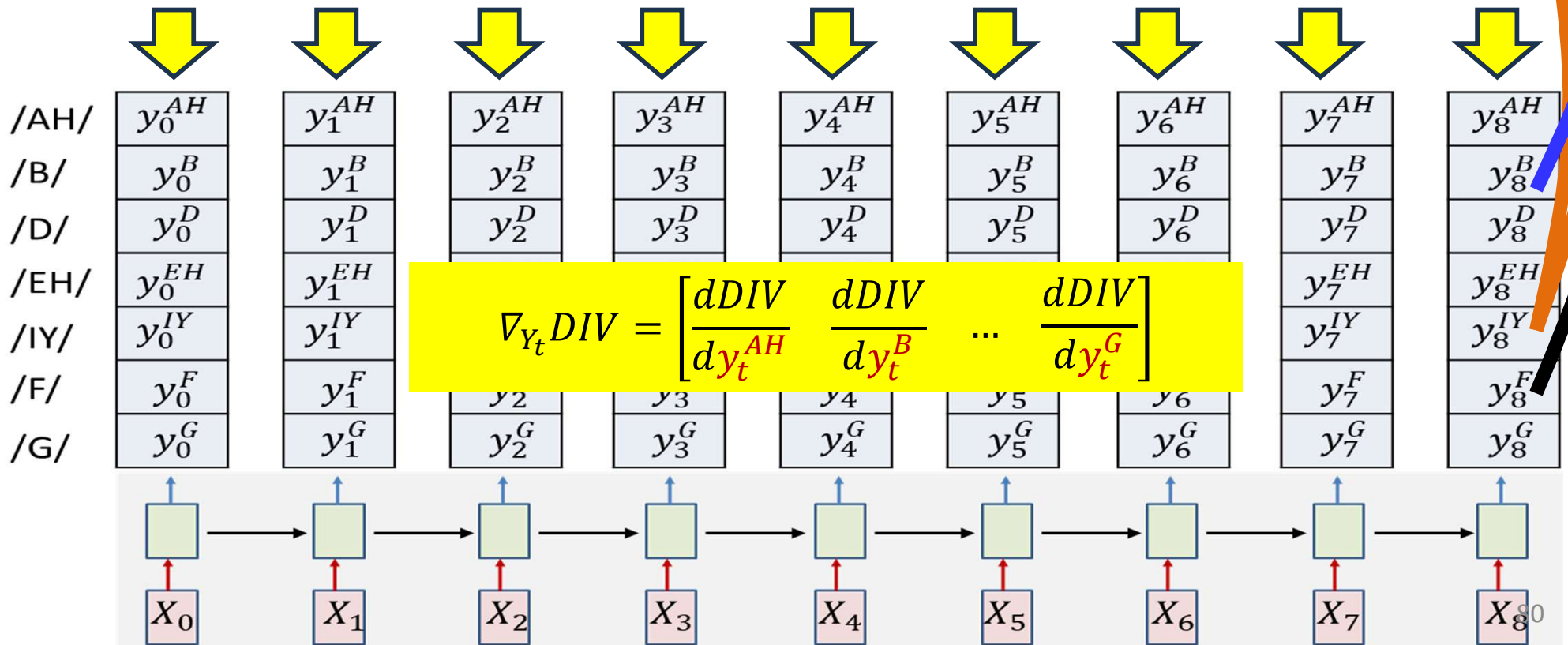
Select all that are true

- The forward-backward algorithm is used to compute the a posteriori probability of aligning each symbol in the compressed sequence to each input
- These probabilities are required to compute the expected divergence across all alignments of the compressed symbol sequence to the input

# Recap: The “aligned” table

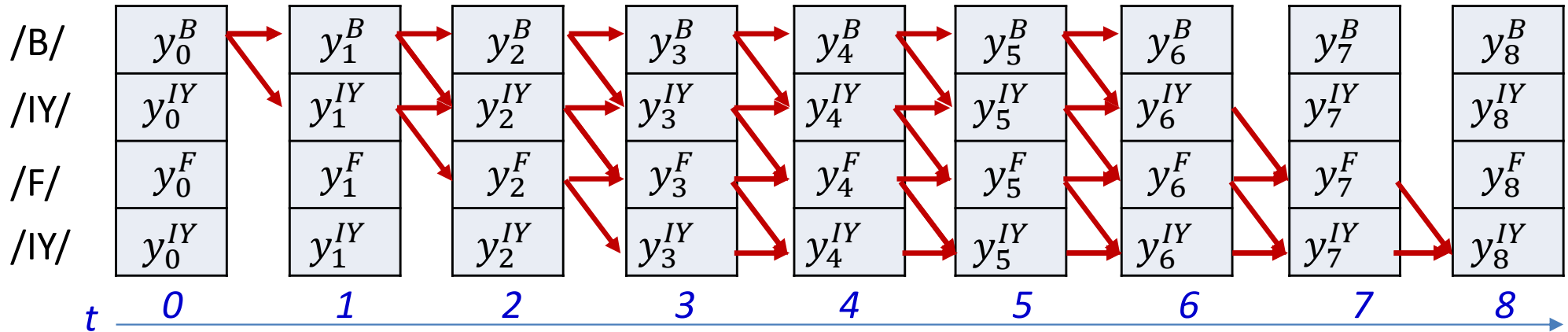


We want loss derivatives for these terms





# The expected divergence



$$DIV = - \sum_t \sum_{s \in S_0 \dots S_{K-1}} P(s_t = s | \mathbf{S}, \mathbf{X}) \log Y(t, s_t = s)$$

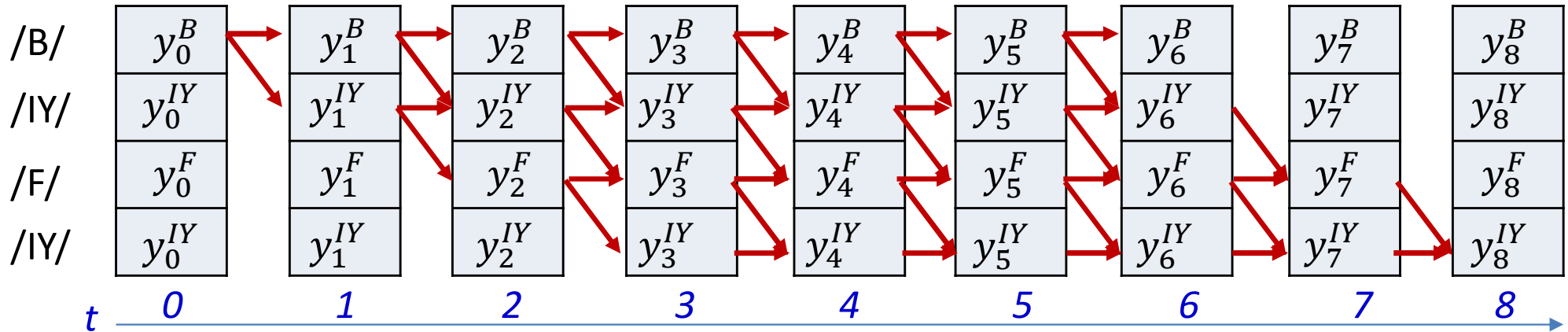
$$DIV = - \sum_t \sum_r \gamma(t, r) \log y_t^{S(r)}$$

- The derivative of the divergence w.r.t the output  $Y_t$  of the net at any time:

$$\nabla_{Y_t} DIV = \left[ \frac{dDIV}{dy_t^{s_0}} \quad \frac{dDIV}{dy_t^{s_1}} \quad \dots \quad \frac{dDIV}{dy_t^{s_{L-1}}} \right]$$

- Components will be non-zero only for symbols that occur in the training instance

# The expected divergence



$$DIV = - \sum_t \sum_{s \in S_0 \dots S_{K-1}} P(s_t = s | \mathbf{S}, \mathbf{X}) \log Y(t, s_t = s)$$

$$DIV = - \sum_t \sum_r \gamma(t, r) \log y_t^{s(r)}$$

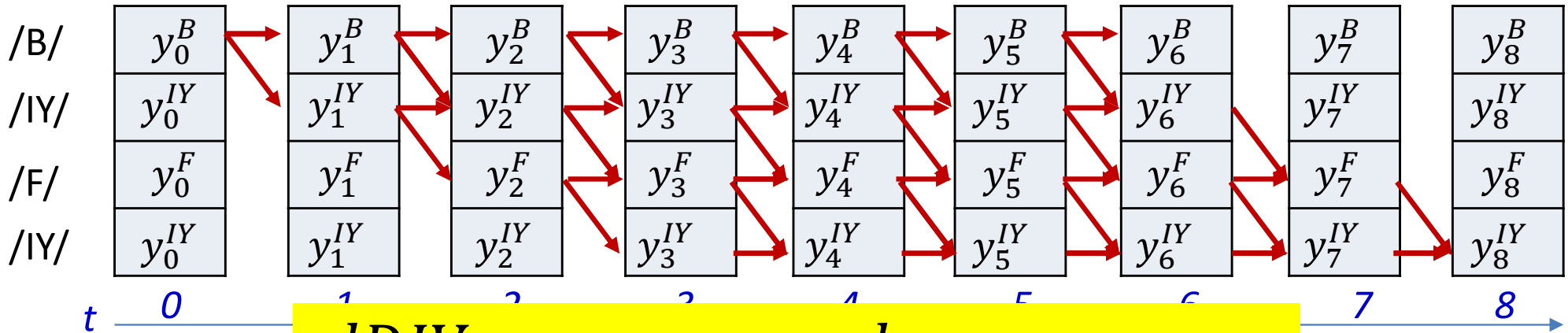
- The derivative of the divergence w.r.t the output  $Y_t$  of the net at any time:

$$\nabla_{Y_t} DIV = \left[ \frac{dDIV}{dy_t^{s_0}} \right] \left[ \frac{dDIV}{dy_t^{s_1}} \right] \dots \left[ \frac{dDIV}{dy_t^{s_{k-1}}} \right]$$

Must compute these terms from here

- Components will be non-zero only for symbols that occur in the training instance

# The expected divergence



$$L \frac{dDIV}{dy_t^l} = - \sum_{r: S(r)=l} \frac{d}{dy_t^l} \gamma(t, r) \log y_t^l$$

$$DIV = - \sum_t \sum_r \gamma(t, r) \log y_t^{S(r)}$$

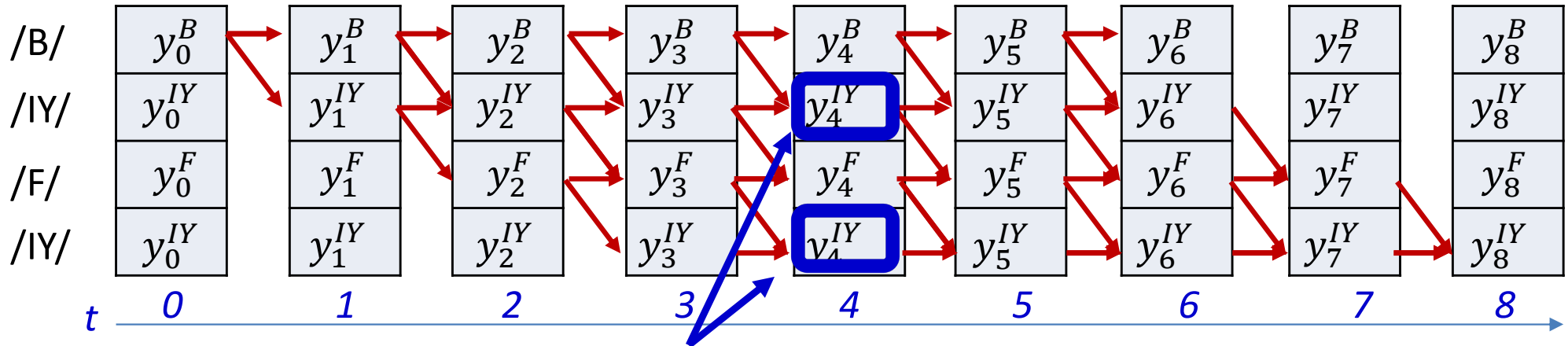
- The derivative of the divergence w.r.t the output  $Y_t$  of the net at any time:

$$\nabla_{Y_t} DIV = \left[ \frac{dDIV}{dy_t^{s_0}} \right] \left[ \frac{dDIV}{dy_t^{s_1}} \right] \dots \left[ \frac{dDIV}{dy_t^{s_{l-1}}} \right]$$

Must compute these terms from here

- Components will be non-zero only for symbols that occur in the training instance

# The expected divergence



The derivatives at both these locations must be summed to get  $\frac{dDIV}{dy_4^{IY}}$

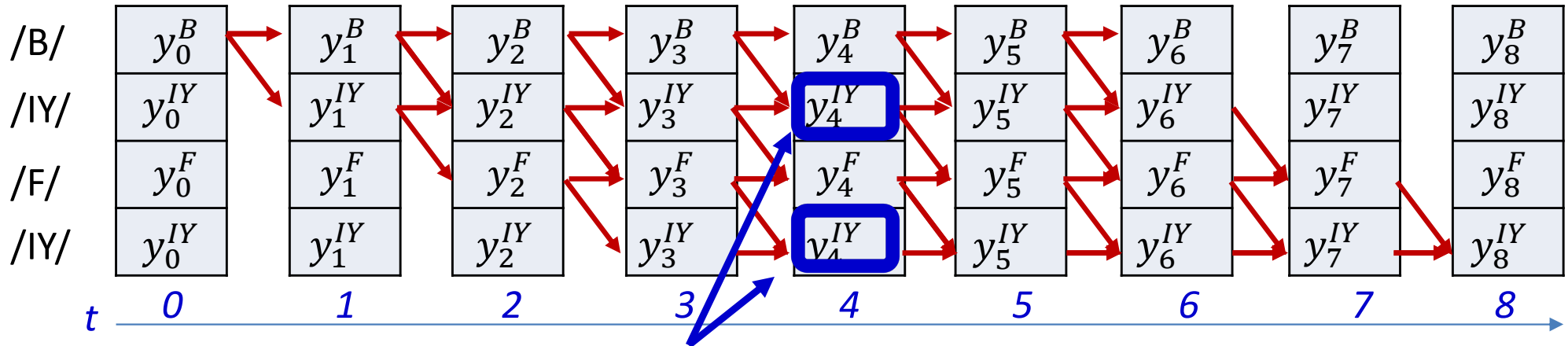
$$\frac{dDIV}{dy_t^l} = - \sum_{r:S(r)=l} \frac{d}{dy_t^l} \gamma(t, r) \log y_t^l$$

- The derivative of the divergence w.r.t the output  $Y_t$  of the net at any time:

$$\nabla_{Y_t} DIV = \left[ \frac{dDIV}{dy_t^{s_0}} \frac{dDIV}{dy_t^{s_1}} \dots \frac{dDIV}{dy_t^{s_{L-1}}} \right]$$

- Components will be non-zero only for symbols that occur in the training instance

# The expected divergence



The derivatives at both these locations must be summed to get  $\frac{dDIV}{dy_4^{IY}}$

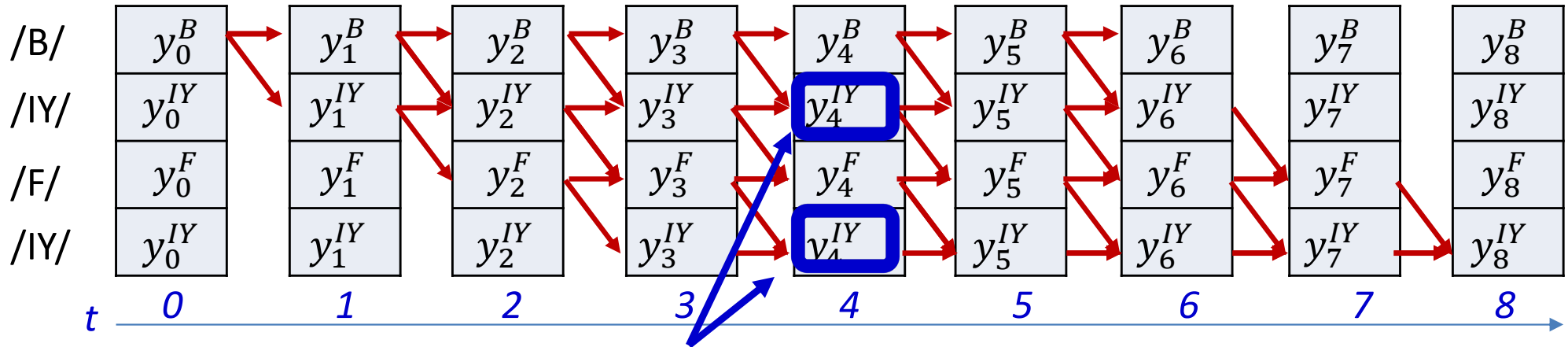
$$\frac{dDIV}{dy_t^l} = - \sum_{r:S(r)=l} \frac{d}{dy_t^l} \gamma(t, r) \log y_t^l$$

- The derivative of the divergence w.r.t the output  $Y_t$  of the net at any time:

$$\nabla_{Y_t} DIV = \left[ \frac{dDIV}{dy_t^{s_0}} \quad \frac{dDIV}{dy_t^{s_1}} \quad \dots \quad \frac{dDIV}{dy_t^{s_{L-1}}} \right]$$

- Components will be non-zero only for symbols that occur in the training instance

# The expected divergence



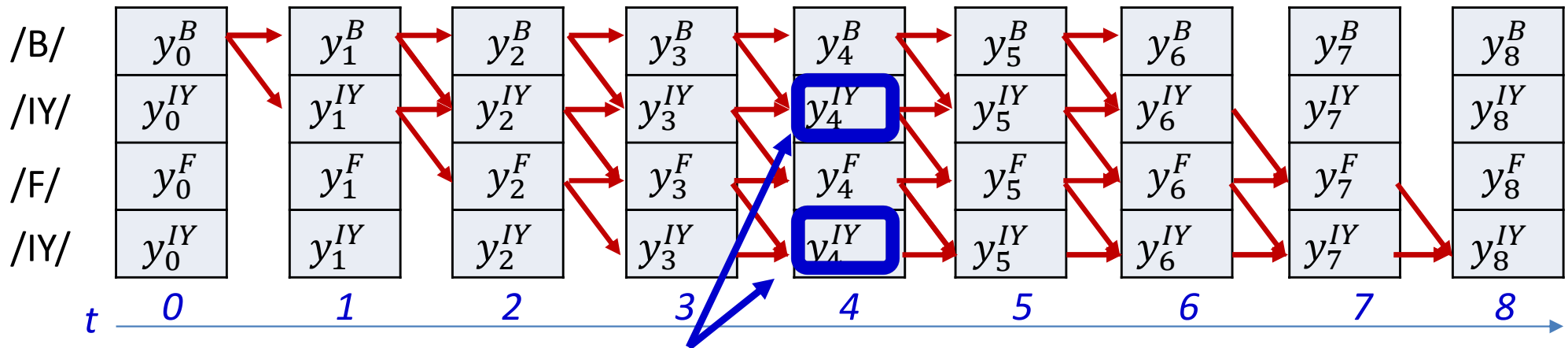
The derivatives at both these locations must be summed to get  $\frac{dDIV}{dy_4^{IY}}$

$$\frac{dDIV}{dy_t^l} = - \sum_{r:S(r)=l} \frac{d}{dy_t^l} \gamma(t, r) \log y_t^l$$

•  $\frac{d}{dy_t^l} \gamma(t, r) \log y_t^l = \frac{\gamma(t, r)}{y_t^l} + \frac{d\gamma(t, r)}{dy_t^l} \log y_t^l$  any time:

– Components will be non-zero only for symbols that occur in the training instance

# The expected divergence



The derivatives at both these locations must be summed to get  $\frac{dDIV}{dy_4^{IY}}$

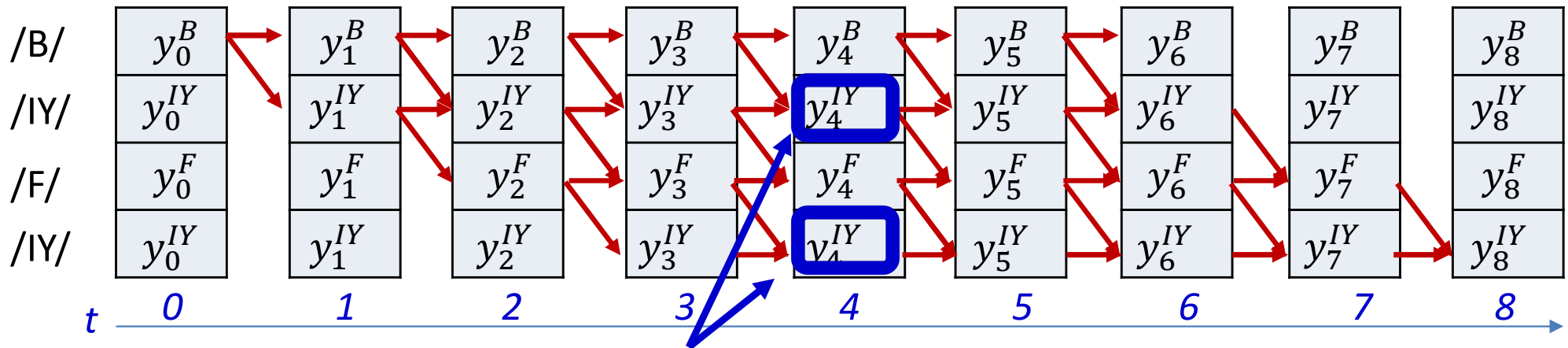
$$\frac{dDIV}{dy_t^l} = - \sum_{r:S(r)=l} \frac{d}{dy_t^l} \gamma(t, r) \log y_t^l$$

- The derivative of  $\frac{d}{dy_t^l} \gamma(t, r) \log y_t^l$  is net at any time:

$$\frac{d}{dy_t^l} \gamma(t, r) \log y_t^l \approx \frac{\gamma(t, r)}{y_t^l}$$

The approximation is exact if we think of this as a maximum-likelihood estimate

# Derivative of the expected divergence



The derivatives at both these locations must be summed to get  $\frac{dDIV}{dy_4^{IY}}$

$$DIV = - \sum_t \sum_r \gamma(t, r) \log y_t^{S(r)}$$

- The derivative of the divergence w.r.t any particular output of the network must sum over all instances of that symbol in the target sequence

$$\frac{dDIV}{dy_t^l} = - \frac{1}{y_t^l} \sum_{r: S(r)=l} \gamma(t, r)$$

- E.g. the derivative w.r.t  $y_t^{IY}$  will sum over both rows representing /IY/ in the above figure



# COMPUTING DERIVATIVES

```
#N is the number of symbols in the target output
#S(i) is the ith symbol in target output
#y(t,i) is the output of the network for the ith symbol at time t
#T = length of input
```

```
#Assuming the forward are completed first
```

```
alpha = forward(y, S) # forward probabilities computed
beta = backward(y, S) # backward probabilities computed
```

```
# Compute posteriors from alpha and beta
```

```
gamma = computeposteriors(alpha, beta)
```

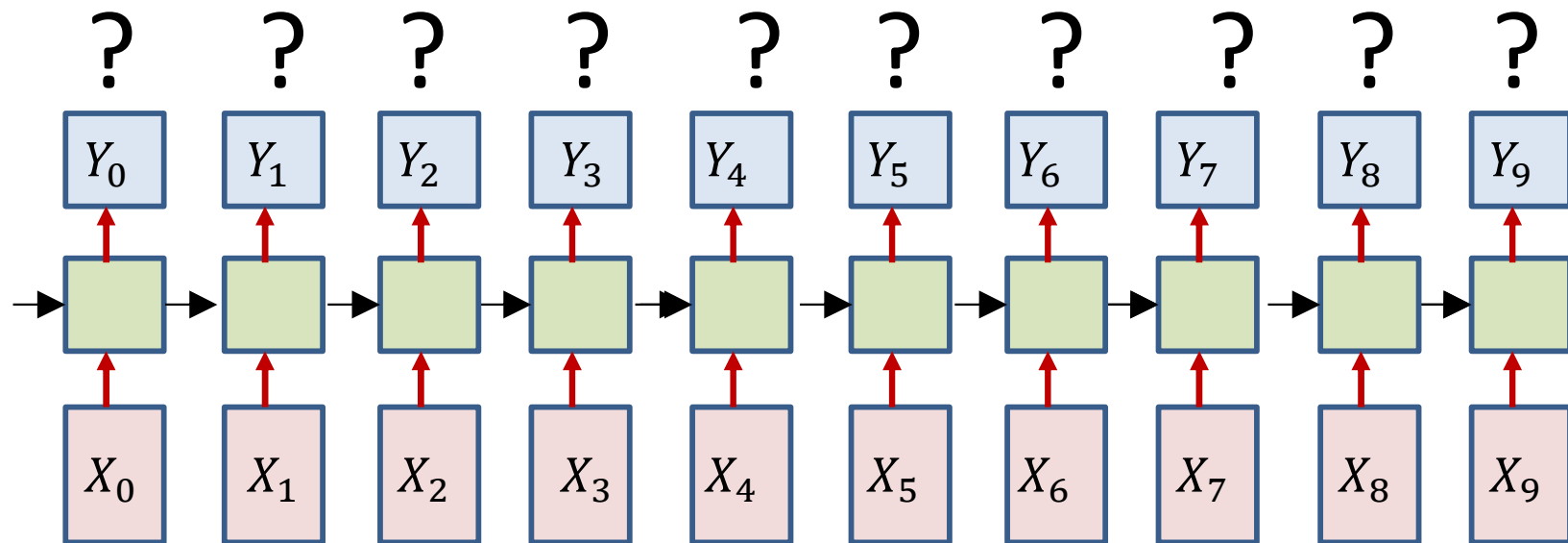
```
#Compute derivatives
```

```
for t = 1:T
    dy(t,1:L) = 0 # Initialize all derivatives at time t to 0
    for i = 1:N
        dy(t,S(i)) -= gamma(t,i) / y(t,S(i))
```

Using 1..N and 1..T indexing, instead of 0..N-1, 0..T-1, for convenience of notation

# Overall training procedure for Seq2Seq case 1

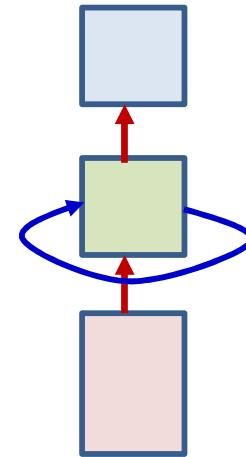
/B/ /IY/ /F/ /IY/



- Problem: Given input and output sequences without alignment, train models

# Overall training procedure for Seq2Seq case 1

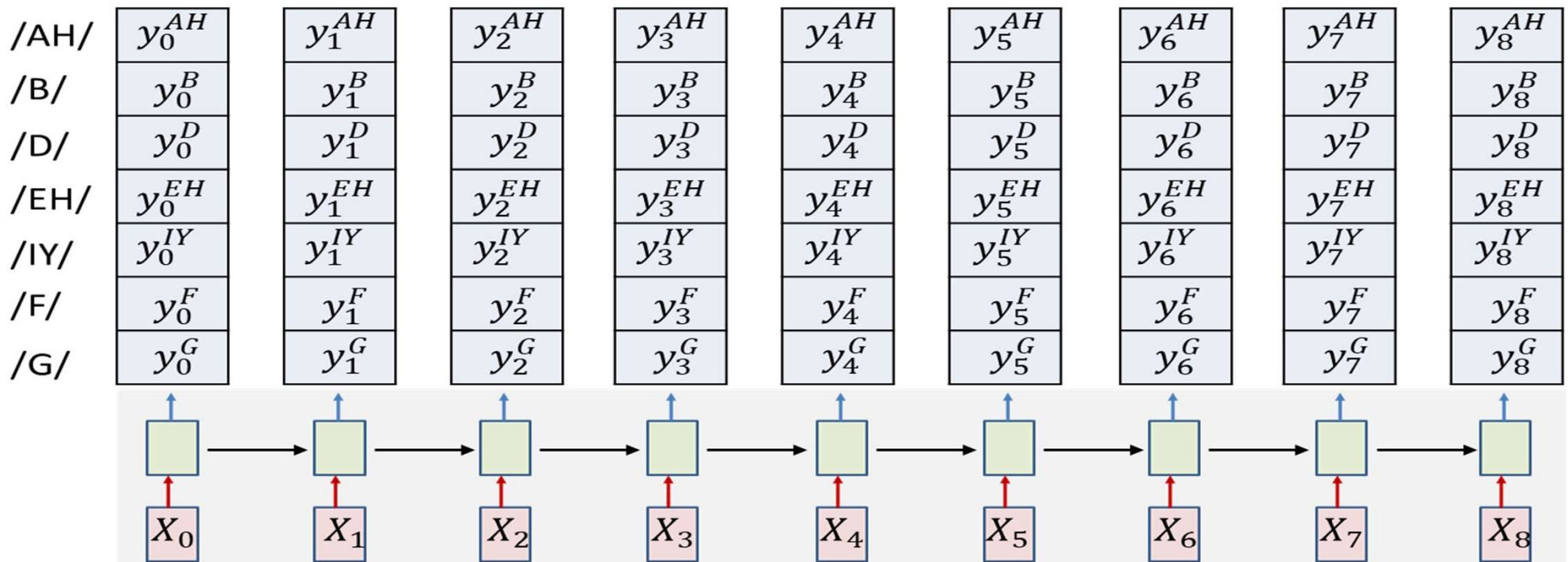
- **Step 1:** Setup the network
  - Typically many-layered LSTM



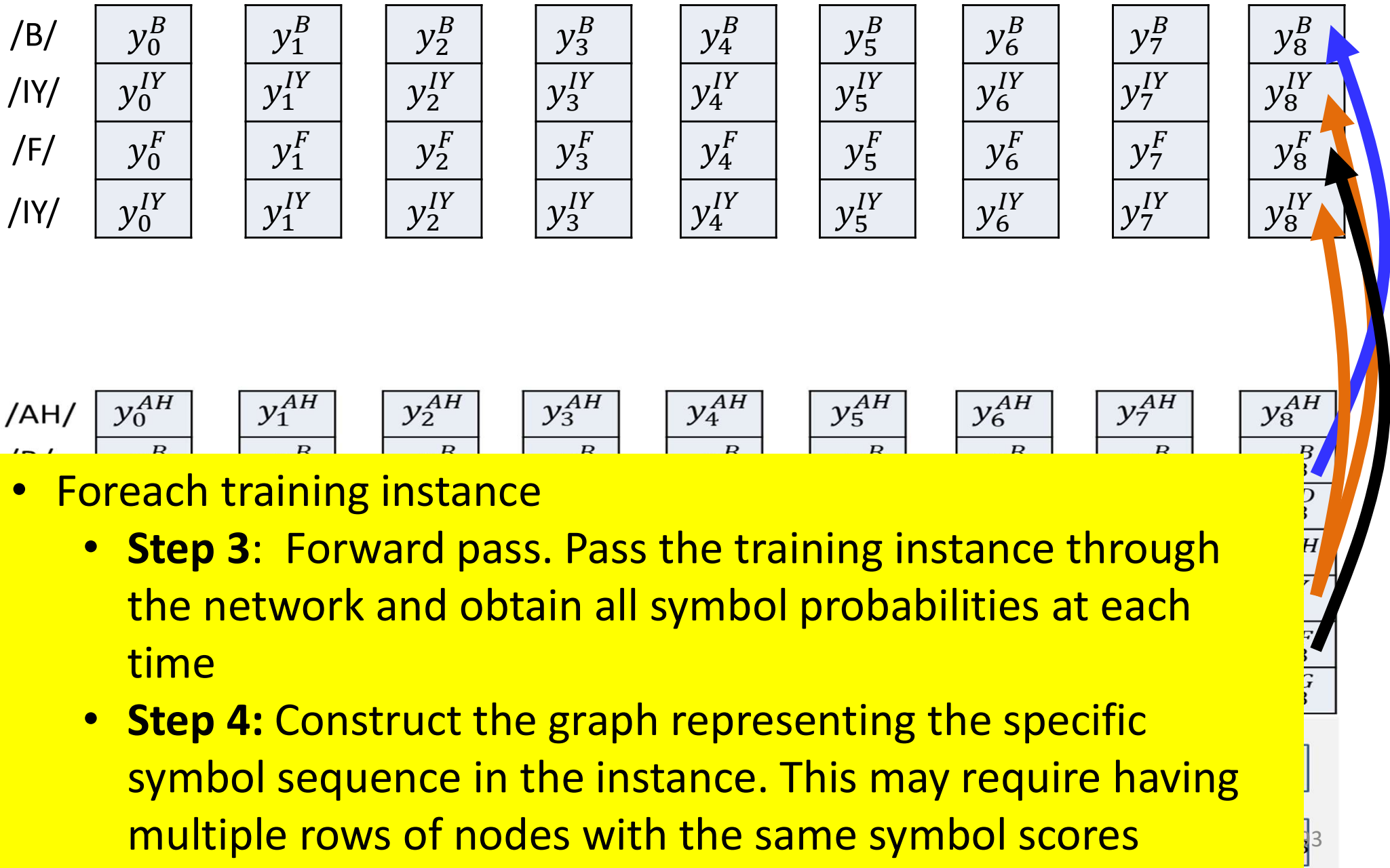
- **Step 2:** Initialize all parameters of the network

# Overall Training: Forward pass

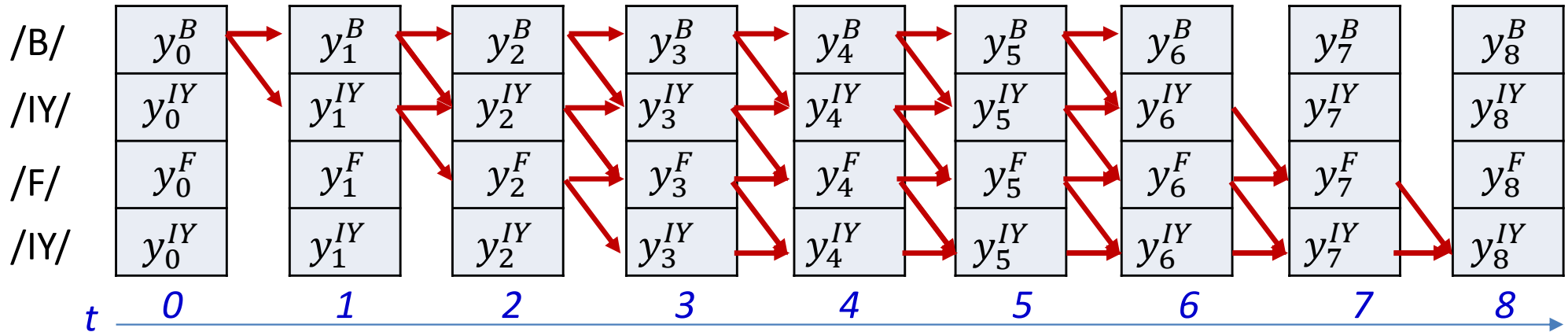
- Foreach training instance
  - **Step 3:** Forward pass. Pass the training instance through the network and obtain all symbol probabilities at each time



# Overall training: Backward pass



# Overall training: Backward pass



- Foreach training instance:
  - **Step 5:** Perform the forward backward algorithm to compute  $\alpha(t, r)$  and  $\beta(t, r)$  at each time, for each row of nodes in the graph. Compute  $\gamma(t, r)$ .
  - **Step 6:** Compute derivative of divergence  $\nabla_{Y_t} DIV$  for each  $Y_t$

# Overall training: Backward pass

- Foreach instance
  - **Step 6:** Compute derivative of divergence  $\nabla_{Y_t} DIV$  for each  $Y_t$

$$\nabla_{Y_t} DIV = \left[ \frac{dDIV}{dy_t^0} \quad \frac{dDIV}{dy_t^1} \quad \dots \quad \frac{dDIV}{dy_t^{L-1}} \right]$$
$$\frac{dDIV}{dy_t^l} = - \sum_{r:S(r)=l} \frac{\gamma(t,r)}{y_t^l}$$

- **Step 7:** Backpropagate  $\frac{dDIV}{dy_t^l}$  and aggregate derivatives over minibatch and update parameters

# Story so far: CTC models

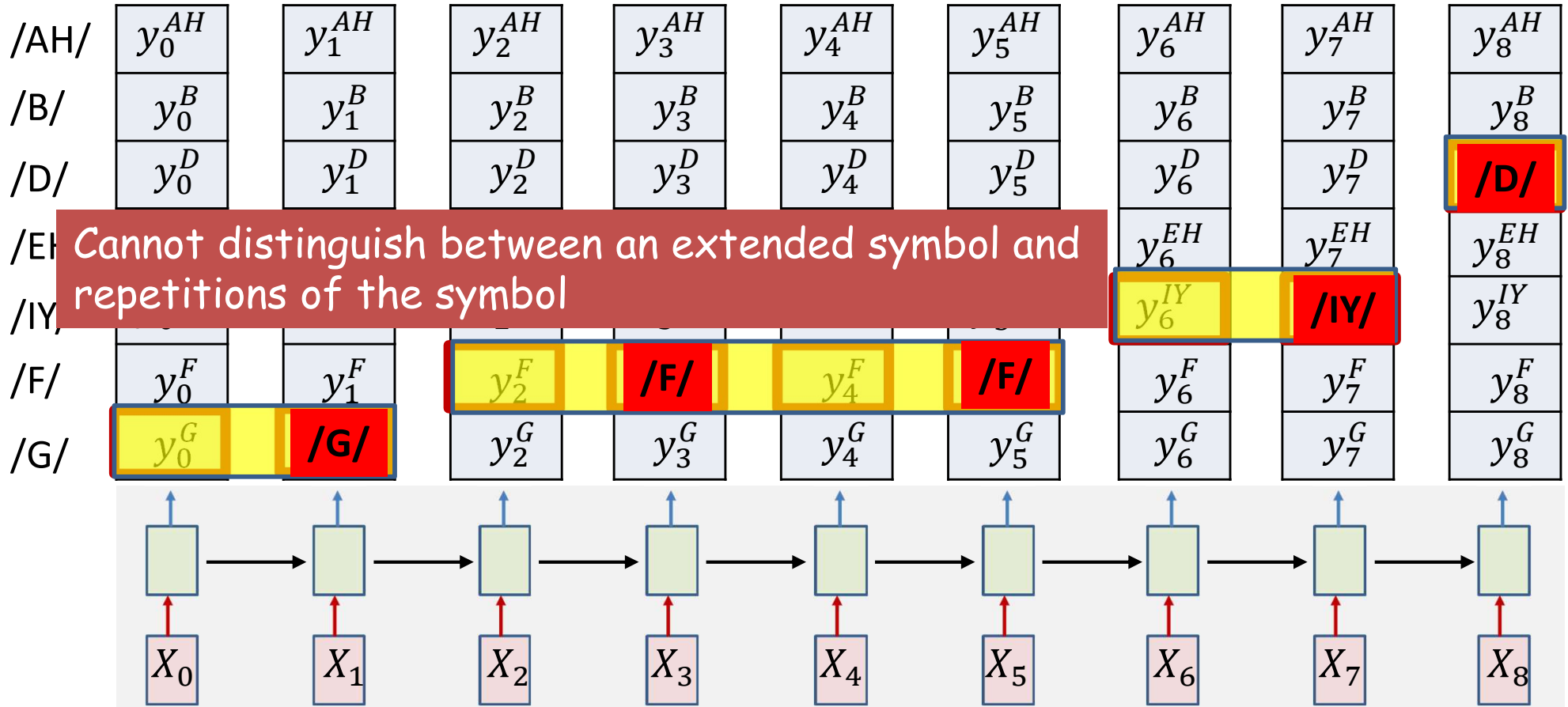
- Sequence-to-sequence networks which irregularly output symbols can be “decoded” by greedy decoding
  - Which assumes that a symbol is output at each time and *merges* adjacent symbols
- They require alignment of the output to the symbol sequence for training
  - This alignment is generally not given
- Training can be performed by iteratively estimating the alignment by Viterbi-alignment and time-synchronous training
- Alternately, it can be performed by optimizing the expected error over *all* possible alignments
  - Posterior probabilities for the expectation can be computed using the forward backward algorithm



# ***A key decoding problem***


- Consider a problem where the output symbols are characters
- We have a decode: R R R E E E E D
- Is this the compressed symbol sequence RED or REED?

# We've seen this before



- /G/ /F/ /F/ /IY/ /D/ or /G/ /F/ /IY/ /D/ ?

# A key *decoding* problem

- We have a decode: R R R E E E E E D
- Is this the symbol sequence RED or REED?
- Solution: Introduce an explicit extra `invisible' symbol which serves to separate discrete versions of a symbol
  - A “blank” (represented by “-”)
  - RRR---EE---DDD = RED
  - RR-E--EED = REED
  - RR-R---EE---D-DD = RREDD
  - R-R-R---E-EDD-DDDD-D = 
    - The next symbol at the end of a sequence of blanks is always a new character
    - When a symbol repeats, there must be at least one blank between the repetitions
- The symbol set recognized by the network must now include the extra blank symbol
  - Which too must be trained

# A key *decoding* problem

- We have a decode: R R R E E E E E D
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  - RRR---EE---DDD = RED
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  - R-R-R---E-EDD-DDDD-D = RRREEDDD
    - The next symbol at the end of a sequence of blanks is always a new character
    - When a symbol repeats, there must be at least one blank between the repetitions
- The symbol set recognized by the network must now include the extra blank symbol
  - Which too must be trained

# Poll 4 (@793)

Which of the following are valid expansions of the character string “BILLY”?

- B B I I L L Y
- B - B I L - L Y
- B - I - L L Y
- B - I - L - L Y Y

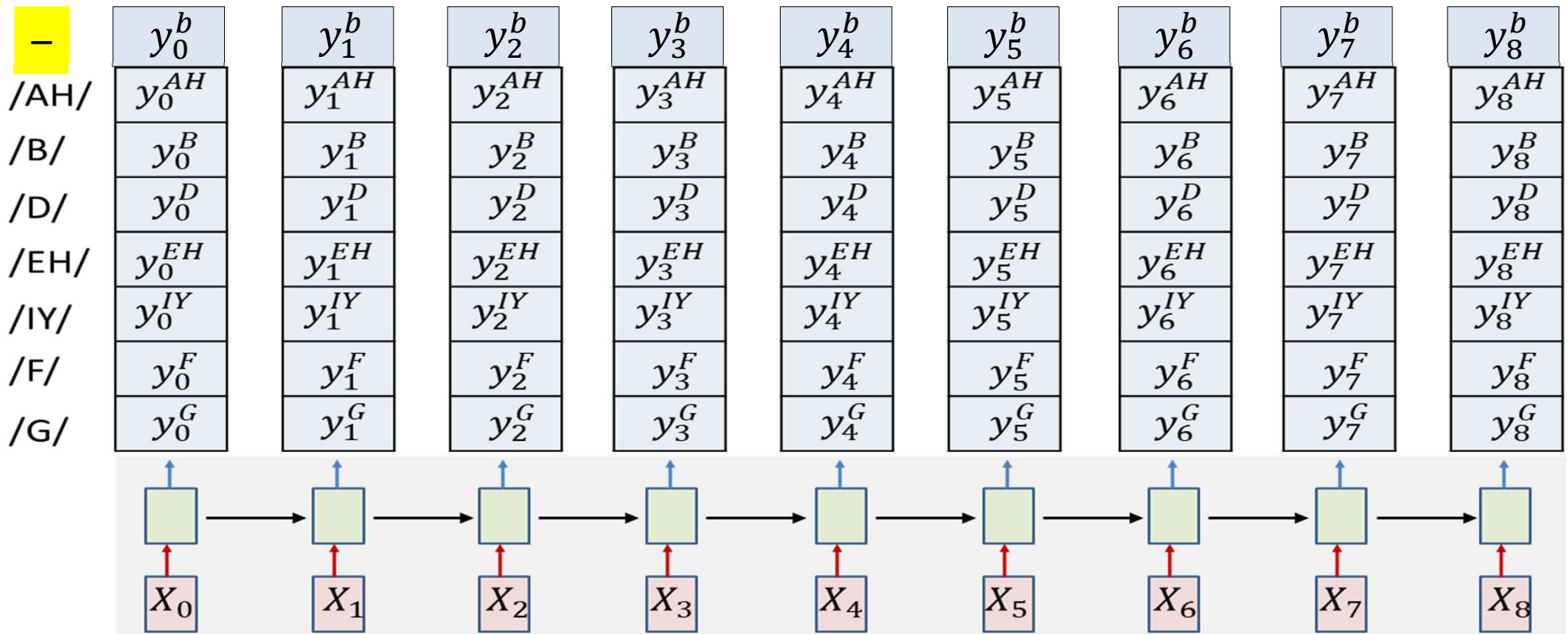
# Poll 4

Which of the following are valid expansions of the character string “BILLY”?

- BBIILLY
- B-BIL-LY
- B-I-LLY
- **B-I-L-LYY**

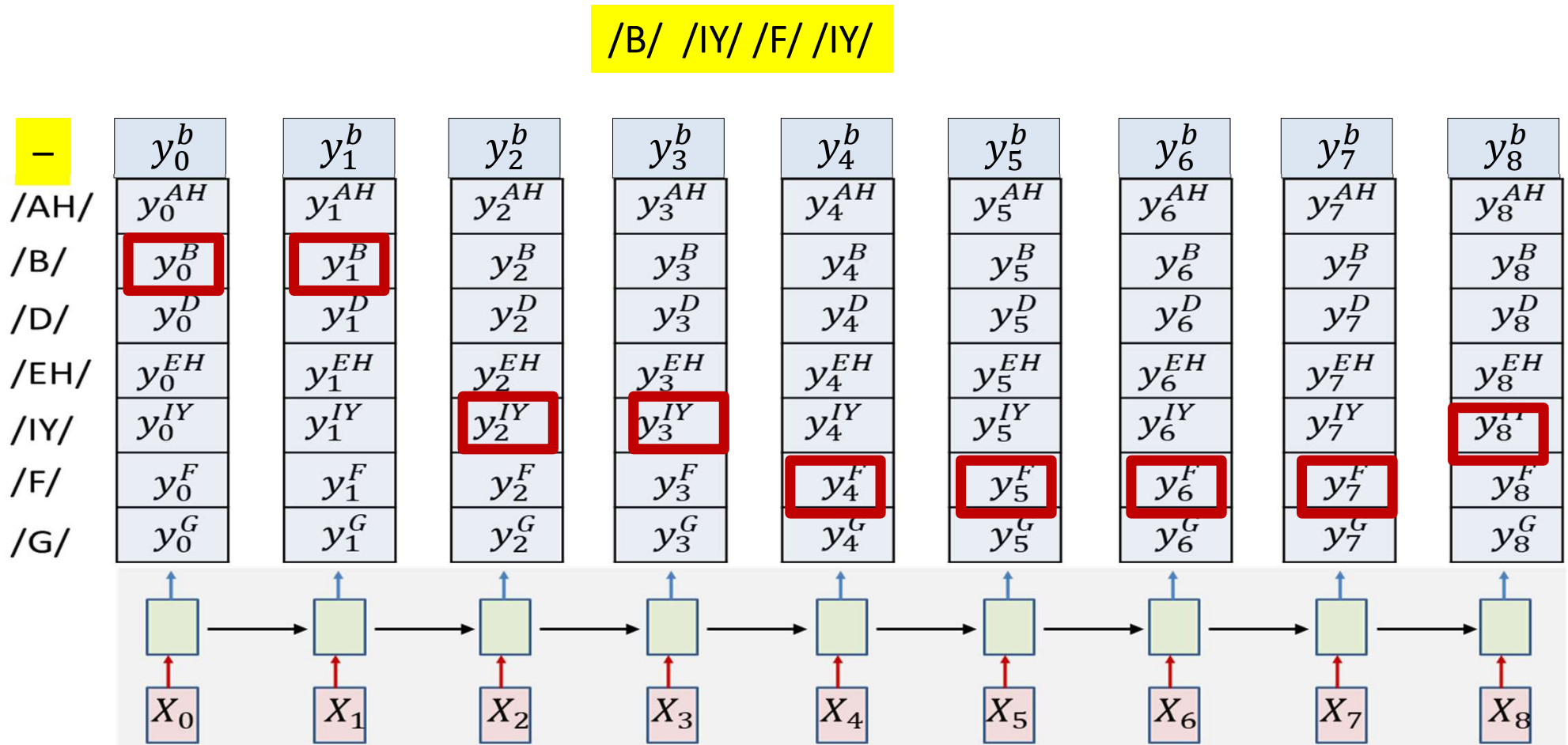
# The modified forward output

- Note the extra “blank” at the output



# The modified forward output

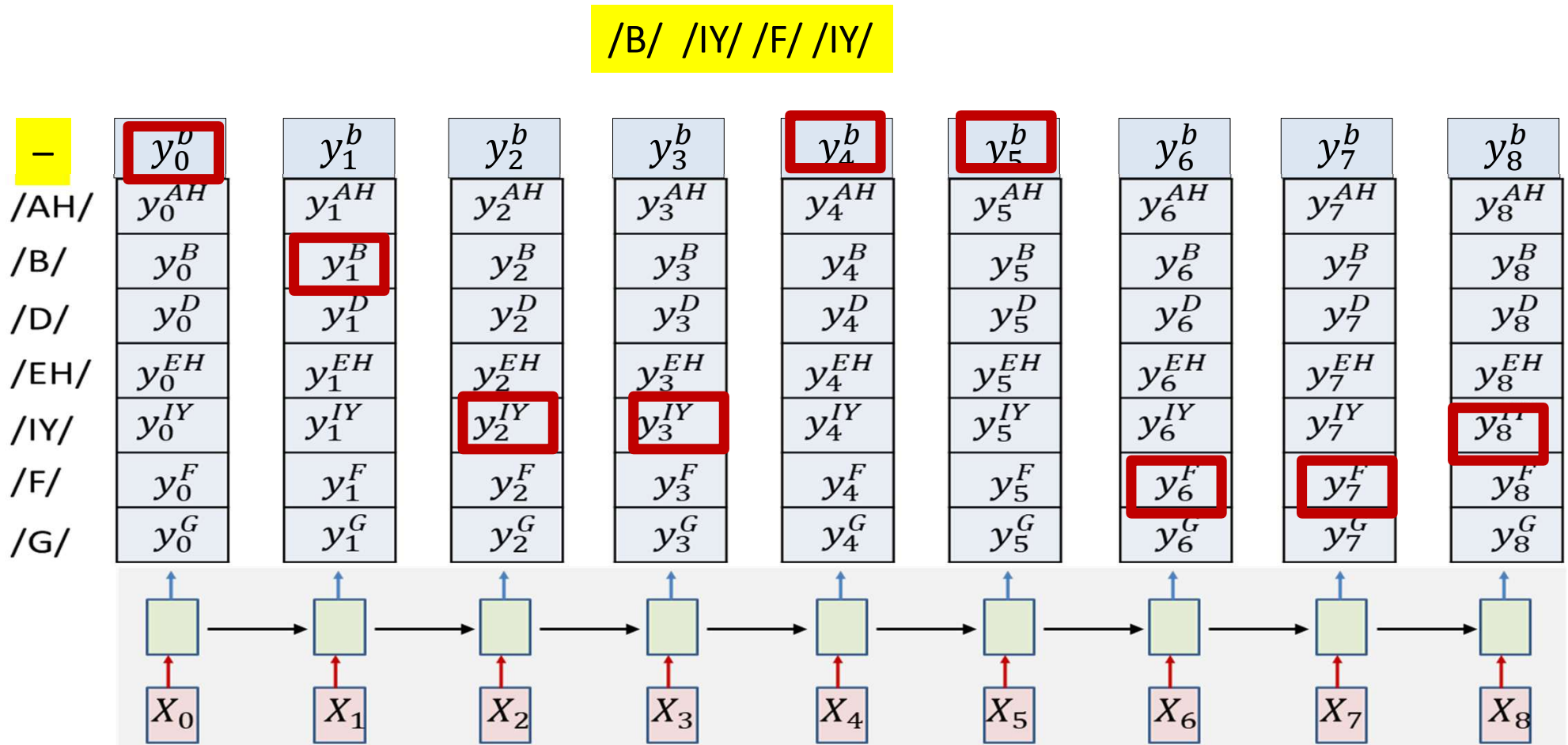
- Note the extra “blank” at the output





# The modified forward output

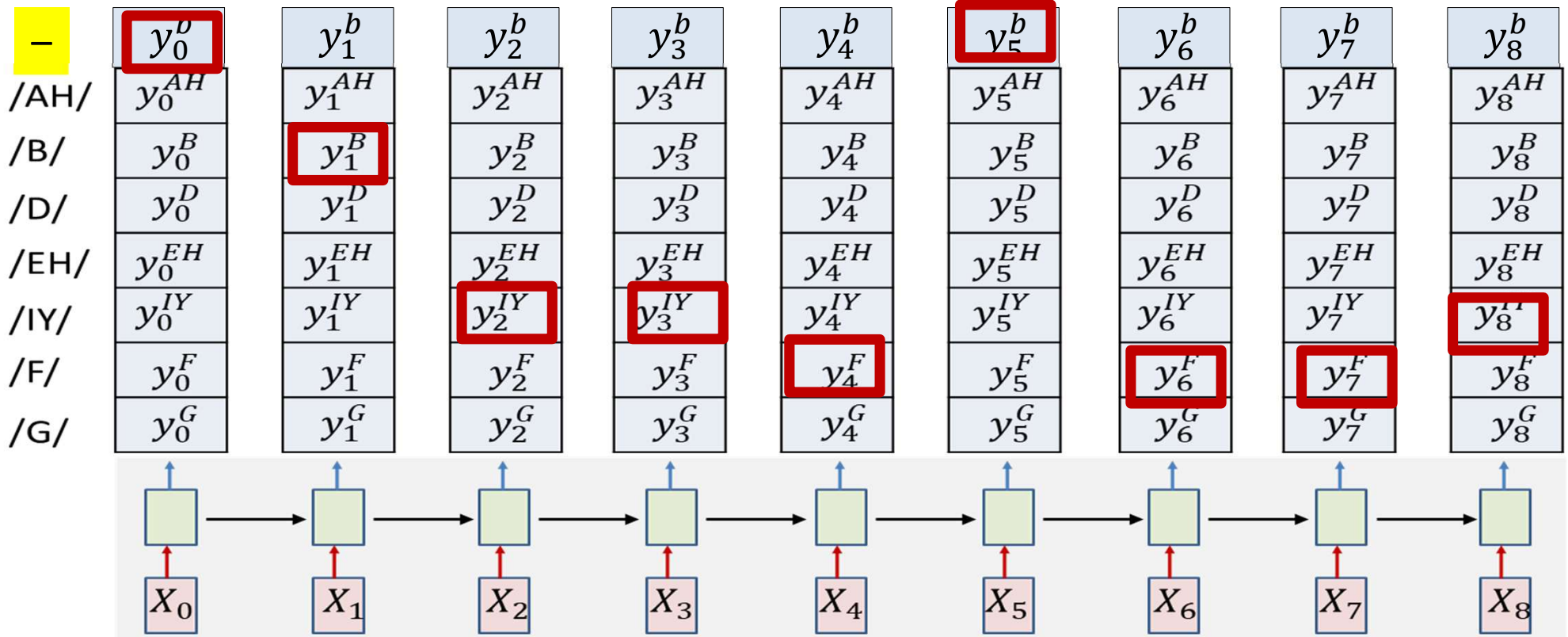
- Note the extra “blank” at the output



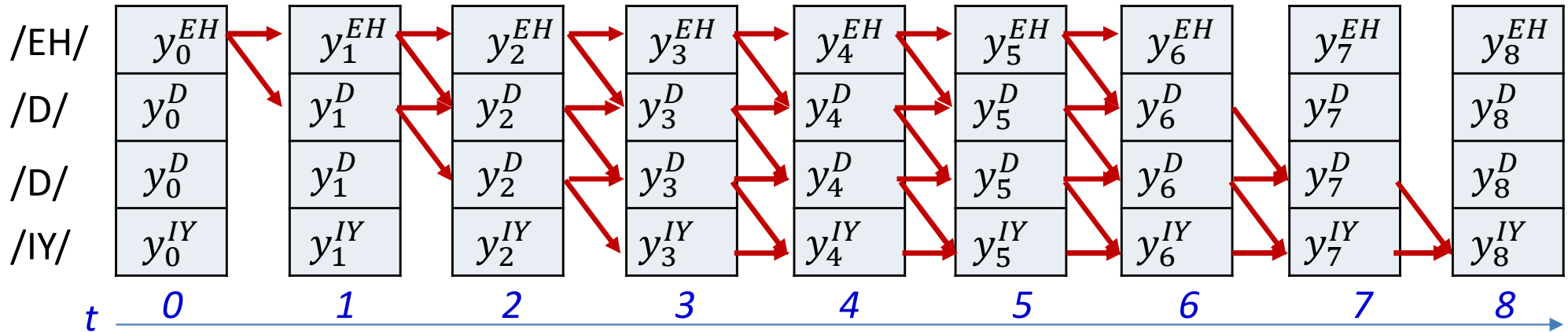
# The modified forward output

- Note the extra “blank” at the output

/B/ /IY/ /F/ /F/ /IY/



# Composing the graph for training

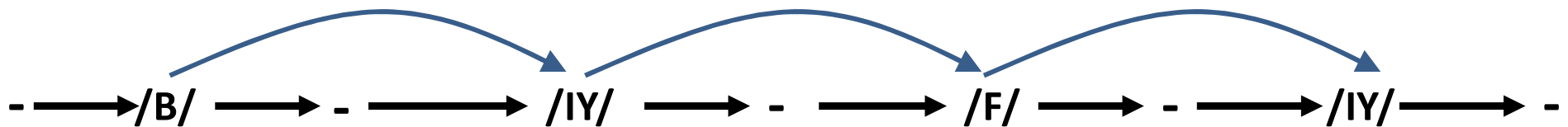


- The original method without blanks
- Changing the example to **/EH/ /D/ /D/ /IY/** for illustration

# How blanks change the graph

/B/ → /IY/ → /F/ → /IY/

- Regular order: Each symbol *must* be followed by the next one

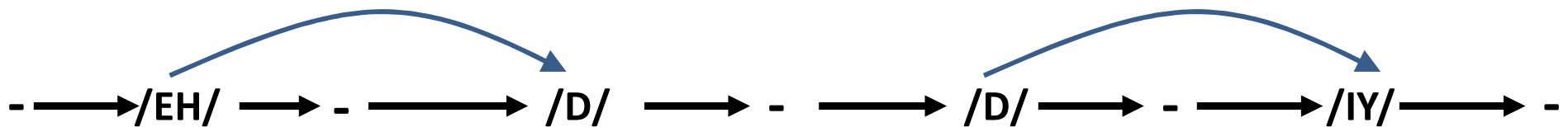


- New pattern: Each symbol may *optionally* be followed by a blank (explicitly shown)

# How blanks change the graph

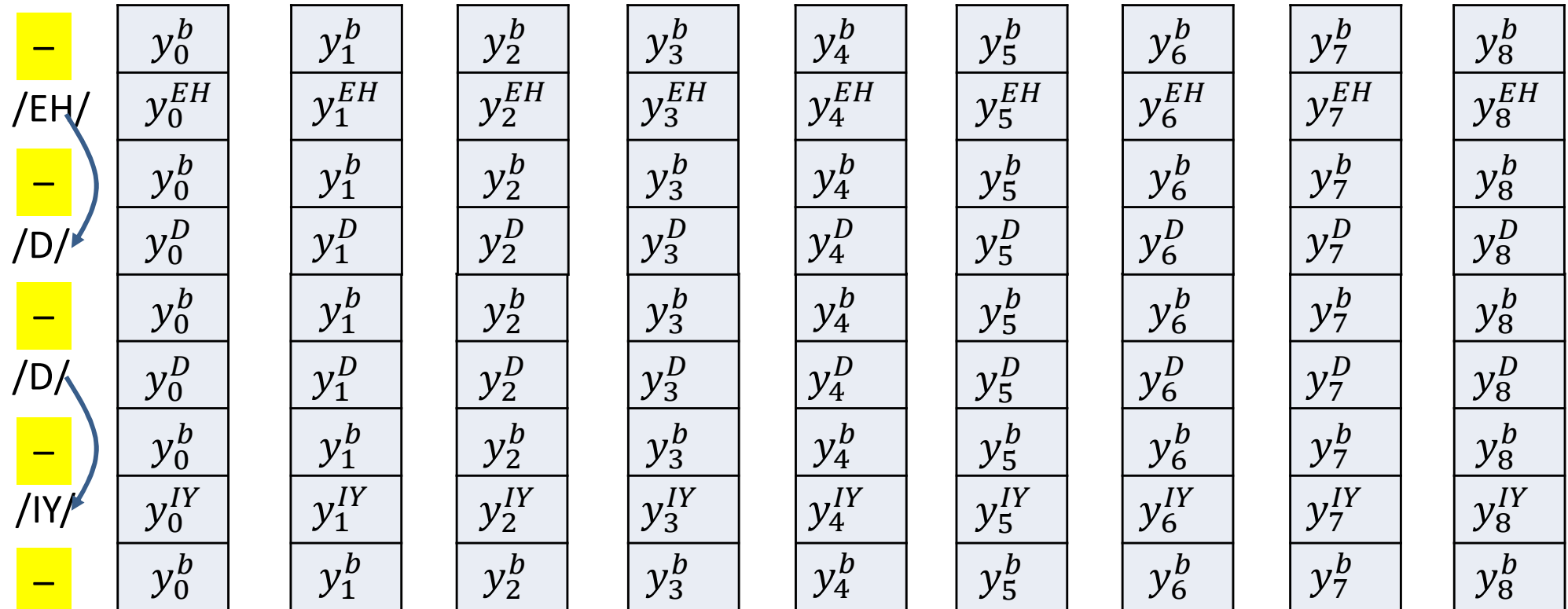
/EH/ → /D/ → /D/ → /IY/

- Regular order: Each symbol *must* be followed by the next one



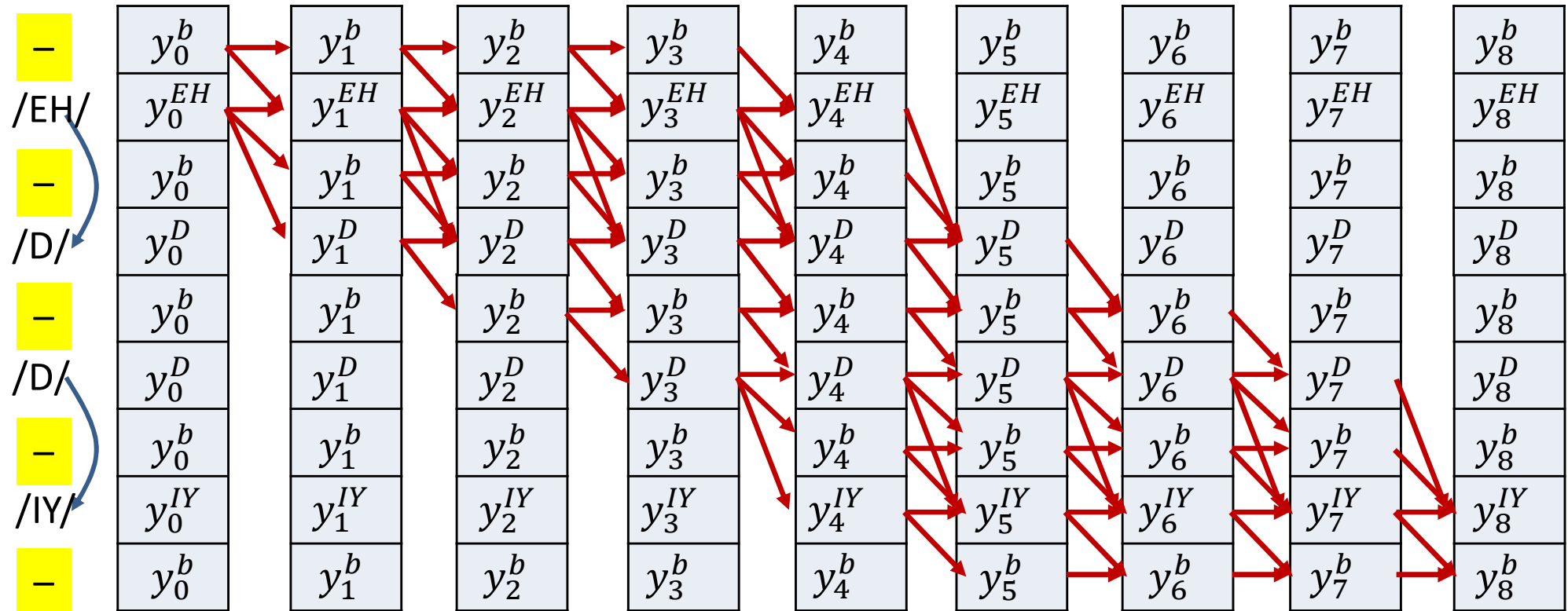
- New pattern: Each symbol may *optionally* be followed by a blank (explicitly shown)
- *Unless the next symbol is the same*
  - E.g. /D/ /D/
  - In this case an intermediate blank is mandatory
  - Blanks may also occur in the first and last positions

# Composing the graph for training



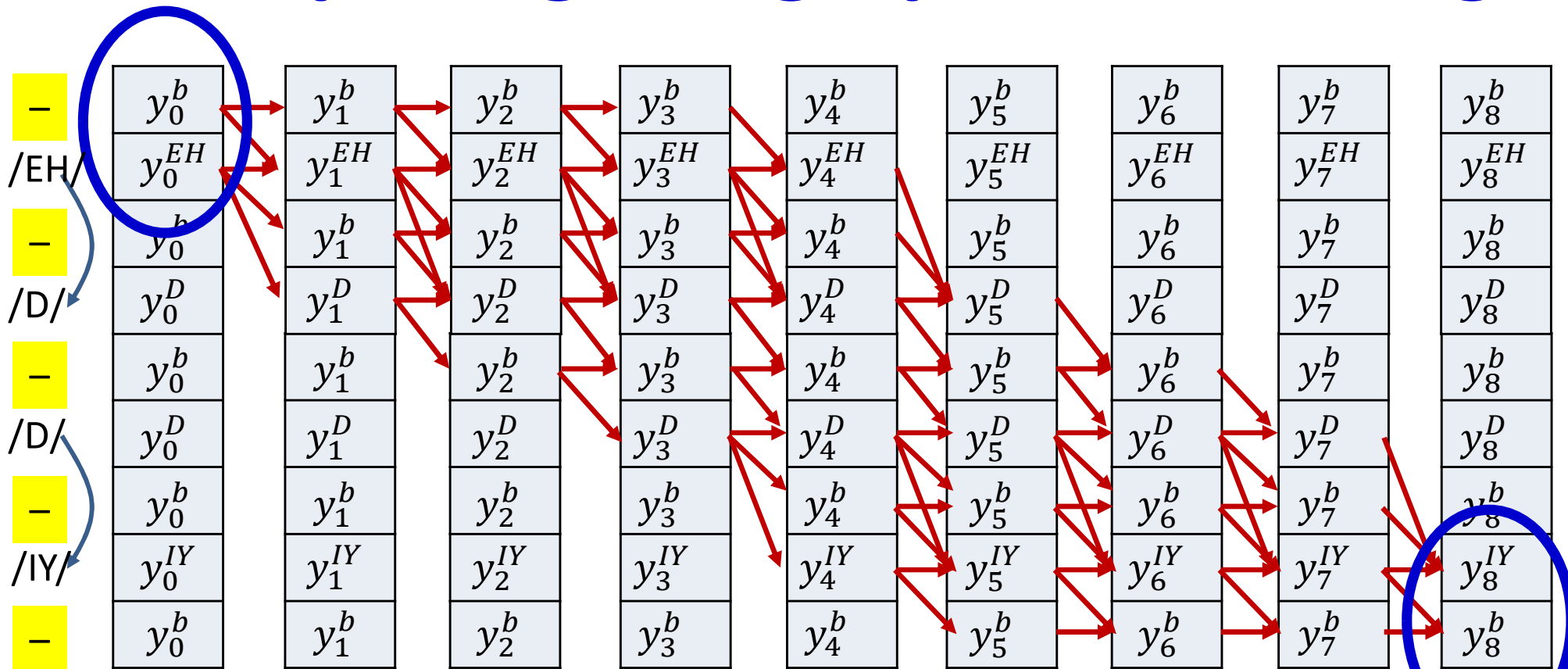
- With blanks
- Follows the graph we just saw
  - Note: a row of blanks between any two symbols
  - Also blanks at the very beginning and the very end

# Composing the graph for training



- Add edges such that all paths from initial node(s) to final node(s) unambiguously represent the target symbol sequence
- If there is an edge on the left graph, there is a corresponding arrow between columns

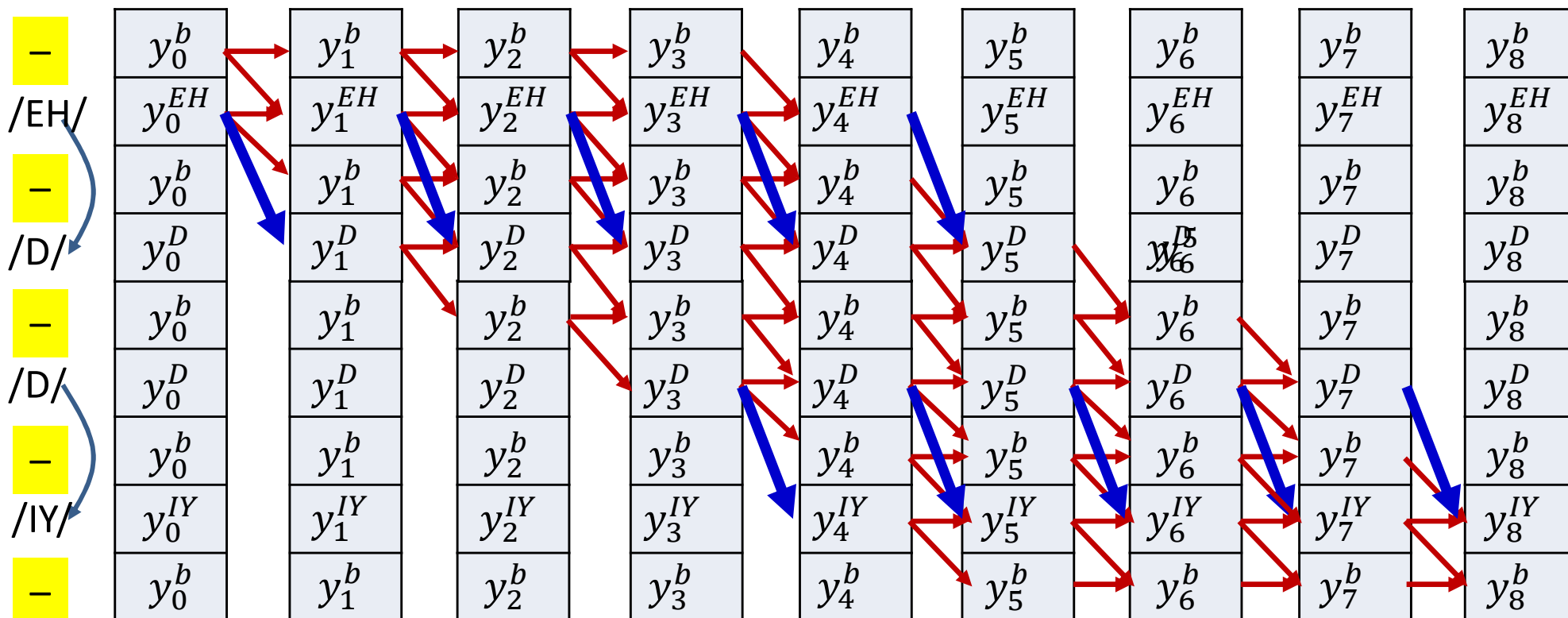
# Composing the graph for training



- The first and last column are allowed to also end at initial and final blanks



# Composing the graph for training



- The first and last column are allowed to also end at initial and final blanks
- Skips are permitted across a blank, but only if the symbols on either side are different
  - Because a blank is *mandatory between repetitions of a symbol* but *not required between distinct symbols*

# Composing the graph

#N is the number of symbols in the target output

#S(i) is the ith symbol in target output

#Compose an *extended* symbol sequence Sext from S, that has the blanks  
#in the appropriate place

#Also keep track of whether an extended symbol Sext(j) is allowed to  
connect

#directly to Sext(j-2) (instead of only to Sext(j-1)) or not

```
function [Sext] = extendedsequencewithblanks(S)
```

```
    j = 1
```

```
    for i = 1:N
```

```
        Sext(j) = 'b' # blank
```

```
        j = j+1
```

```
        Sext(j) = S(i)
```

```
        j = j+1
```

```
    end
```

```
    Sext(j) = 'b'
```

```
    return Sext
```

Using 1..N and 1..T indexing, instead of 0..N-1, 0..T-1, for convenience of notation

## Example of using blanks for alignment: Viterbi alignment with blanks

### MODIFIED VITERBI ALIGNMENT WITH BLANKS

```
[Sext] = extendedsequencewithblanks(S)
```

```
N = length(Sext)    # length of extended sequence
```

```
# Viterbi starts here
```

```
BP(1,1) = -1
```

```
Bscr(1,1) = y(1, Sext(1))    # Blank
```

```
Bscr(1,2) = y(1, Sext(2))
```

```
Bscr(1,2:N) = -inf
```

```
for t = 2:T
```

```
    BP(t,1) = BP(t-1,1);
```

```
    Bscr(t,1) = Bscr(t-1,1)*y(t, Sext(1))
```

```
    for i = 2:N
```

```
        if (i > 2 && Sext(i) != Sext(i-2))
```

```
            BP(t,i) = argmax_i(Bscr(t-1,i), Bscr(t-1,i-1), Bscr(t-1,i-2))
```

```
        else
```

```
            BP(t,i) = argmax_i(Bscr(t-1,i), Bscr(t-1,i-1))
```

```
            Bscr(t,i) = Bscr(t-1, BP(t,i)) * y(t, Sext(i))
```

```
# Backtrace
```

```
AlignedSymbol(T) = Bscr(T,N) > Bscr(T,N-1) ? N, N-1;
```

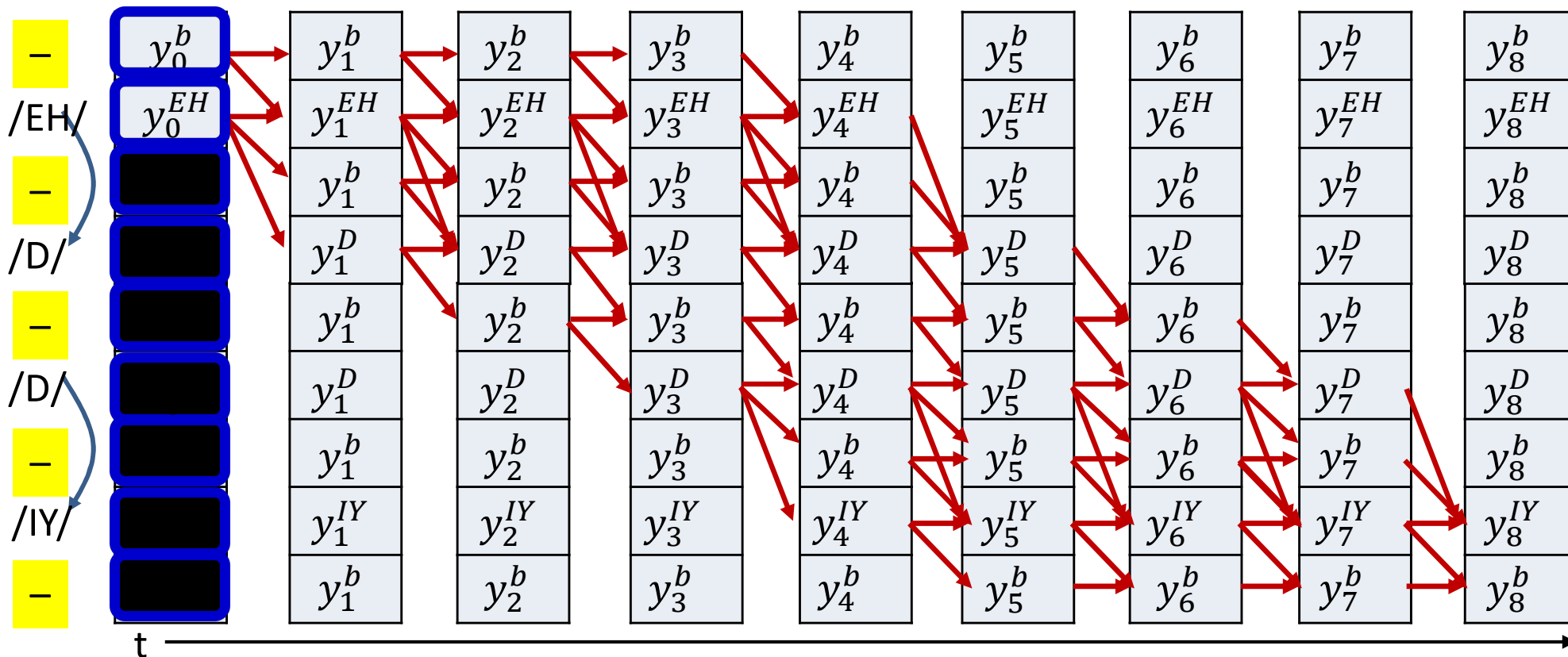
```
for t = T downto 1
```

```
    AlignedSymbol(t-1) = BP(t, AlignedSymbol(t))
```

Without explicit construction of output table

Using 1..N and 1..T indexing, instead of 0..N-1, 0..T-1, for convenience of notation

# Modified Forward Algorithm

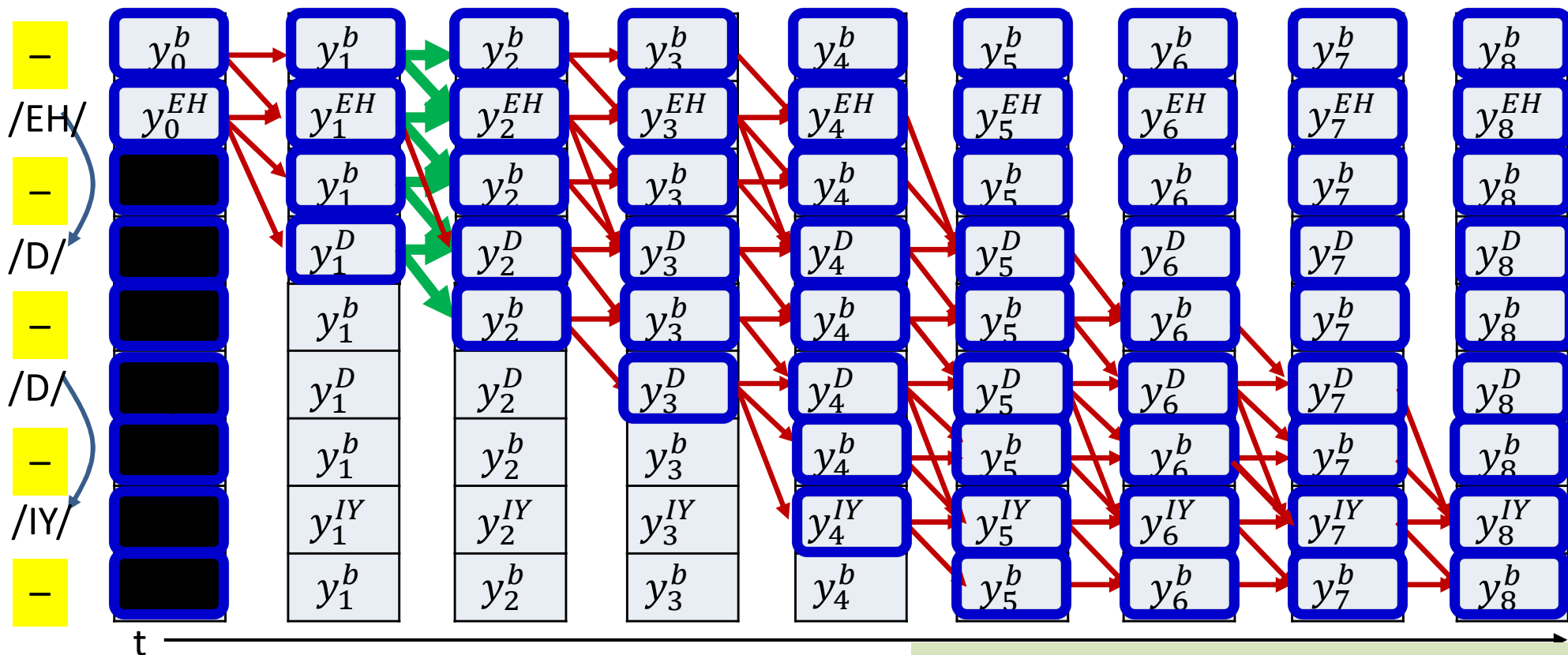


- Initialization:

$$- \alpha(0,0) = y_0^b, \alpha(0,1) = y_0^{S(1)}, \alpha(0,r) = 0 \quad r > 1$$

$S(k)$  refers to the *extended* sequence with blanks included

# Modified Forward Algorithm

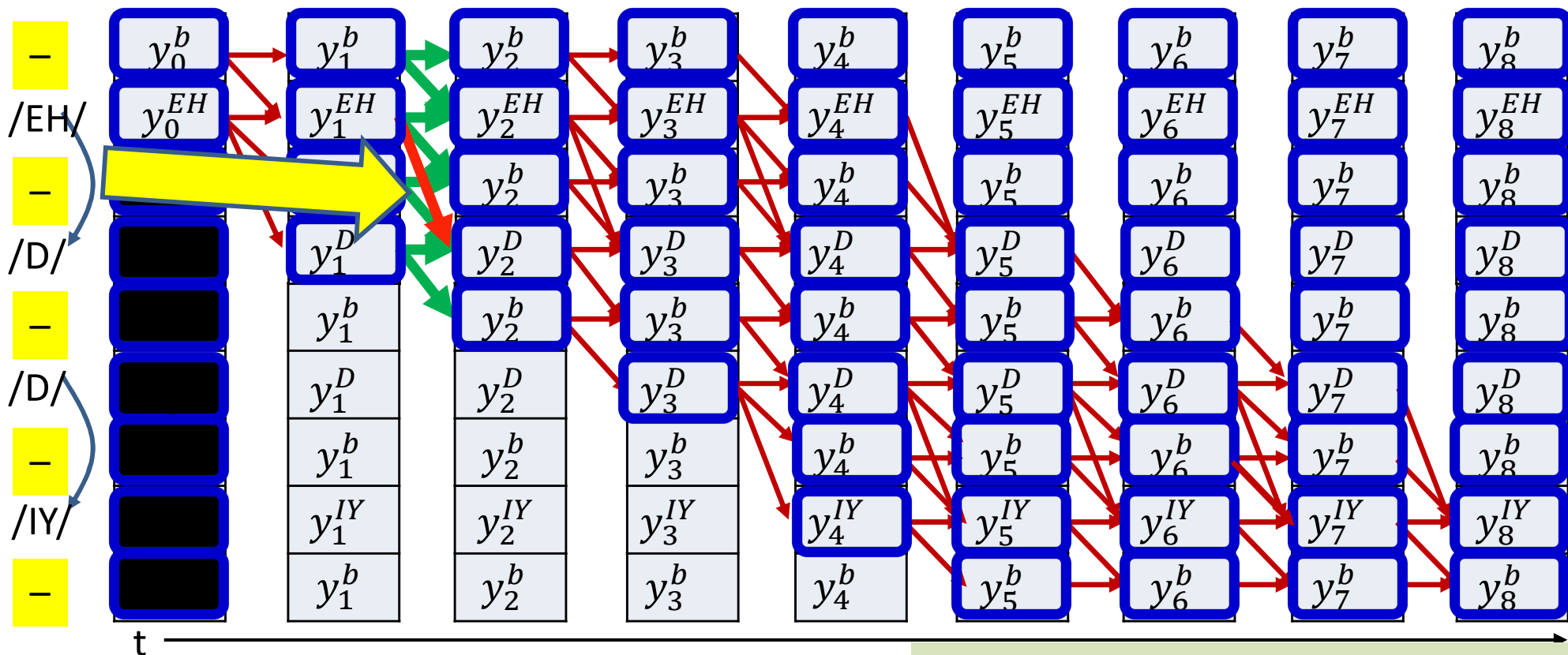


- Iteration  $t = 1:N$ :

$$\alpha(t, r) = \alpha(t - 1, r) + \alpha(t - 1, r - 1)$$

$$\alpha(t, r) = Y_t^{S(r)} \sum_{q: S_q \in \text{pred}(S_r)} \alpha(t - 1, q)$$

# Modified Forward Algorithm



- Iteration  $t = 1:N$ :

$$\alpha(t, r) = \alpha(t - 1, r) + \alpha(t - 1, r - 1)$$

- If  $S(r) \neq S(r - 2)$

$$\alpha(t, r) += \alpha(t - 1, r - 2)$$

$$\alpha(t, r) *= y_t^{S(r)}$$

$$\alpha(t, r) = Y_t^{S(r)} \sum_{q: S_q \in \text{pred}(S_r)} \alpha(t - 1, q)$$

# FORWARD ALGORITHM (with blanks)

**[Sext] = extendedsequencewithblanks (S)**

N = length(Sext) # Length of *extended* sequence

## **#The forward recursion**

# First, at  $t = 1$

alpha(1,1) = y(1,Sext(1)) #This is the blank

alpha(1,2) = y(1,Sext(2))

alpha(1,3:N) = 0

for t = 2:T

    alpha(t,1) = alpha(t-1,1)\*y(t,Sext(1))

    for i = 2:N

        alpha(t,i) = alpha(t-1,i) + alpha(t-1,i-1)

        if (i > 2 && Sext(i) != Sext(i-2))

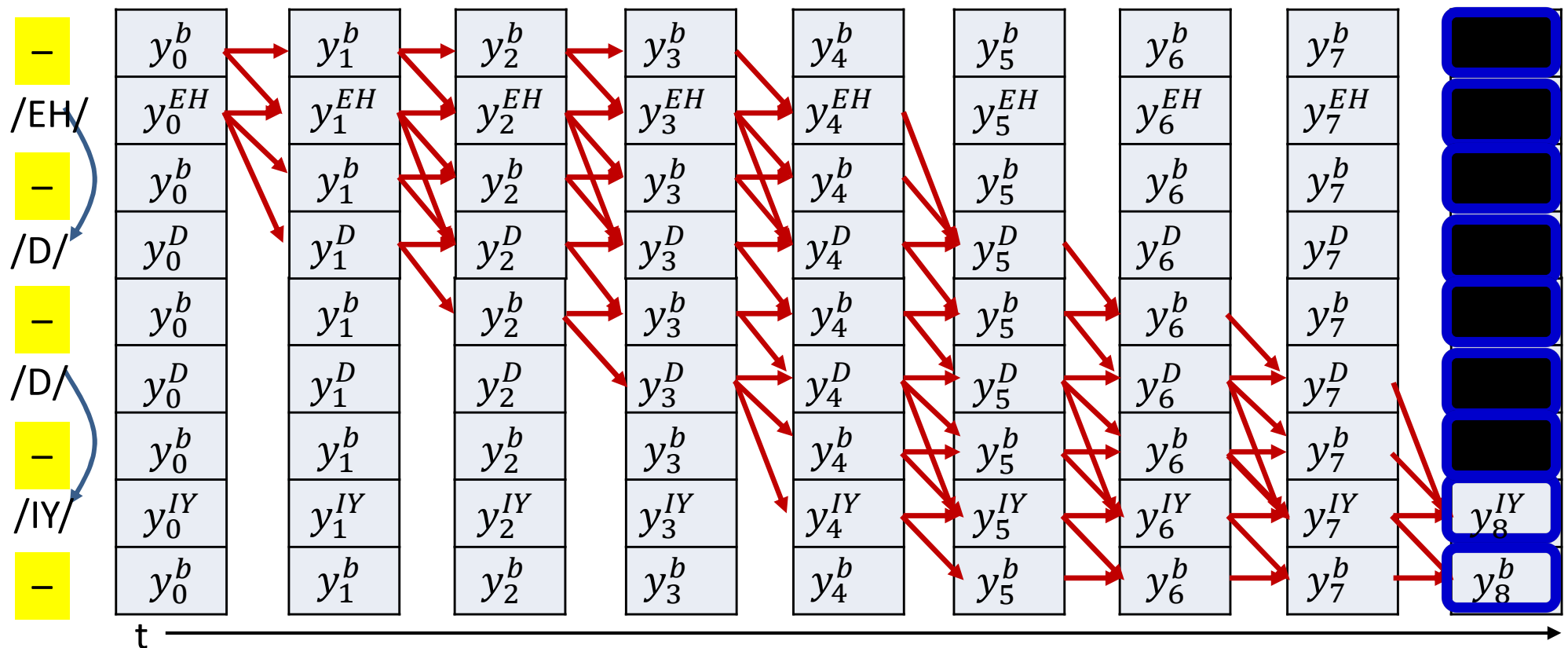
            alpha(t,i) += alpha(t-1,i-2)

        alpha(t,i) \*= y(t,Sext(i))

Without explicitly composing the output table

Using 1..N and 1..T indexing, instead of 0..N-1, 0..T-1, for convenience of notation

# Modified Backward Algorithm



- Initialization:

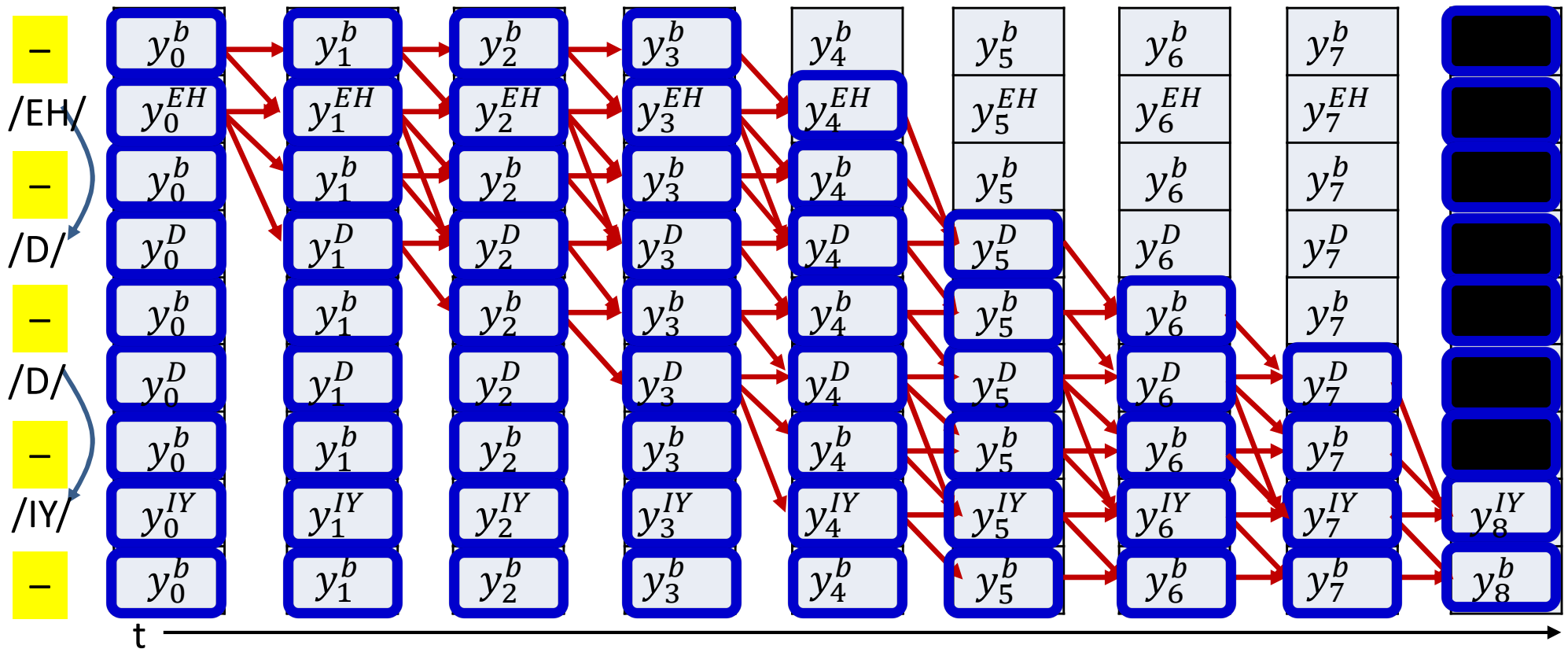
$$\hat{\beta}(T-1, 2K-1) = y_{T-1}^b; \hat{\beta}(T-1, 2K-2) = y_{T-1}^{S(2K-1)}$$

$$\hat{\beta}(T-1, r) = 0 \quad r < 2K-2$$

$S(k)$  refers to the *extended* sequence with blanks included



# Modified Backward Algorithm



- Iteration:

$$\hat{\beta}(t, r) = \hat{\beta}(t + 1, r) + \hat{\beta}(t + 1, r + 1)$$

- If  $S(r) \neq S(r + 2)$

$$\hat{\beta}(t, r) += \hat{\beta}(t + 1, r + 2)$$

$$\hat{\beta}(t, r) *= y_t^{S(r)}$$

- $\beta(t, r) = \hat{\beta}(t, r) / y_t^{S(r)}$

$$\hat{\beta}(t, r) = y_t^{S_r} \sum_{q: S_q \in \text{succ}(S_r)} \hat{\beta}(t + 1, q)$$

# BACKWARD ALGORITHM WITH BLANKS

```
[Sext] = extendedsequencewithblanks(S)
```

```
N = length(Sext) # Length of extended sequence
```

## #The backward recursion

```
# First, at t = T
```

```
betahat(T,N) = y(T,Sext(N))
```

```
betahat(T,N-1) = y(T,Sext(N-1))
```

```
betahat(T,1:N-2) = 0
```

```
for t = T-1 downto 1
```

```
    betahat(t,N) = betahat(t+1,N)*y(t,Sext(N))
```

```
    for i = N-1 downto 1
```

```
        betahat(t,i) = betahat(t+1,i) + betahat(t+1,i+1))
```

```
        if (i<=N-2 && Sext(i) != Sext(i+2))
```

```
            betahat(t,i) += betahat(t+1,i+2)
```

```
        betahat(t,i) *= y(t,Sext(i))
```

## #Compute beta from betahat

```
for t = T downto 1
```

```
    for i = N downto 1
```

```
        beta(t,i) = betahat(t,i)/y(t,Sext(i))
```

Without explicitly composing the output table

Using 1..N and 1..T indexing, instead of 0..N-1, 0..T-1, for convenience of notation

# The rest of the computation

- Posteriors and derivatives are computed exactly as before
- But using the extended graphs with blanks

# COMPUTING POSTERIORIS

```
[Sext, skipconnect] = extendedsequencewithblanks(S)
```

```
N = length(Sext) # Length of extended sequence
```

```
#Assuming the forward are completed first
```

```
alpha = forward(y, Sext) # forward probabilities computed
```

```
beta = backward(y, Sext) # backward probabilities computed
```

```
#Now compute the posteriors
```

```
for t = 1:T
```

```
    sumgamma(t) = 0
```

```
    for i = 1:N
```

```
        gamma(t,i) = alpha(t,i) * beta(t,i)
```

```
        sumgamma(t) += gamma(t,i)
```

```
    end
```

```
    for i=1:N
```

```
        gamma(t,i) = gamma(t,i) / sumgamma(t)
```

Using 1..N and 1..T indexing, instead of 0..N-1, 0..T-1, for convenience of notation

# COMPUTING DERIVATIVES

```
[Sext, skipconnect] = extendedsequencewithblanks(S)
N = length(Sext) # Length of extended sequence

#Assuming the forward are completed first
alpha = forward(y, Sext) # forward probabilities computed
beta = backward(y, Sext) # backward probabilities computed

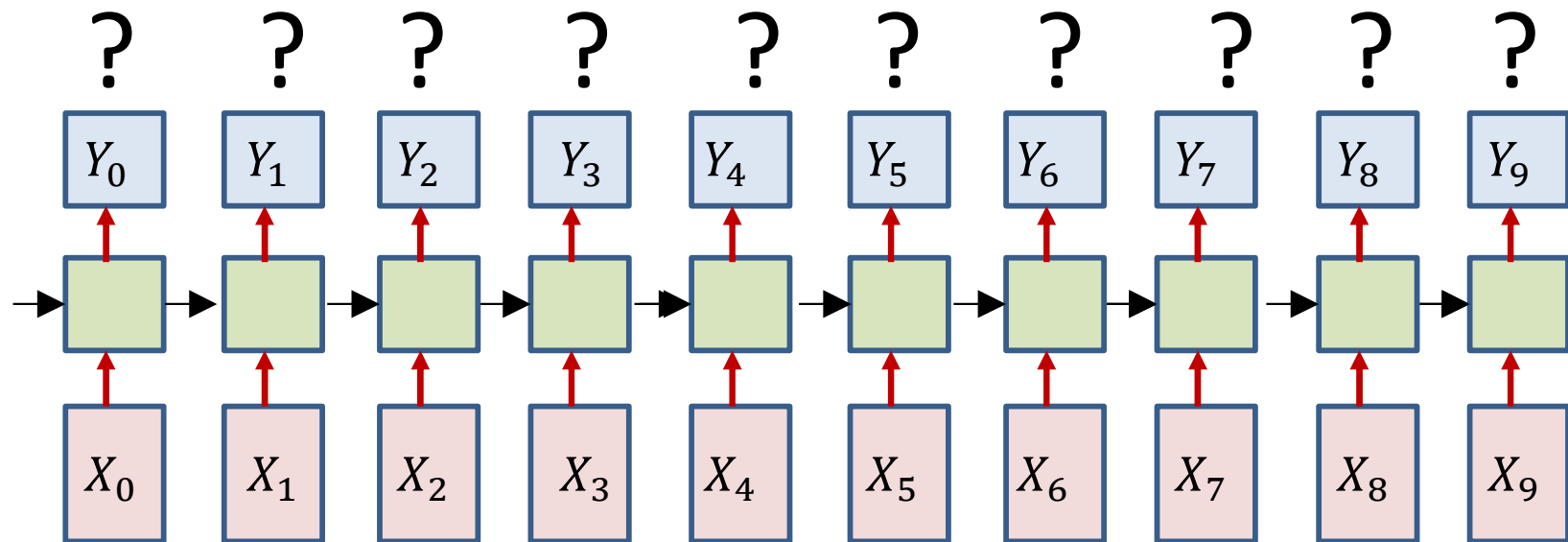
# Compute posteriors from alpha and beta
gamma = computeposteriors(alpha, beta)

#Compute derivatives
for t = 1:T
    dy(t,1:L) = 0 #Initialize all derivatives at time t to 0
    for i = 1:N
        dy(t,Sext(i)) -= gamma(t,i) / y(t,Sext(i))
    end
end
```

Using 1..N and 1..T indexing, instead of 0..N-1, 0..T-1, for convenience of notation

# Overall training procedure for Seq2Seq with blanks

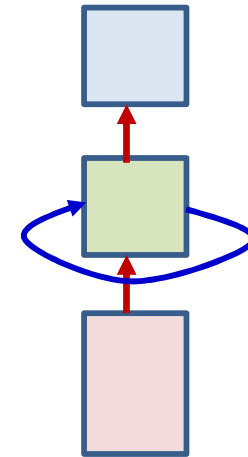
*/EH/ /D/ /D/ /IY/*



- Problem: Given input and output sequences without alignment, train models

# Overall training procedure

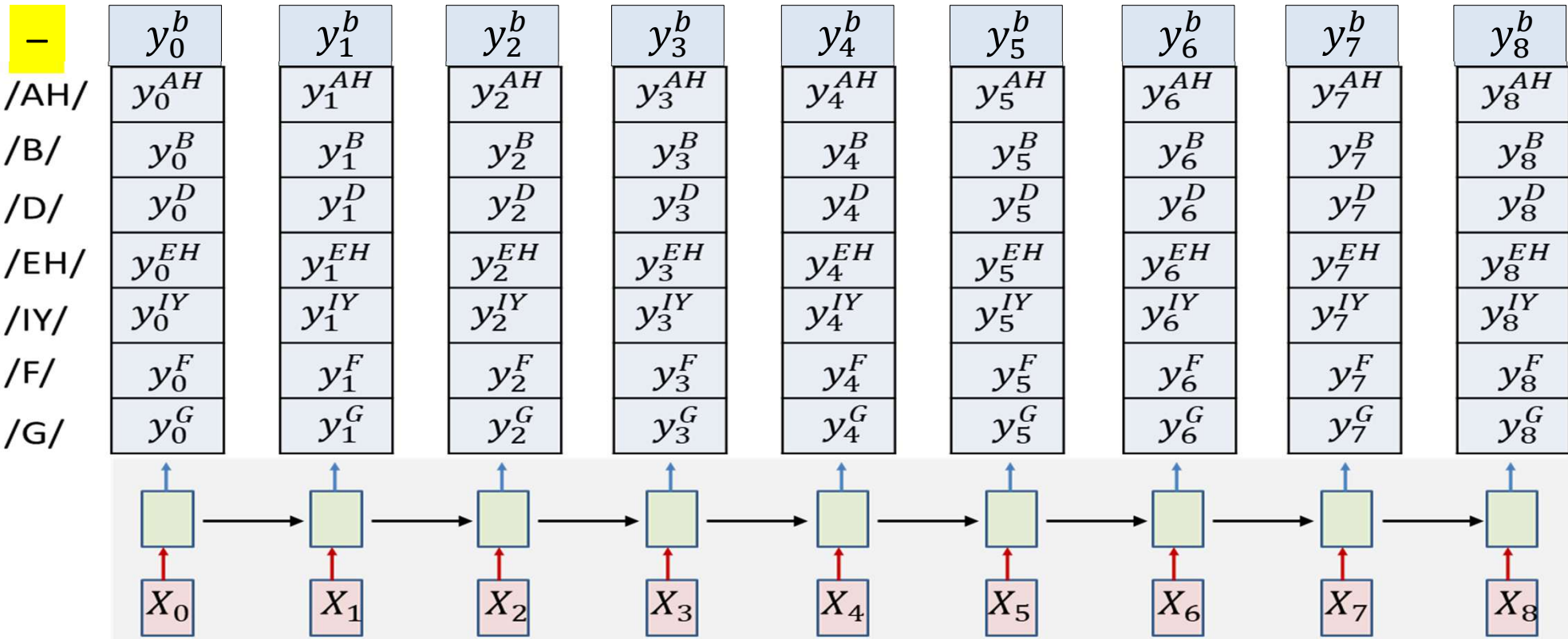
- **Step 1:** Setup the network
  - Typically many-layered LSTM



- **Step 2:** Initialize all parameters of the network
  - Include a “blank” symbol in vocabulary

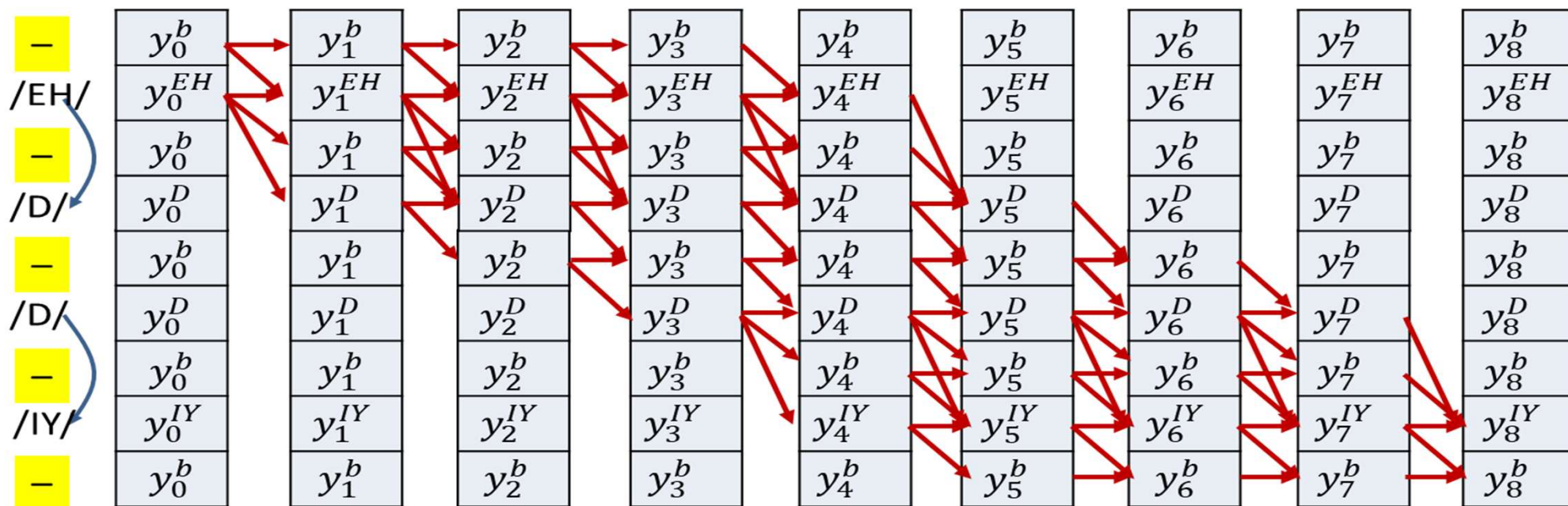
# Overall Training: Forward pass

- Foreach training instance
  - **Step 3:** Forward pass. Pass the training instance through the network and obtain all symbol probabilities at each time, including blanks



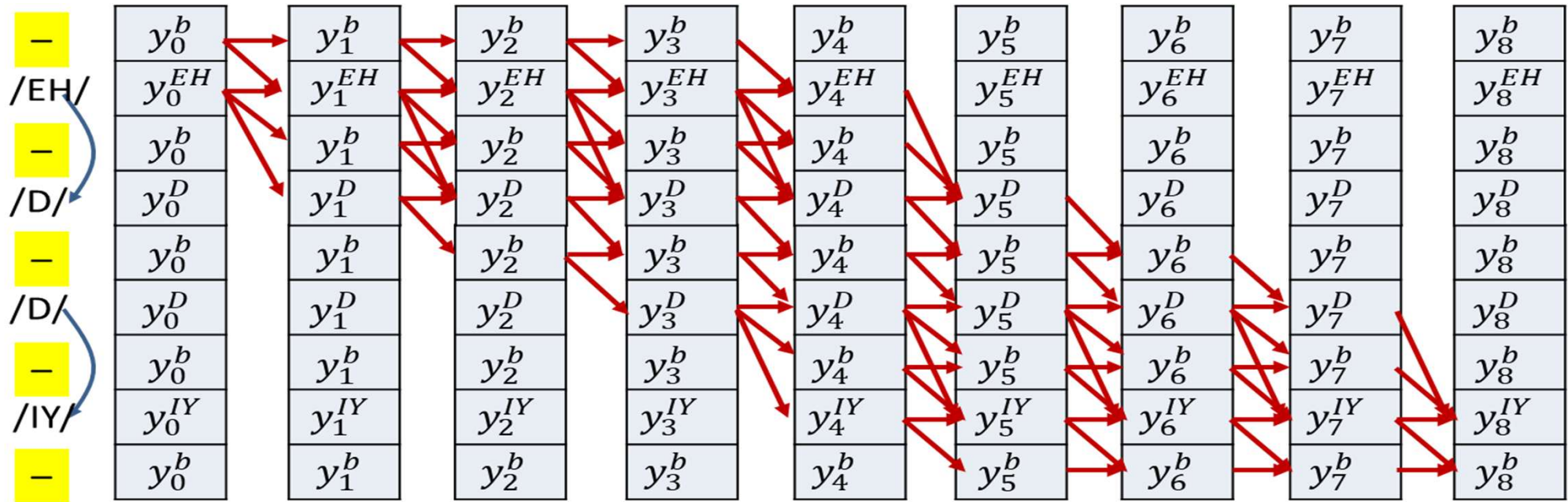


# Overall training: Backward pass



- Foreach training instance
  - **Step 3:** Forward pass. Pass the training instance through the network and obtain all symbol probabilities at each time
  - **Step 4:** Construct the graph representing the specific symbol sequence in the instance. Use appropriate connections if blanks are included

# Overall training: Backward pass



- Foreach training instance:
  - **Step 5:** Perform the forward backward algorithm to compute  $\alpha(t, r)$  and  $\beta(t, r)$  at each time, for each row of nodes in the graph using the modified forward-backward equations. Compute a posteriori probabilities  $\gamma(t, r)$  from them
  - **Step 6:** Compute derivative of divergence  $\nabla_{Y_t} DIV$  for each  $Y_t$

# Overall training: Backward pass

- Foreach instance
  - **Step 6:** Compute derivative of divergence  $\nabla_{Y_t} DIV$  for each  $Y_t$

$$\nabla_{Y_t} DIV = \left[ \frac{dDIV}{dy_t^0} \quad \frac{dDIV}{dy_t^1} \quad \dots \quad \frac{dDIV}{dy_t^{L-1}} \right]$$

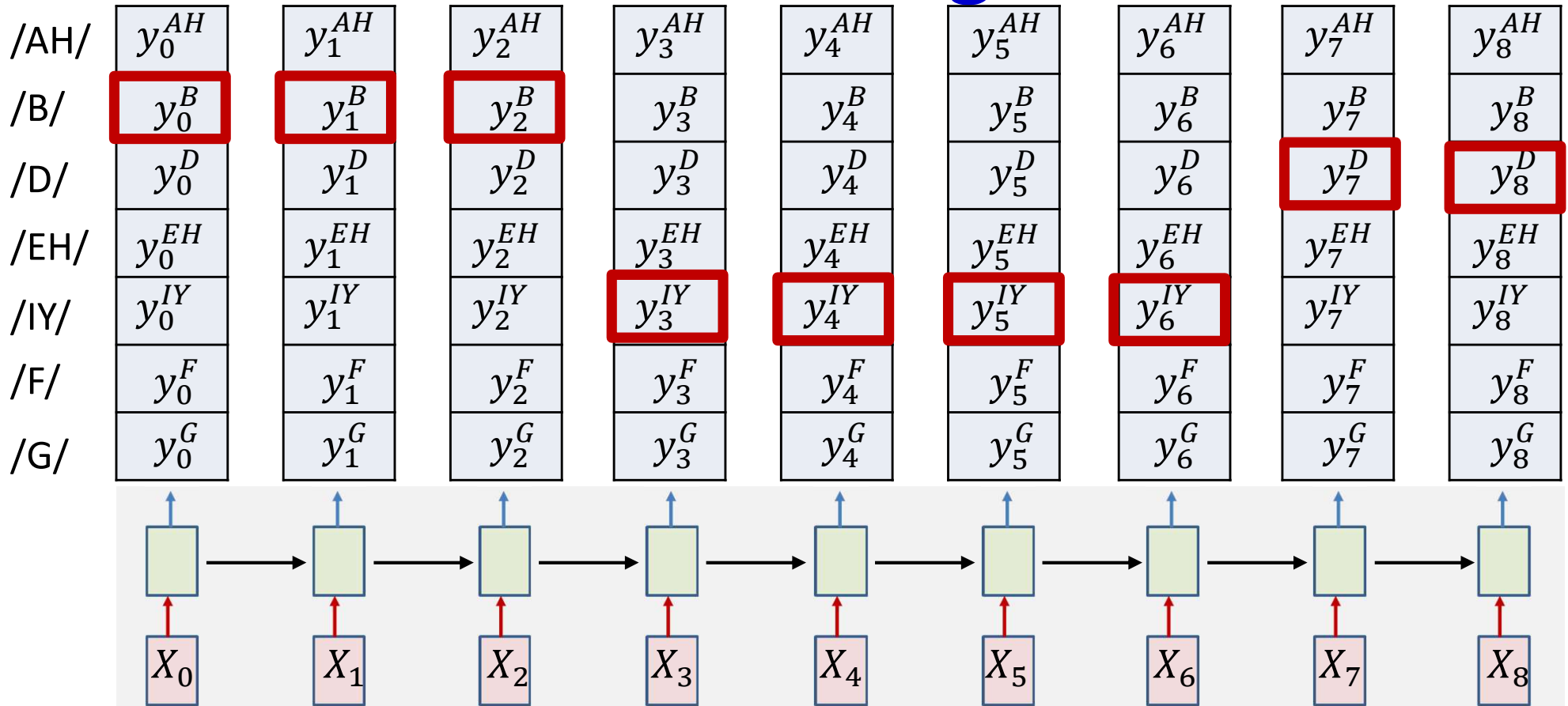
$$\frac{dDIV}{dy_t^l} = - \sum_{r:S(r)=l} \frac{\gamma(t,r)}{y_t^{S(r)}}$$

- **Step 7:** Backpropagate  $\frac{dDIV}{dy_t^l}$  and aggregate derivatives over minibatch and update parameters

# CTC: Connectionist Temporal Classification

- The overall framework we saw is referred to as CTC
- Applies to models that output order-aligned, but time-asynchronous outputs

# Returning to an old problem: Decoding



- The greedy decode computes its output by finding the most likely symbol at each time and merging repetitions in the sequence
- This is in fact a *suboptimal* decode that actually finds the most likely *time-synchronous* output sequence
  - Which is not necessarily the most likely *order-synchronous* sequence

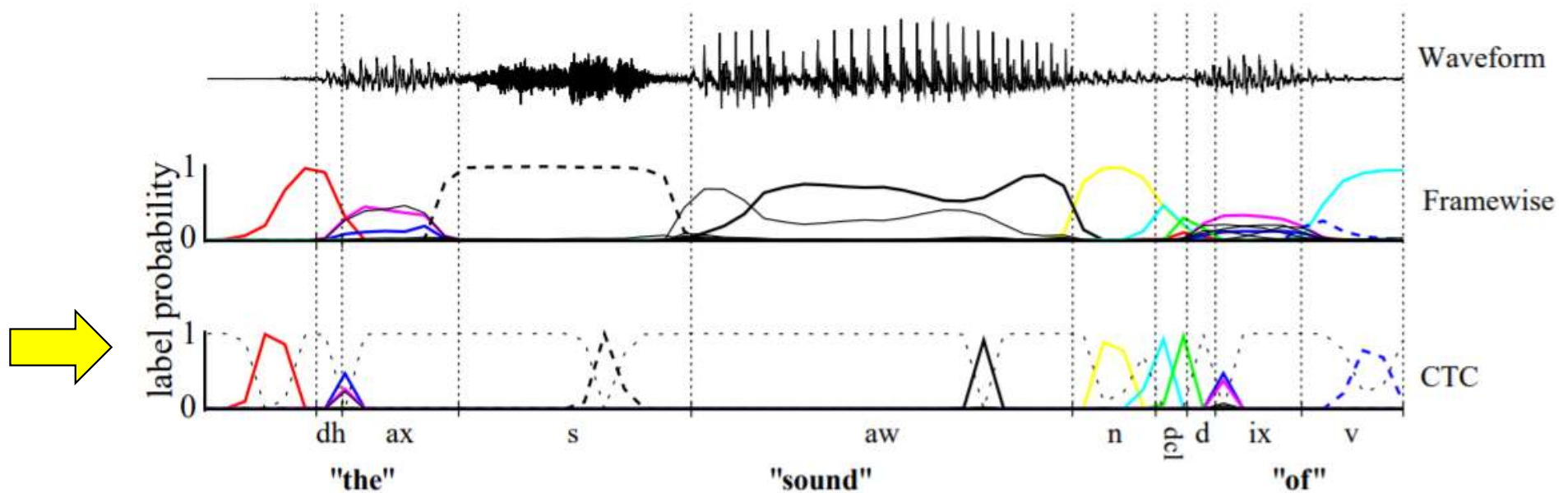
# Greedy decodes are suboptimal

- Consider the following candidate decodes
  - R R – E E D (RED, 0.7)
  - R R – – E D (RED, 0.68)
  - R R E E E D (RED, 0.69)
  - T T E E E D (TED, 0.71)
  - T T – E E D (TED, 0.3)
  - T T – – E D (TED, 0.29)
- A greedy decode picks the most likely output: TED
- A decode that considers the sum of all alignments of the same final output will select RED
- Which is more reasonable?

# Greedy decodes are suboptimal

- Consider the following candidate decodes
  - R R – E E D (RED, 0.7)
  - R R – – E D (RED, 0.68)
  - R R E E E D (RED, 0.69)
  - T T E E E D (TED, 0.71)
  - T T – E E D (TED, 0.3)
  - T T – – E D (TED, 0.29)
- A greedy decode picks the most likely output: TED
- A decode that considers the sum of all alignments of the same final output will select RED
- Which is more reasonable?
- *And yet, remarkably, greedy decoding can be surprisingly effective, when using decoding with blanks*

# What a CTC system outputs



*Figure 1. Framewise and CTC networks classifying a speech signal. The shaded lines are the output activations, corresponding to the probabilities of observing phonemes at particular times. The CTC network predicts only the sequence of phonemes (typically as a series of spikes, separated by 'blanks', or null predictions), while the framewise network attempts to align them with the manual segmentation (vertical lines). The framewise network receives an error for misaligning the segment boundaries, even if it predicts the correct phoneme (e.g. 'dh'). When one phoneme always occurs beside another (e.g. the closure 'dcl' with the stop 'd'), CTC tends to predict them together in a double spike. The choice of labelling can be read directly from the CTC outputs (follow the spikes), whereas the predictions of the framewise network must be post-processed before use.*

- Ref: Graves
- Symbol outputs peak at the ends of the sounds
  - Typical output: -- R --- E --- D
  - Model output naturally eliminates alignment ambiguities
- But this is still suboptimal..

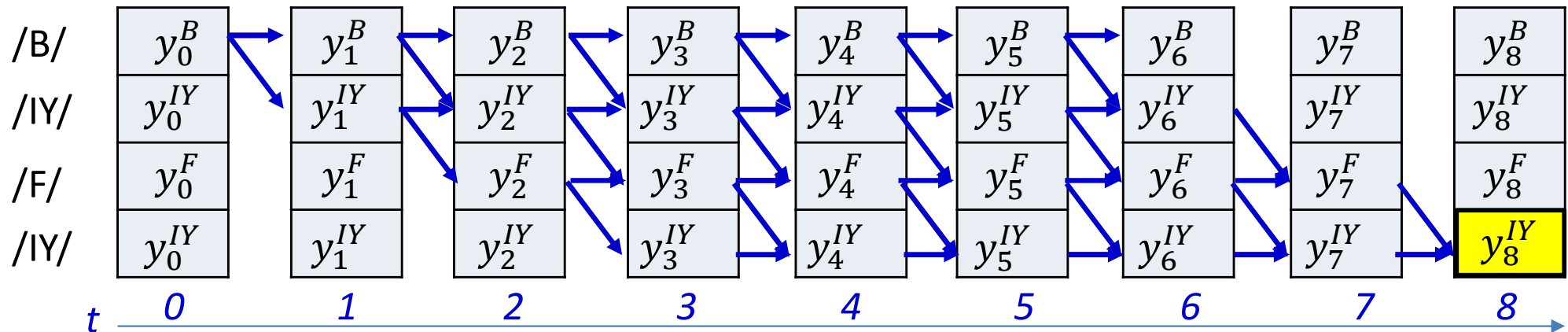


# Actual objective of decoding

- Want to find most likely order-aligned symbol sequence
  - **RED**
  - What greedy decode finds: most likely time synchronous symbol sequence
    - **- /R/ /R/ -- /EH//EH//D/**
    - Which must be compressed
- Find the order-aligned symbol sequence  $\mathcal{S} = S_0, \dots, S_{K-1}$ , given an input  $\mathbf{X} = X_0, \dots, X_{T-1}$ , that is most likely

$$= \underset{\mathcal{S}}{\operatorname{argmax}} P(S_0, \dots, S_{K-1} | \mathbf{X})$$

# Recall: The forward probability $\alpha(t, r)$



$$\alpha_{S_0 \dots S_{K-1}}(T-1, K-1) = P(S_0 \dots S_{K-1} | \mathbf{X})$$

- The probability of the entire symbol sequence is the alpha at the bottom right node

# Actual decoding objective

- Find the most likely (asynchronous) symbol sequence

$$\hat{\mathbf{S}} = \operatorname{argmax}_{\mathbf{S}} \alpha_{\mathbf{S}}(S_{K-1}, T - 1)$$

# Poll 5 (@794, @795)

The actual objective of decoding is to identify the compressed/unaligned sequence that has the highest probability given the input

- True
- False

This is the same as finding the compressed sequence with the highest forward probability (alpha) for aligning the final symbol in the sequence to the final input

- True
- False

# Poll 5

The actual objective of decoding is to identify the compressed/unaligned sequence that has the highest probability given the input

- **True**
- False

This is the same as finding the compressed sequence with the highest forward probability (alpha) for aligning the final symbol in the sequence to the final input

- **True**
- False

# Actual decoding objective

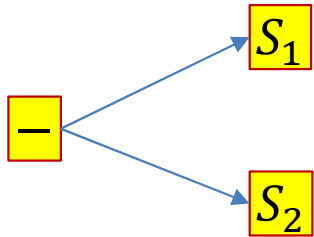
- Find the most likely (asynchronous) symbol sequence

$$\hat{\mathbf{S}} = \underset{\mathbf{S}}{\operatorname{argmax}} \alpha_{\mathbf{S}}(S_{K-1}, T - 1)$$

- Unfortunately, explicit computation of this will require evaluate of an exponential number of symbol sequences
- Solution: Organize all possible symbol sequences as a (semi)tree

# Hypothesis semi-tree

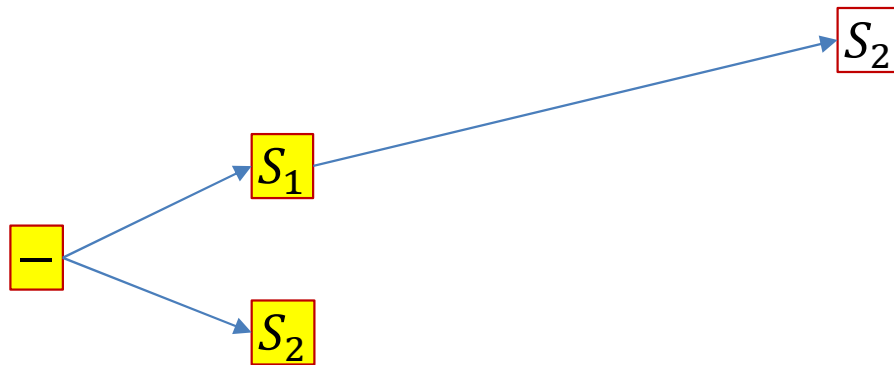
Highlighted boxes represent possible symbols for first frame



- The semi tree of hypotheses (assuming only 3 symbols in the vocabulary)
- Every symbol connects to every symbol other than itself
  - It also connects to a blank, which connects to every symbol including itself
- The simple structure repeats recursively
- Each node represents a unique (partial) symbol sequence!

# Hypothesis semi-tree

Highlighted boxes represent possible symbols for first frame

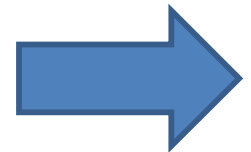
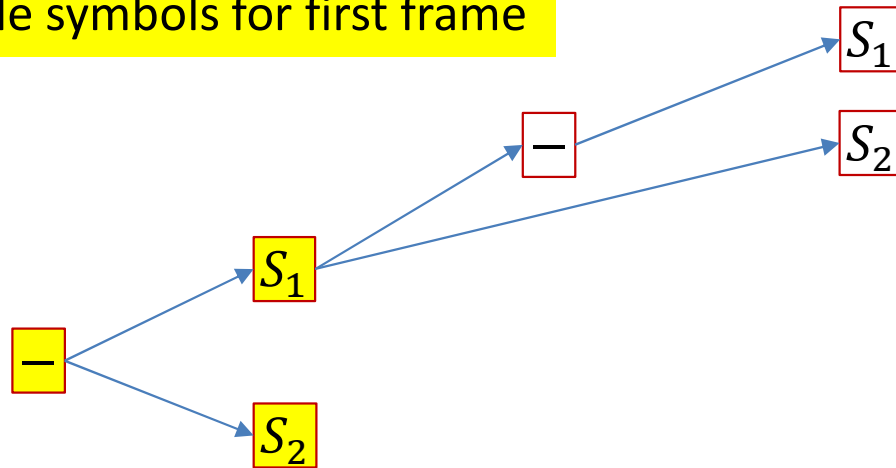


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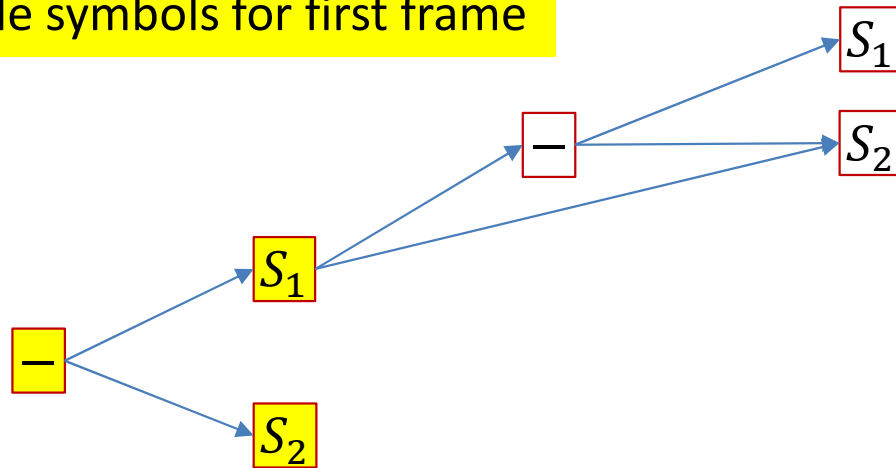
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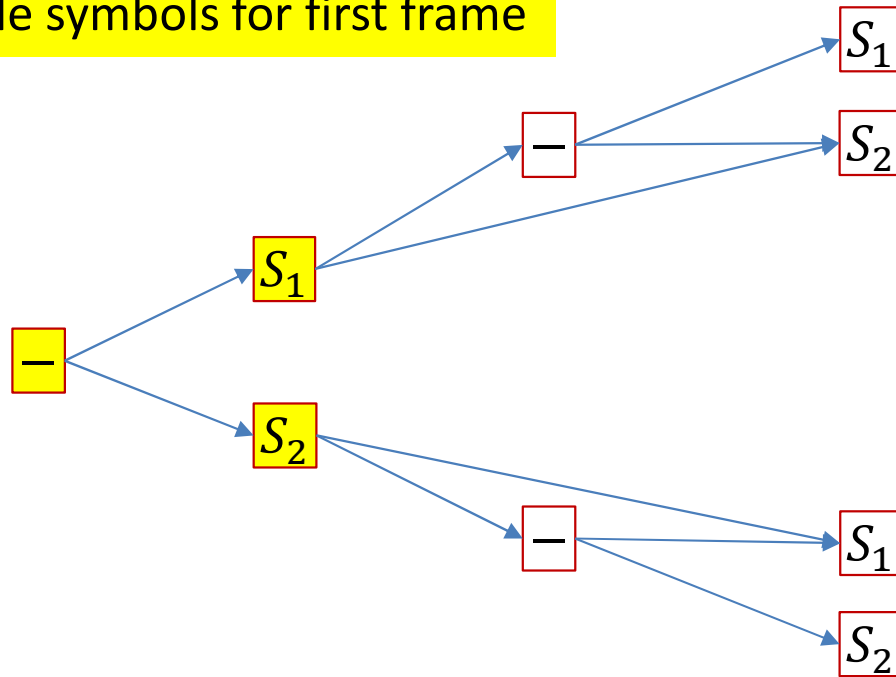
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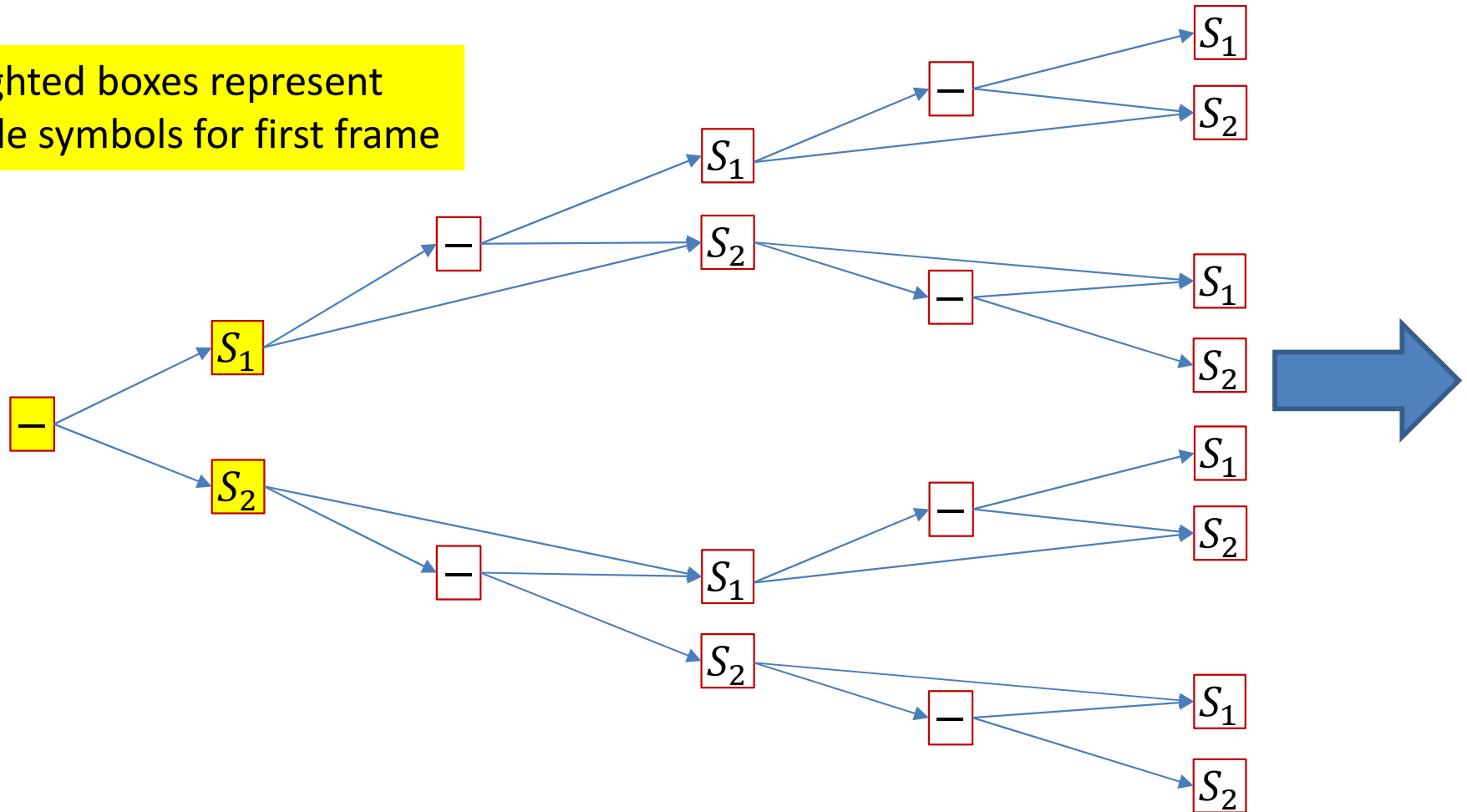
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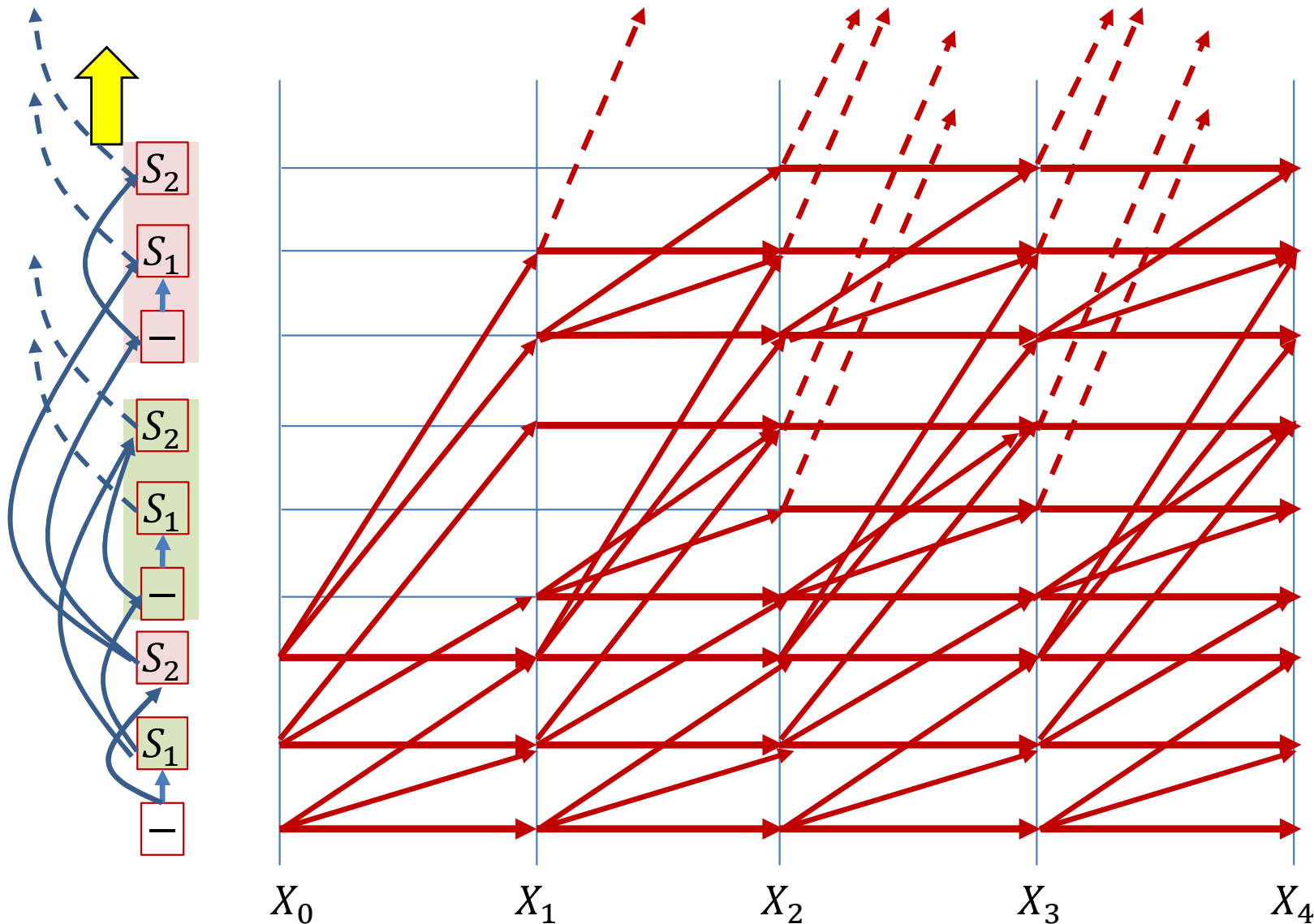
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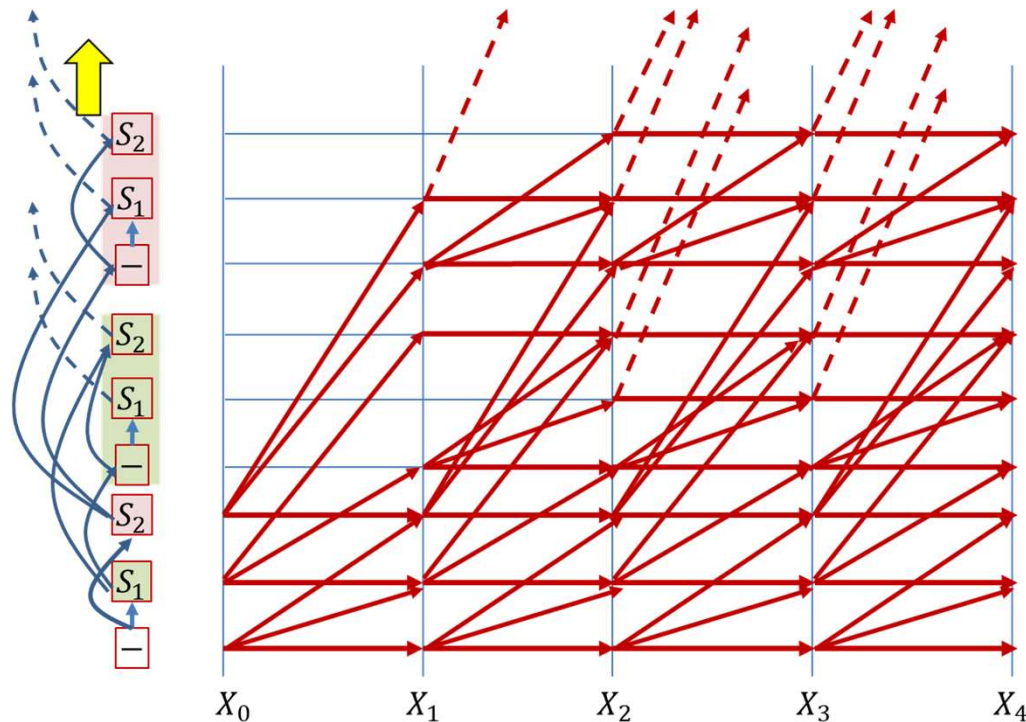
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# The decoding graph for the tree



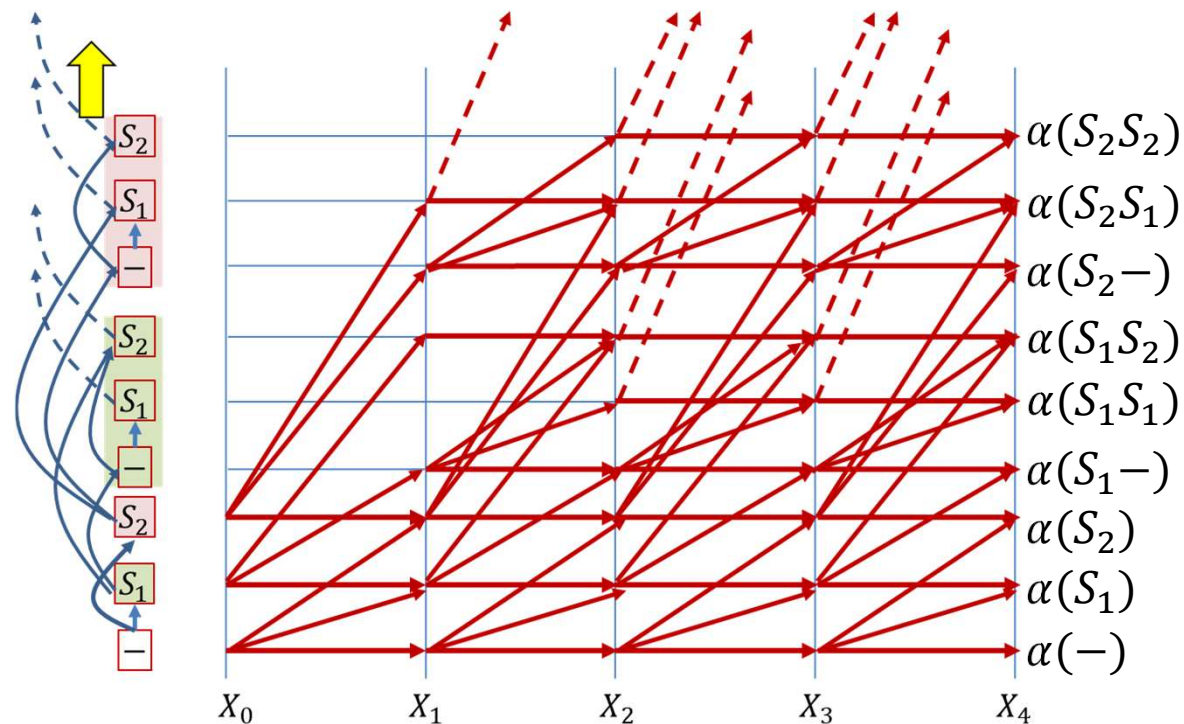
- Graph with more than 2 symbols will be similar but much more cluttered and complicated

# The decoding graph for the tree



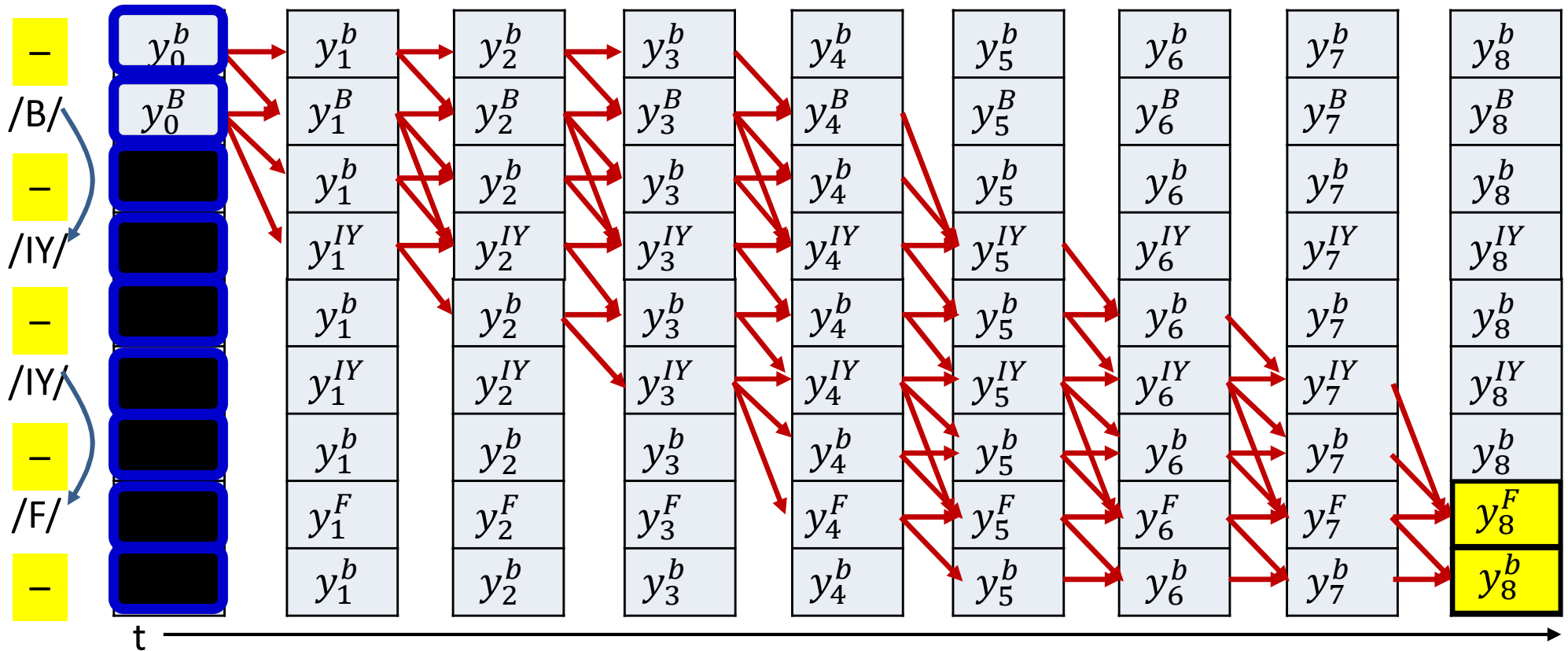
- The figure to the left is the tree, drawn in a vertical line
- The graph is just the tree unrolled over time
  - For a vocabulary of  $V$  symbols, every node connects out to  $V$  other nodes at the next time
- Every node in the graph represents a unique symbol sequence

# The decoding graph for the tree



- The forward score  $\alpha(r, T)$  at the final time represents the full forward score for a unique symbol sequence (including sequences terminating in blanks)
- Select the symbol sequence with the largest alpha at the final time

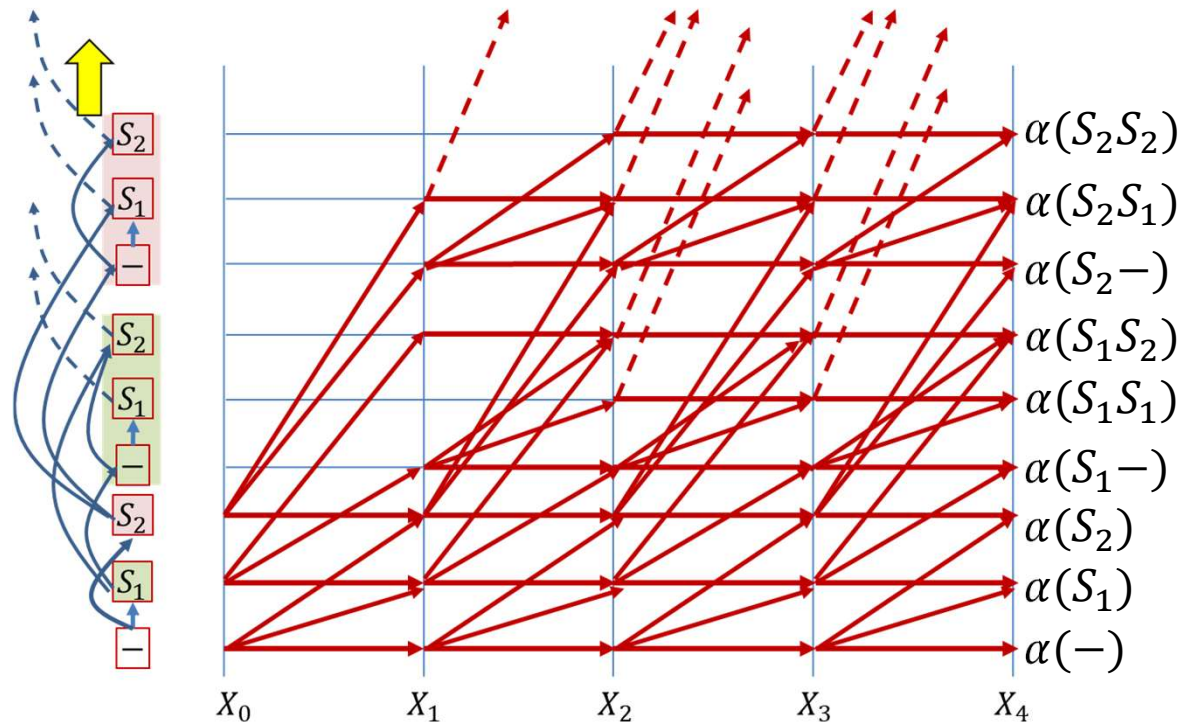
# Recall: Forward Algorithm



- $$P(S_0, \dots, S_{K-1} | \mathbf{X}) = \alpha(T-1, 2K) + \alpha(T-1, 2K+1)$$

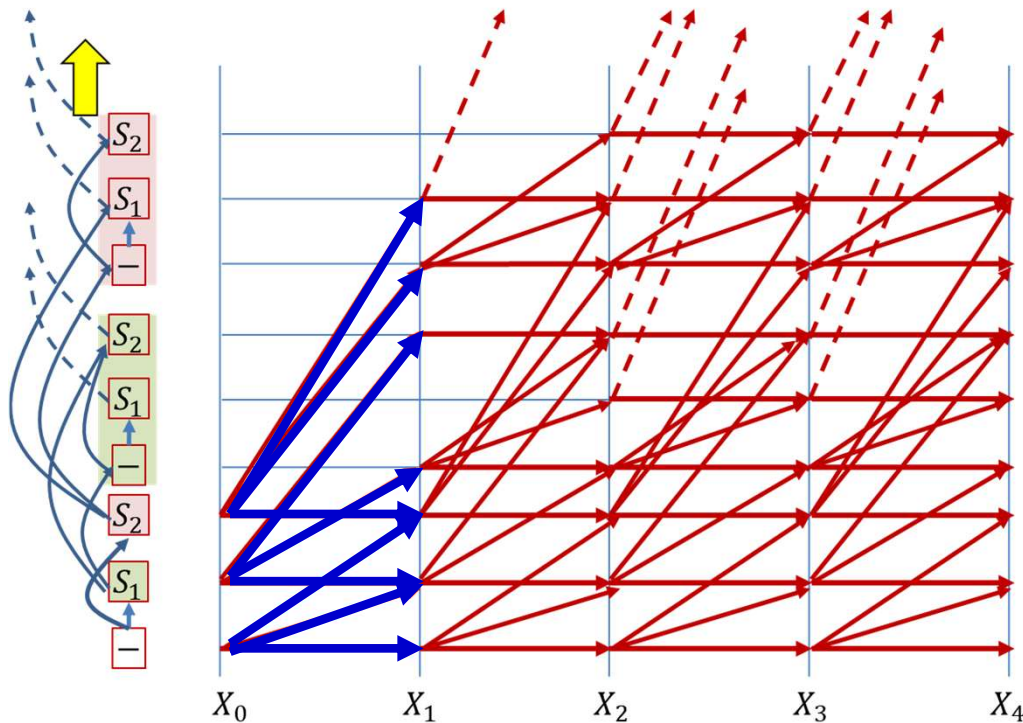


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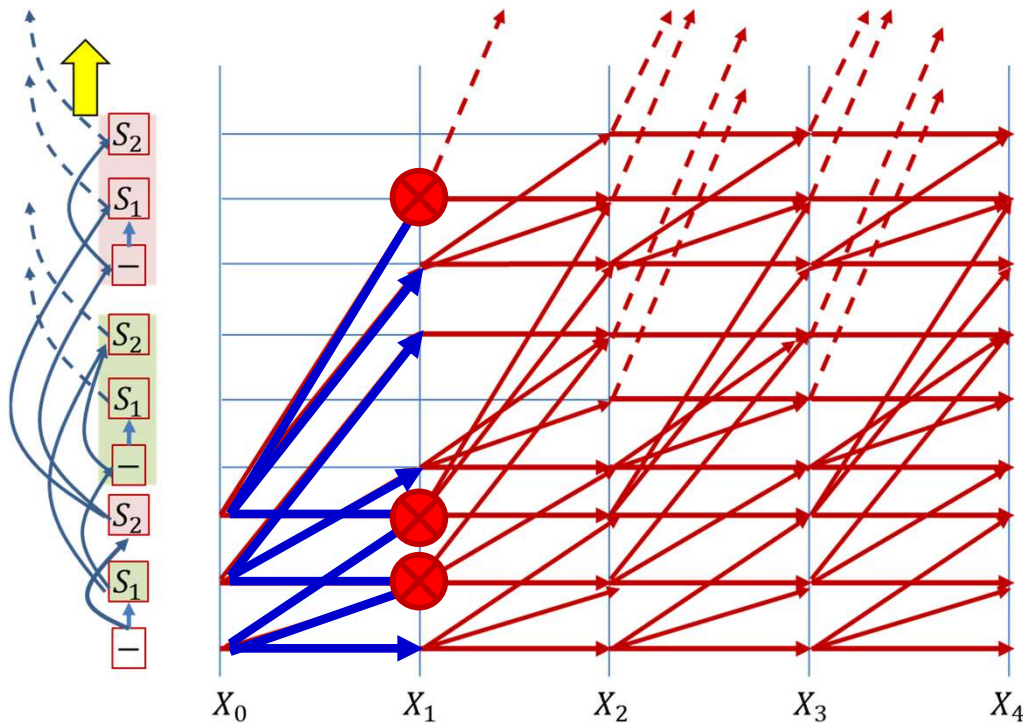
- The forward score  $\alpha(r, T)$  at the final time represents the full forward score for a unique symbol sequence (including sequences terminating in blanks)
- Select the symbol sequence with the largest alpha
  - Sequences may have two alphas, one for the sequence itself, one for the sequence followed by a blank
  - Add the alphas before selecting the most likely

# CTC decoding



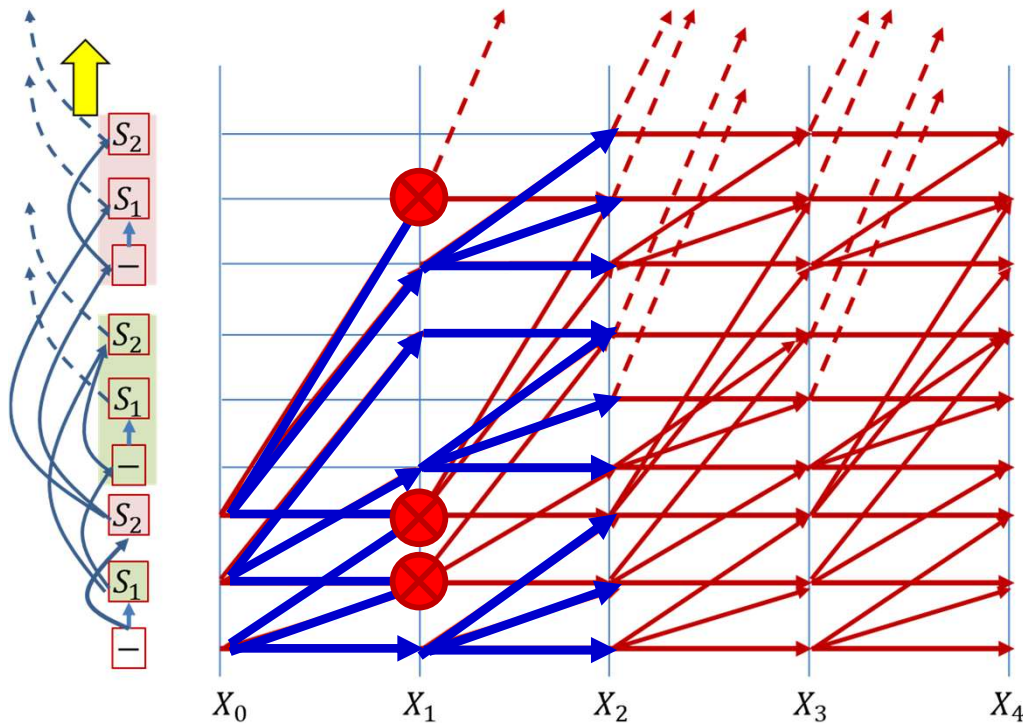
- This is the “theoretically correct” CTC decoder
- In practice, the graph gets exponentially large very quickly
- To prevent this pruning strategies are employed to keep the graph (and computation) manageable
  - This may cause suboptimal decodes, however
  - The fact that CTC scores peak at symbol terminations minimizes the damage due to pruning

# CTC decoding



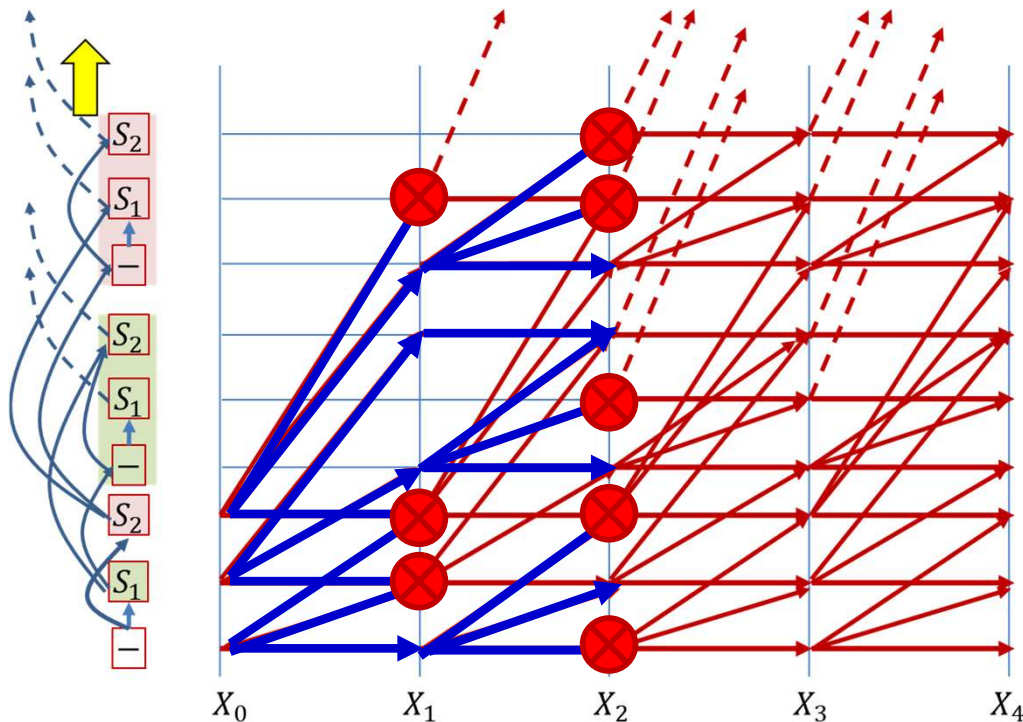
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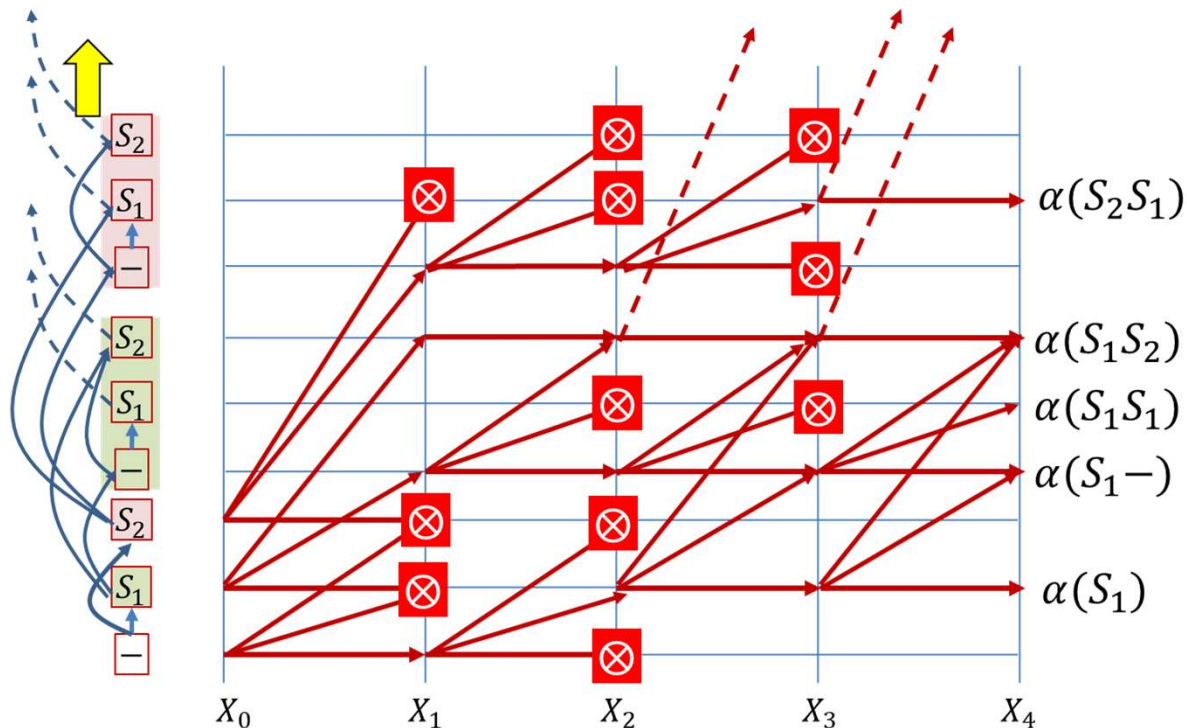
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# CTC decoding



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  - The fact that CTC scores peak at symbol terminations minimizes the damage due to pruning

# Beamsearch Pseudocode Notes

- Retaining separate lists of paths and pathscores for paths terminating in blanks, and those terminating in valid symbols
  - Since blanks are special
  - Do not explicitly represent blanks in the partial decode strings
- Pseudocode takes liberties (particularly w.r.t null strings)
  - I.e. you must be careful if you convert this to code
- Key
  - **PathScore** : array of scores for paths ending with symbols
  - **BlankPathScore** : array of scores for paths ending with blanks
  - **SymbolSet** : A list of symbols *not* including the blank

## BEAM SEARCH

```
Global PathScore = [], BlankPathScore = []

# First time instant: Initialize paths with each of the symbols,
# including blank, using score at time t=1
NewPathsWithTerminalBlank, NewPathsWithTerminalSymbol, NewBlankPathScore, NewPathScore =
    InitializePaths(SymbolSet, y[:,0])

# Subsequent time steps
for t = 1:T
    # Prune the collection down to the BeamWidth
    PathsWithTerminalBlank, PathsWithTerminalSymbol, BlankPathScore, PathScore =
        Prune(NewPathsWithTerminalBlank, NewPathsWithTerminalSymbol,
              NewBlankPathScore, NewPathScore, BeamWidth)

    # First extend paths by a blank
    NewPathsWithTerminalBlank, NewBlankPathScore = ExtendWithBlank(PathsWithTerminalBlank,
                                                                    PathsWithTerminalSymbol, y[:,t])

    # Next extend paths by a symbol
    NewPathsWithTerminalSymbol, NewPathScore = ExtendWithSymbol(PathsWithTerminalBlank,
                                                                PathsWithTerminalSymbol, SymbolSet, y[:,t])

end

# Merge identical paths differing only by the final blank
MergedPaths, FinalPathScore = MergeIdenticalPaths(NewPathsWithTerminalBlank, NewBlankPathScore
                                                  NewPathsWithTerminalSymbol, NewPathScore)

# Pick best path
BestPath = argmax(FinalPathScore) # Find the path with the best score
```

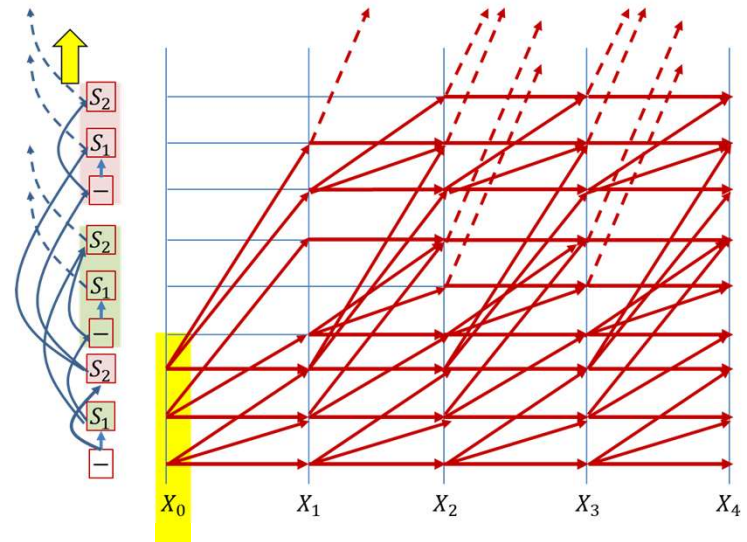
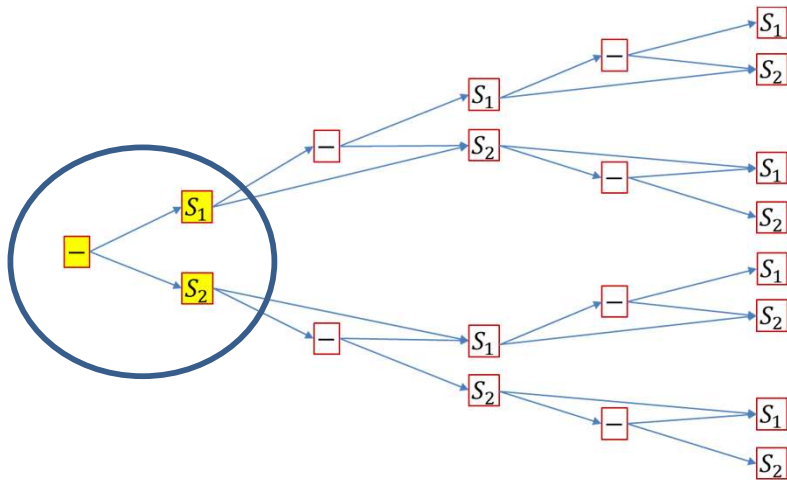


# BEAM SEARCH

```
Global PathScore = [], BlankPathScore = []
```

```
# First time instant: Initialize paths with each of the symbols,  
# including blank, using score at time t=1
```

```
NewPathsWithTerminalBlank, NewPathsWithTerminalSymbol, NewBlankPathScore, NewPathScore =  
InitializePaths(SymbolSet, y[:,0])
```



# BEAM SEARCH

```
Global PathScore = [], BlankPathScore = []
```

```
# First time instant: Initialize paths with each of the symbols,  
# including blank, using score at time t=1
```

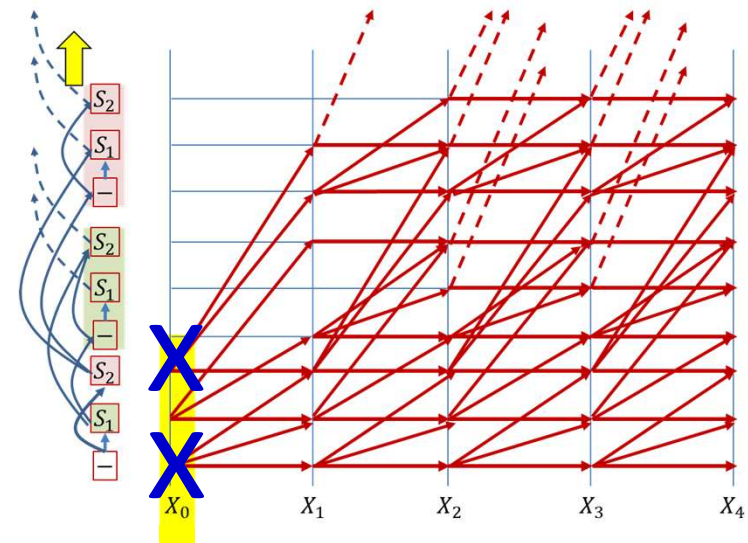
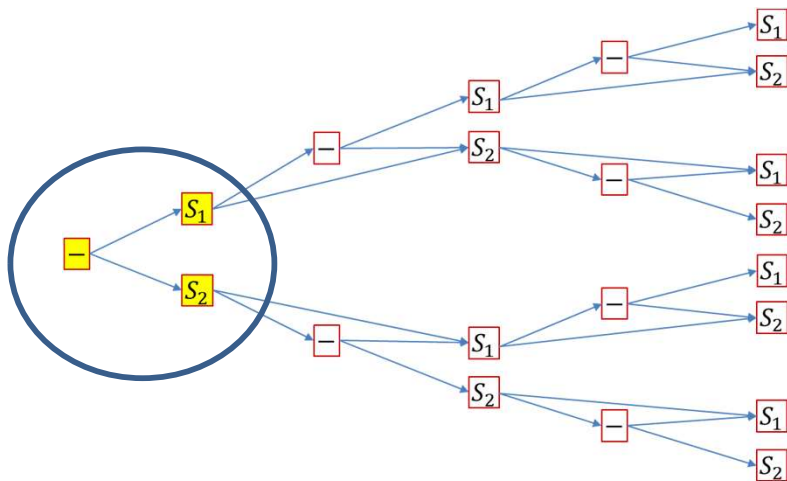
```
NewPathsWithTerminalBlank, NewPathsWithTerminalSymbol, NewBlankPathScore, NewPathScore =  
    InitializePaths(SymbolSet, y[:,0])
```

```
# Subsequent time steps
```

```
for t = 1:T
```

```
    # Prune the collection down to the BeamWidth
```

```
    PathsWithTerminalBlank, PathsWithTerminalSymbol, BlankPathScore, PathScore =  
        Prune(NewPathsWithTerminalBlank, NewPathsWithTerminalSymbol,  
              NewBlankPathScore, NewPathScore, BeamWidth)
```



# BEAM SEARCH

```
Global PathScore = [], BlankPathScore = []
```

```
# First time instant: Initialize paths with each of the symbols,  
# including blank, using score at time t=1
```

```
NewPathsWithTerminalBlank, NewPathsWithTerminalSymbol, NewBlankPathScore, NewPathScore =  
    InitializePaths(SymbolSet, y[:,0])
```

```
# Subsequent time steps
```

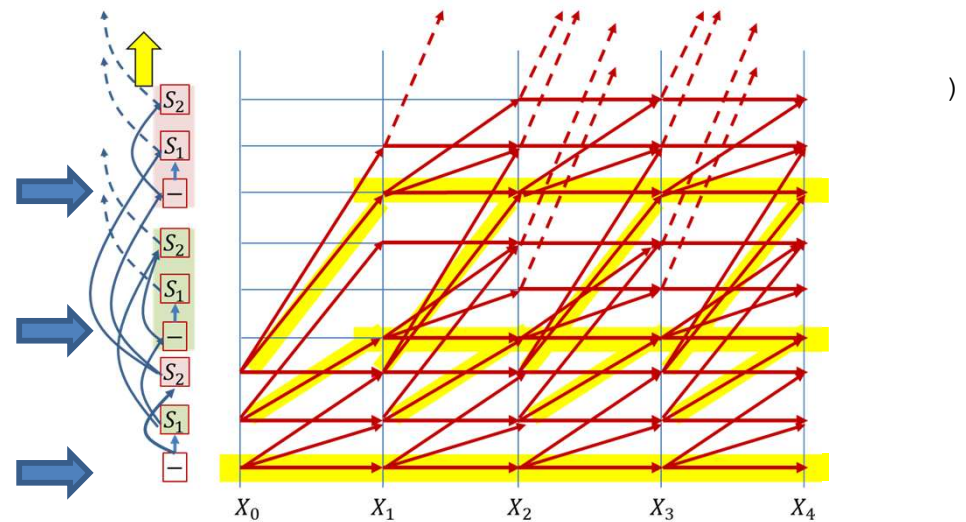
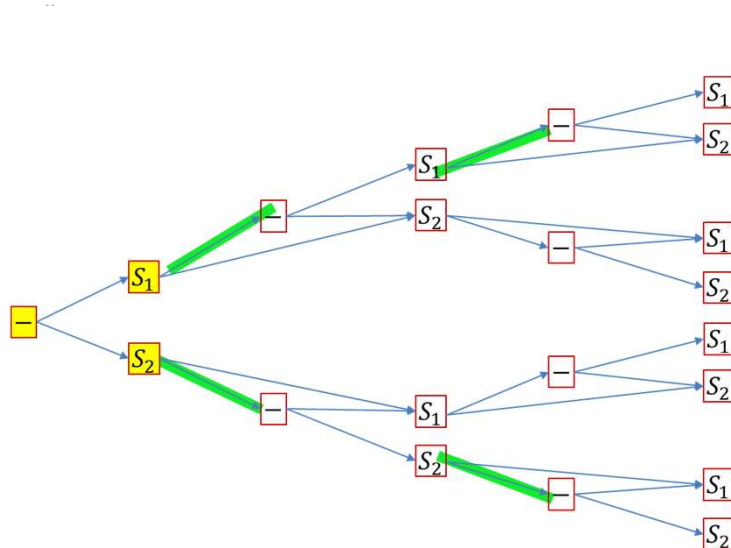
```
for t = 1:T
```

```
    # Prune the collection down to the BeamWidth
```

```
    PathsWithTerminalBlank, PathsWithTerminalSymbol, BlankPathScore, PathScore =  
        Prune(NewPathsWithTerminalBlank, NewPathsWithTerminalSymbol,  
            NewBlankPathScore, NewPathScore, BeamWidth)
```

```
    # First extend paths by a blank
```

```
    NewPathsWithTerminalBlank, NewBlankPathScore = ExtendWithBlank(PathsWithTerminalBlank,  
        PathsWithTerminalSymbol, y[:,t])
```



# BEAM SEARCH

```
Global PathScore = [], BlankPathScore = []
```

```
# First time instant: Initialize paths with each of the symbols,  
# including blank, using score at time t=1
```

```
NewPathsWithTerminalBlank, NewPathsWithTerminalSymbol, NewBlankPathScore, NewPathScore =  
    InitializePaths(SymbolSet, y[:,0])
```

```
# Subsequent time steps
```

```
for t = 1:T
```

```
    # Prune the collection down to the BeamWidth
```

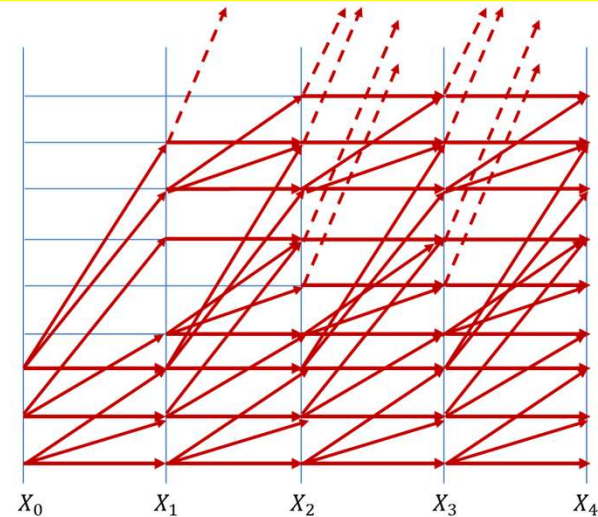
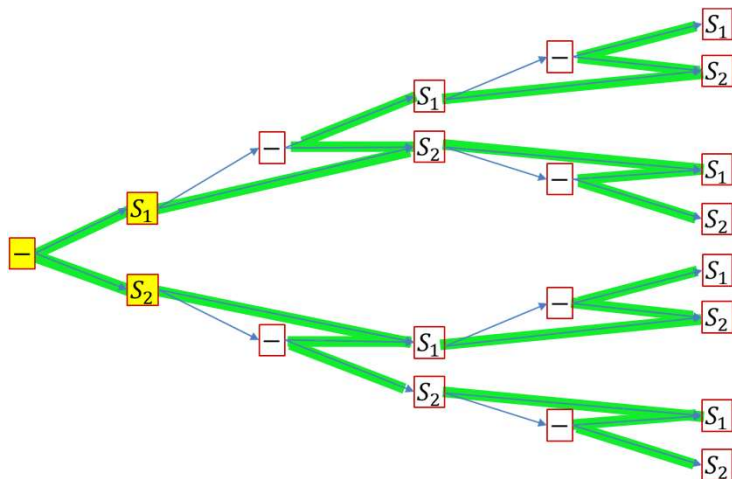
```
    PathsWithTerminalBlank, PathsWithTerminalSymbol, BlankPathScore, PathScore =  
        Prune(NewPathsWithTerminalBlank, NewPathsWithTerminalSymbol,  
            NewBlankPathScore, NewPathScore, BeamWidth)
```

```
    # First extend paths by a blank
```

```
    NewPathsWithTerminalBlank, NewBlankPathScore = ExtendWithBlank(PathsWithTerminalBlank,  
        PathsWithTerminalSymbol, y[:,t])
```

```
    # Next extend paths by a symbol
```

```
    NewPathsWithTerminalSymbol, NewPathScore = ExtendWithSymbol(PathsWithTerminalBlank,  
        PathsWithTerminalSymbol, SymbolSet, y[:,t])
```



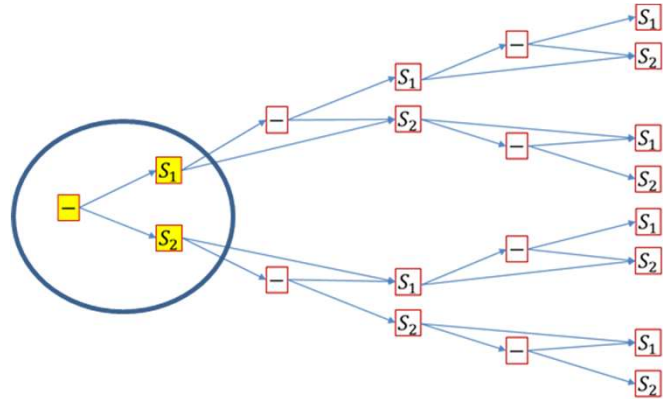
## BEAM SEARCH InitializePaths: **FIRST TIME INSTANT**

```
function InitializePaths(SymbolSet, y)

InitialBlankPathScore = [], InitialPathScore = []
# First push the blank into a path-ending-with-blank stack. No symbol has been invoked yet
path = null
InitialBlankPathScore[path] = y[blank] # Score of blank at t=1
InitialPathsWithFinalBlank = {path}

# Push rest of the symbols into a path-ending-with-symbol stack
InitialPathsWithFinalSymbol = {}
for c in SymbolSet # This is the entire symbol set, without the blank
    path = c
    InitialPathScore[path] = y[c] # Score of symbol c at t=1
    InitialPathsWithFinalSymbol += path # Set addition
end

return InitialPathsWithFinalBlank, InitialPathsWithFinalSymbol,
       InitialBlankPathScore, InitialPathScore
```



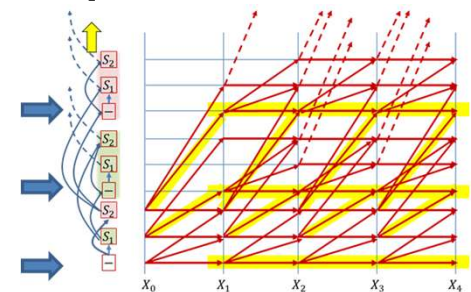
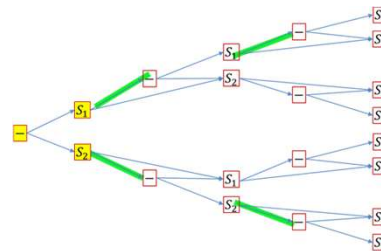
# BEAM SEARCH: Extending with blanks

Global PathScore, BlankPathScore

```
function ExtendWithBlank(PathsWithTerminalBlank, PathsWithTerminalSymbol, y)
  UpdatedPathsWithTerminalBlank = {}
  UpdatedBlankPathScore = []
  # First work on paths with terminal blanks
  #(This represents transitions along horizontal trellis edges for blanks)
  for path in PathsWithTerminalBlank:
    # Repeating a blank doesn't change the symbol sequence
    UpdatedPathsWithTerminalBlank += path # Set addition
    UpdatedBlankPathScore[path] = BlankPathScore[path]*y[blank]
  end

  # Then extend paths with terminal symbols by blanks
  for path in PathsWithTerminalSymbol:
    # If there is already an equivalent string in UpdatesPathsWithTerminalBlank
    # simply add the score. If not create a new entry
    if path in UpdatedPathsWithTerminalBlank
      UpdatedBlankPathScore[path] += Pathscore[path]* y[blank]
    else
      UpdatedPathsWithTerminalBlank += path # Set addition
      UpdatedBlankPathScore[path] = PathScore[path] * y[blank]
    end
  end
end

return UpdatedPathsWithTerminalBlank,
       UpdatedBlankPathScore
```



# BEAM SEARCH: Extending with symbols

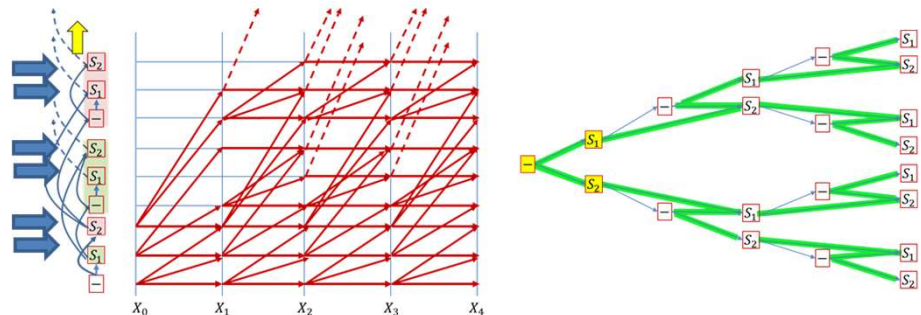
Global PathScore, BlankPathScore

```
function ExtendWithSymbol(PathsWithTerminalBlank, PathsWithTerminalSymbol, SymbolSet, y)
    UpdatedPathsWithTerminalSymbol = {}
    UpdatedPathScore = []

    # First extend the paths terminating in blanks. This will always create a new sequence
    for path in PathsWithTerminalBlank:
        for c in SymbolSet: # SymbolSet does not include blanks
            newpath = path + c # Concatenation
            UpdatedPathsWithTerminalSymbol += newpath # Set addition
            UpdatedPathScore[newpath] = BlankPathScore[path] * y(c)
        end
    end

    # Next work on paths with terminal symbols
    for path in PathsWithTerminalSymbol:
        # Extend the path with every symbol other than blank
        for c in SymbolSet: # SymbolSet does not include blanks
            newpath = (c == path[end]) ? path : path + c # Horizontal transitions don't extend the sequence
            if newpath in UpdatedPathsWithTerminalSymbol: # Already in list, merge paths
                UpdatedPathScore[newpath] += PathScore[path] * y[c]
            else # Create new path
                UpdatedPathsWithTerminalSymbol += newpath # Set addition
                UpdatedPathScore[newpath] = PathScore[path] * y[c]
            end
        end
    end
end
```

```
return UpdatedPathsWithTerminalSymbol,
       UpdatedPathScore
```



## BEAM SEARCH: Pruning low-scoring entries

Global PathScore, BlankPathScore

```
function Prune(PathsWithTerminalBlank, PathsWithTerminalSymbol, BlankPathScore, PathScore, BeamWidth)
    PrunedBlankPathScore = []
    PrunedPathScore = []
    # First gather all the relevant scores
    i = 1
    for p in PathsWithTerminalBlank
        scorelist[i] = BlankPathScore[p]
        i++
    end
    for p in PathsWithTerminalSymbol
        scorelist[i] = PathScore[p]
        i++
    end

    # Sort and find cutoff score that retains exactly BeamWidth paths
    sort(scorelist) # In decreasing order
    cutoff = BeamWidth < length(scorelist) ? scorelist[BeamWidth] : scorelist[end]

    PrunedPathsWithTerminalBlank = {}
    for p in PathsWithTerminalBlank
        if BlankPathScore[p] >= cutoff
            PrunedPathsWithTerminalBlank += p # Set addition
            PrunedBlankPathScore[p] = BlankPathScore[p]
        end
    end

    PrunedPathsWithTerminalSymbol = {}
    for p in PathsWithTerminalSymbol
        if PathScore[p] >= cutoff
            PrunedPathsWithTerminalSymbol += p # Set addition
            PrunedPathScore[p] = PathScore[p]
        end
    end

    return PrunedPathsWithTerminalBlank, PrunedPathsWithTerminalSymbol, PrunedBlankPathScore, PrunedPathScore
end
```



## BEAM SEARCH: Merging final paths

# Note : not using global variable here

```
function MergeIdenticalPaths (PathsWithTerminalBlank, BlankPathScore,  
                             PathsWithTerminalSymbol, PathScore)
```

```
  # All paths with terminal symbols will remain
```

```
  MergedPaths = PathsWithTerminalSymbol
```

```
  FinalPathScore = PathScore
```

```
  # Paths with terminal blanks will contribute scores to existing identical paths from
```

```
  # PathsWithTerminalSymbol if present, or be included in the final set, otherwise
```

```
  for p in PathsWithTerminalBlank
```

```
    if p in MergedPaths
```

```
      FinalPathScore[p] += BlankPathScore[p]
```

```
    else
```

```
      MergedPaths += p # Set addition
```

```
      FinalPathScore[p] = BlankPathScore[p]
```

```
    end
```

```
  end
```

```
  return MergedPaths, FinalPathScore
```

# Story so far: CTC models

- Sequence-to-sequence networks which irregularly produce output symbols can be trained by
  - Iteratively aligning the target output to the input and time-synchronous training
  - Optimizing the expected error over *all* possible alignments: CTC training
- Distinct repetition of symbols can be disambiguated from repetitions representing the extended output of a single symbol by the introduction of blanks
- Decoding the models can be performed by
  - Best-path decoding, i.e. greedy decoding
  - Optimal CTC decoding based on the application of the forward algorithm to a tree-structured representation of all possible output strings

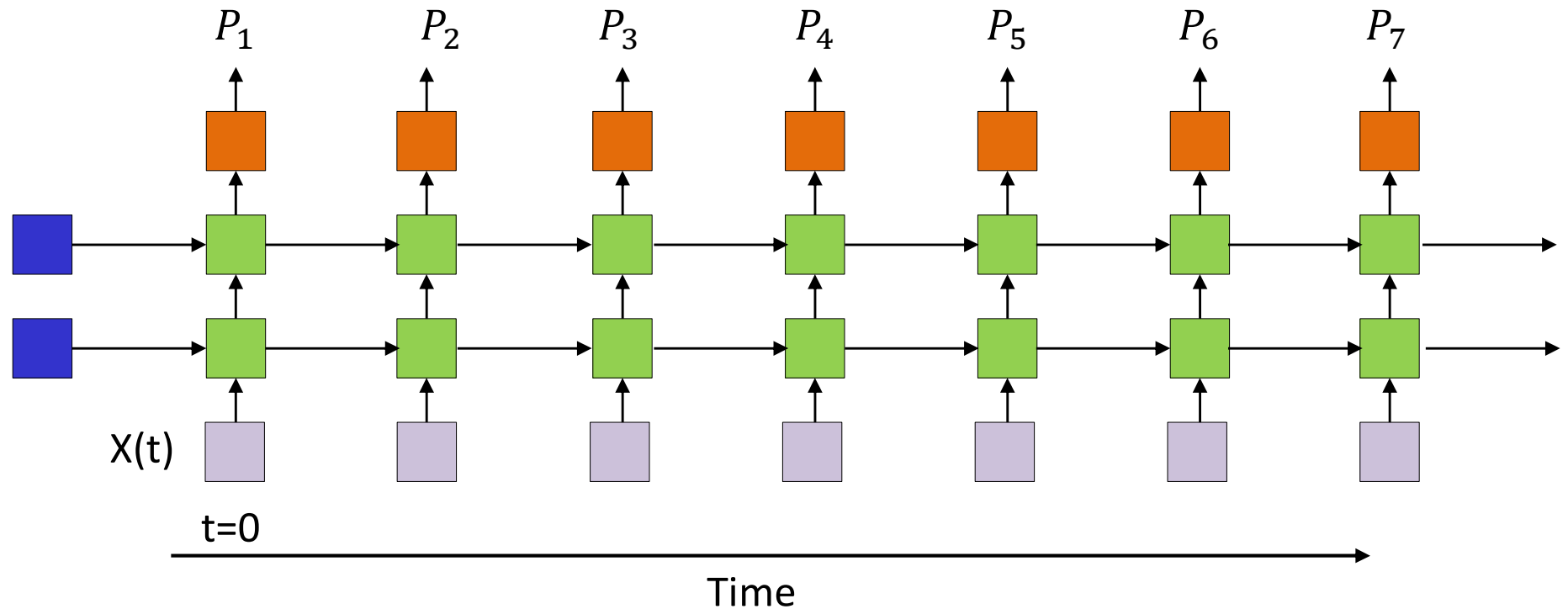
# CTC caveats

- The “blank” structure (with concurrent modifications to the forward-backward equations) is only one way to deal with the problem of repeating symbols
- Possible variants:
  - Symbols partitioned into two or more sequential subunits
    - No blanks are required, since subunits must be visited in order
  - Symbol-specific blanks
    - Doubles the “vocabulary”
  - CTC can use *bidirectional* recurrent nets
    - And frequently does
  - Other variants possible..

# Most common CTC applications

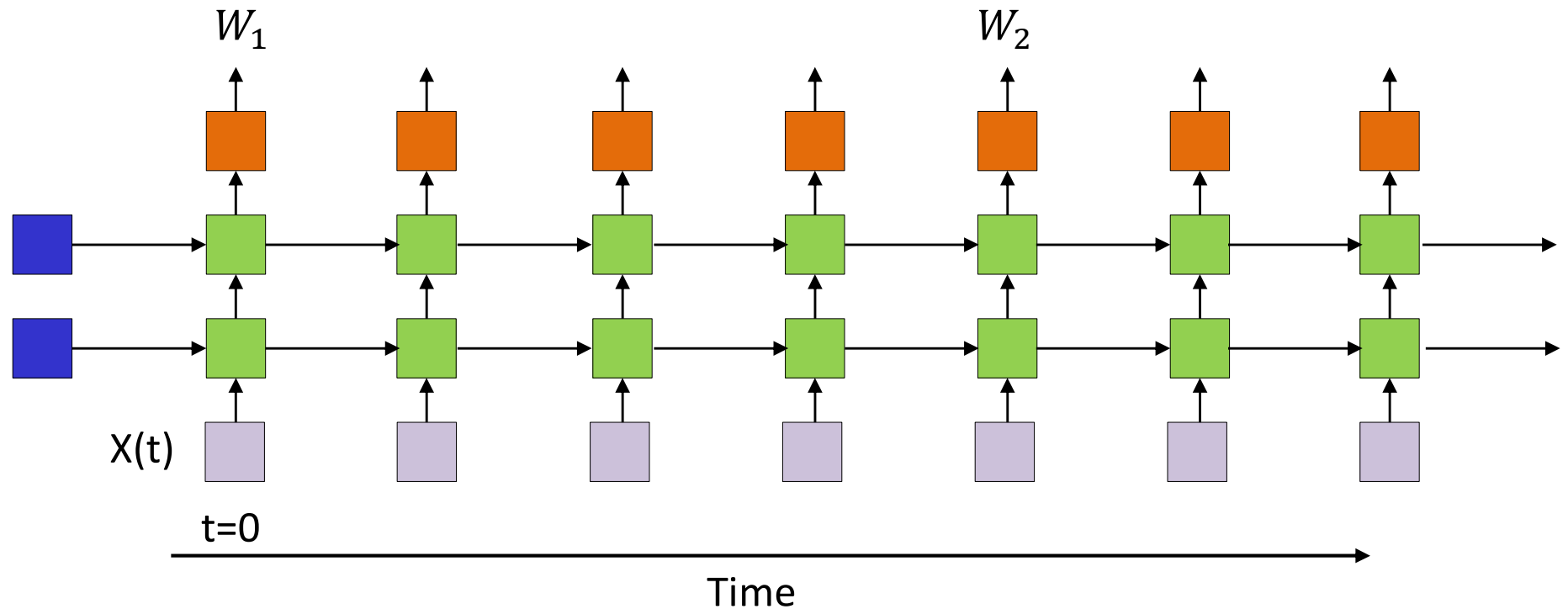
- Speech recognition
  - Speech in, phoneme sequence out
  - Speech in, character sequence (spelling out)
- Handwriting recognition

# Speech recognition using Recurrent Nets



- Recurrent neural networks (with LSTMs) can be used to perform speech recognition
  - Input: Sequences of audio feature vectors
  - Output: Phonetic label of each vector

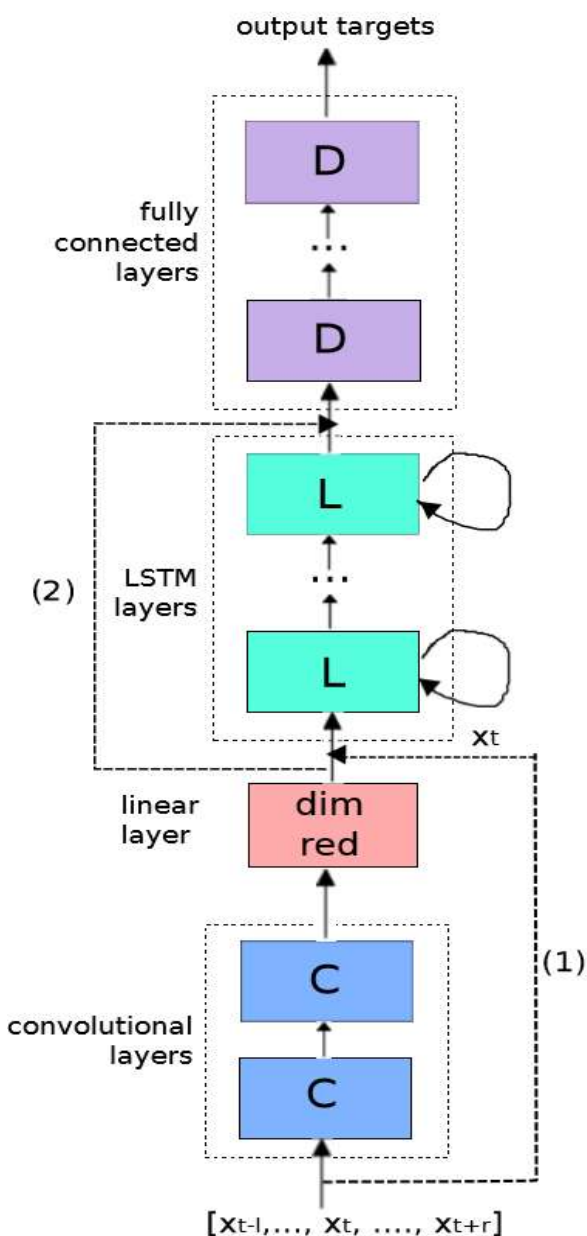
# Speech recognition using Recurrent Nets



- Alternative: Directly output phoneme, character or word sequence

# Next up: Attention models

# CNN-LSTM-DNN for speech recognition



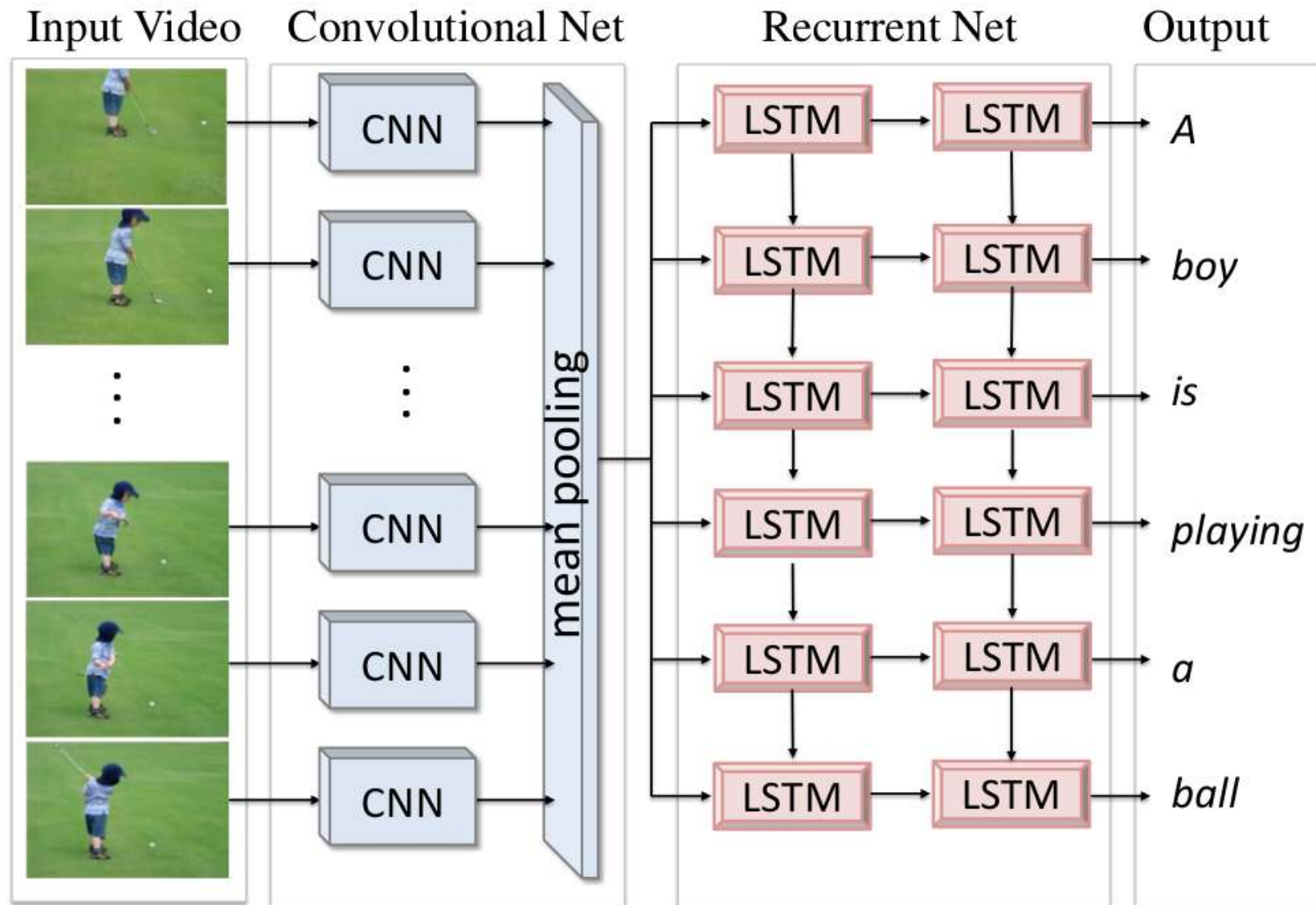
## Ensembles of RNN/LSTM, DNN, & Conv Nets (CNN) :

T. Sainath, O. Vinyals, A. Senior, H. Sak.  
“Convolutional, Long Short-Term Memory, Fully Connected Deep Neural Networks,”  
ICASSP 2015.

Fig. 1. CLDNN Architecture



# Translating Videos to Natural Language Using Deep Recurrent Neural Networks



Translating Videos to Natural Language Using Deep Recurrent Neural Networks

Subhashini Venugopalan, Huijun Xu, Jeff Donahue, Marcus Rohrbach, Raymond Mooney, Kate Saenko<sup>177</sup>  
North American Chapter of the Association for Computational Linguistics, Denver, Colorado, June 2015.



"man in black shirt is playing guitar."



"construction worker in orange safety vest is working on road."



"two young girls are playing with lego toy."



"boy is doing backflip on wakeboard."



"a young boy is holding a baseball bat."



"a cat is sitting on a couch with a remote control."



"a woman holding a teddy bear in front of a mirror."



"a horse is standing in the middle of a road."

# Not explained

- Can be combined with CNNs
  - Lower-layer CNNs to extract features for RNN
- Can be used in tracking
  - Incremental prediction